Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

5-Inverse-trig-functions/5.5-Inverse-secant/156-5.5.1-u-a+b-arcsec-c-x-^n

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CHAPTER 1

Introduction

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [174]. This is test number [156].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (174)	0.00 (0)
Mathematica	100.00 (174)	0.00 (0)
Maple	79.89 (139)	20.11 (35)
Fricas	71.26 (124)	28.74 (50)
Giac	55.75 (97)	44.25 (77)
Sympy	40.23 (70)	59.77 (104)
Maxima	35.63 (62)	64.37 (112)
Mupad	30.46 (53)	69.54 (121)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

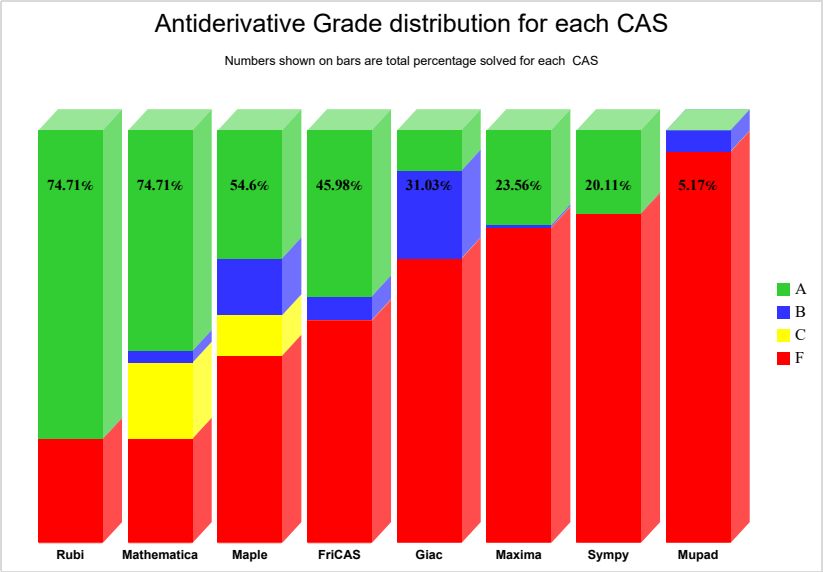
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

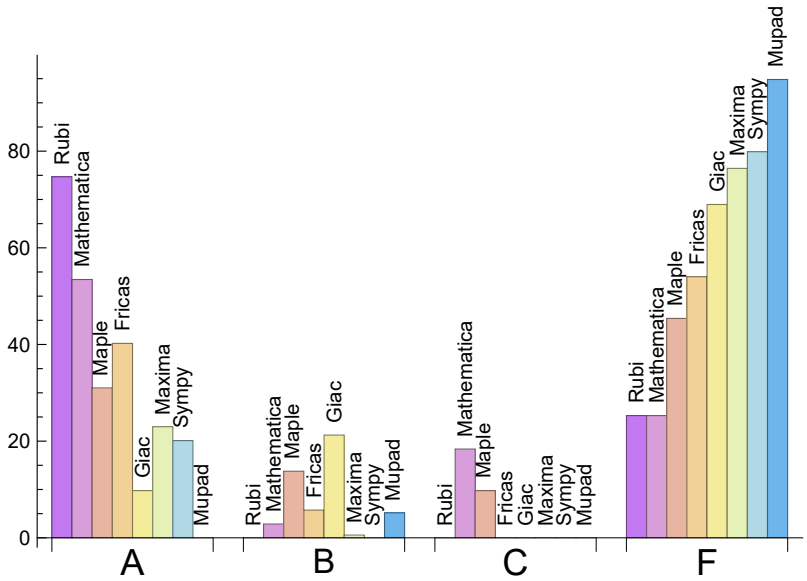
System	% A grade	% B grade	% C grade	% F grade
Rubi	74.713	0.000	0.000	25.287
Mathematica	53.448	2.874	18.391	25.287
Fricas	40.230	5.747	0.000	54.023
Maple	31.034	13.793	9.770	45.402
Maxima	22.989	0.575	0.000	76.437
Sympy	20.115	0.000	0.000	79.885
Giac	9.770	21.264	0.000	68.966
Mupad	0.000	5.172	0.000	94.828

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima

and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	35	100.00	0.00	0.00
Fricas	50	98.00	2.00	0.00
Giac	77	61.04	5.19	33.77
Maxima	112	46.43	0.00	53.57
Sympy	104	72.12	27.88	0.00
Mupad	121	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.33
Rubi	0.65
Mupad	1.11
Maxima	1.95
Mathematica	3.69
Giac	4.09
Maple	5.13
Sympy	20.67

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	31.77	1.25	27.00	1.17
Sympy	123.46	1.35	33.00	1.18
Rubi	201.26	1.00	129.00	1.00
Mathematica	249.16	1.13	124.00	1.09
Maxima	256.45	12.72	140.50	1.42
Fricas	256.83	1.90	93.50	1.23
Maple	294.71	1.46	138.00	1.14
Giac	1786.87	12.64	65.00	1.20

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

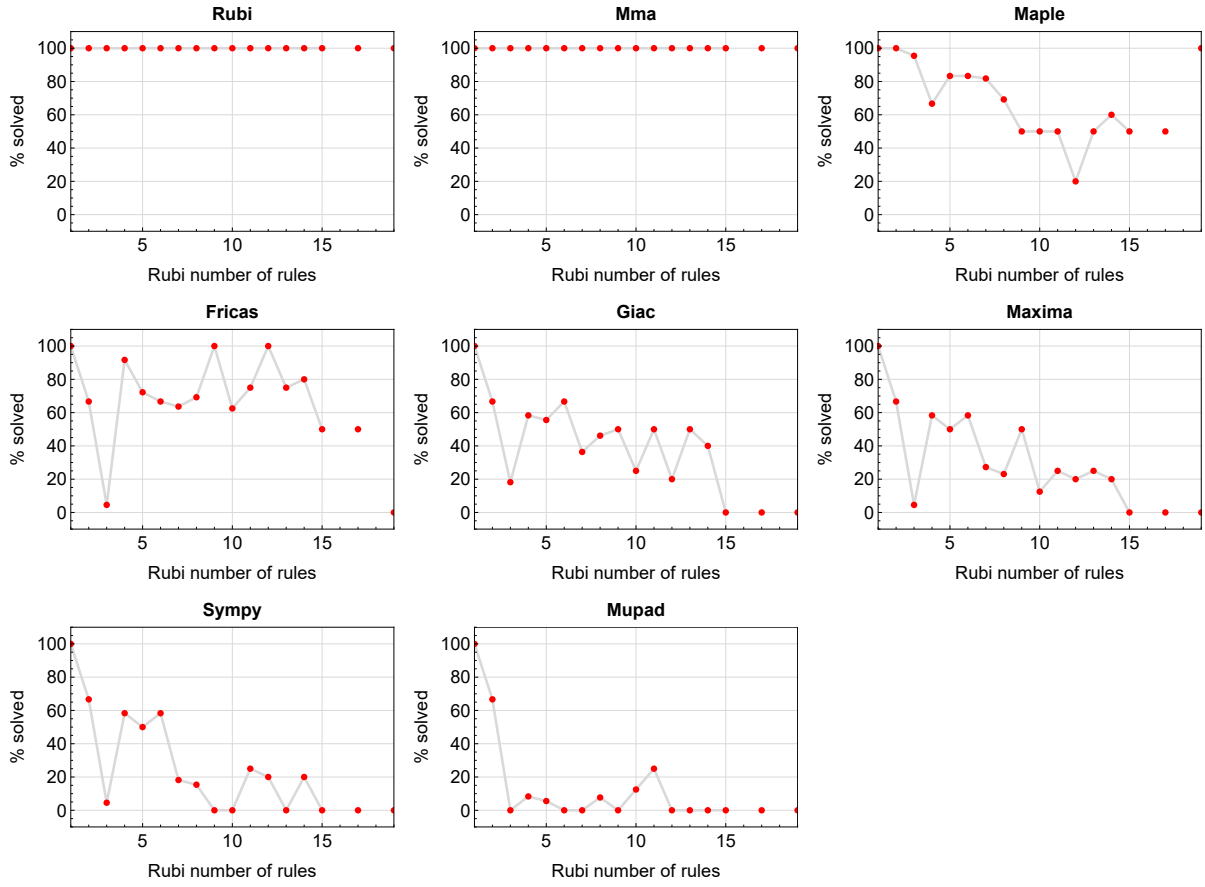


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

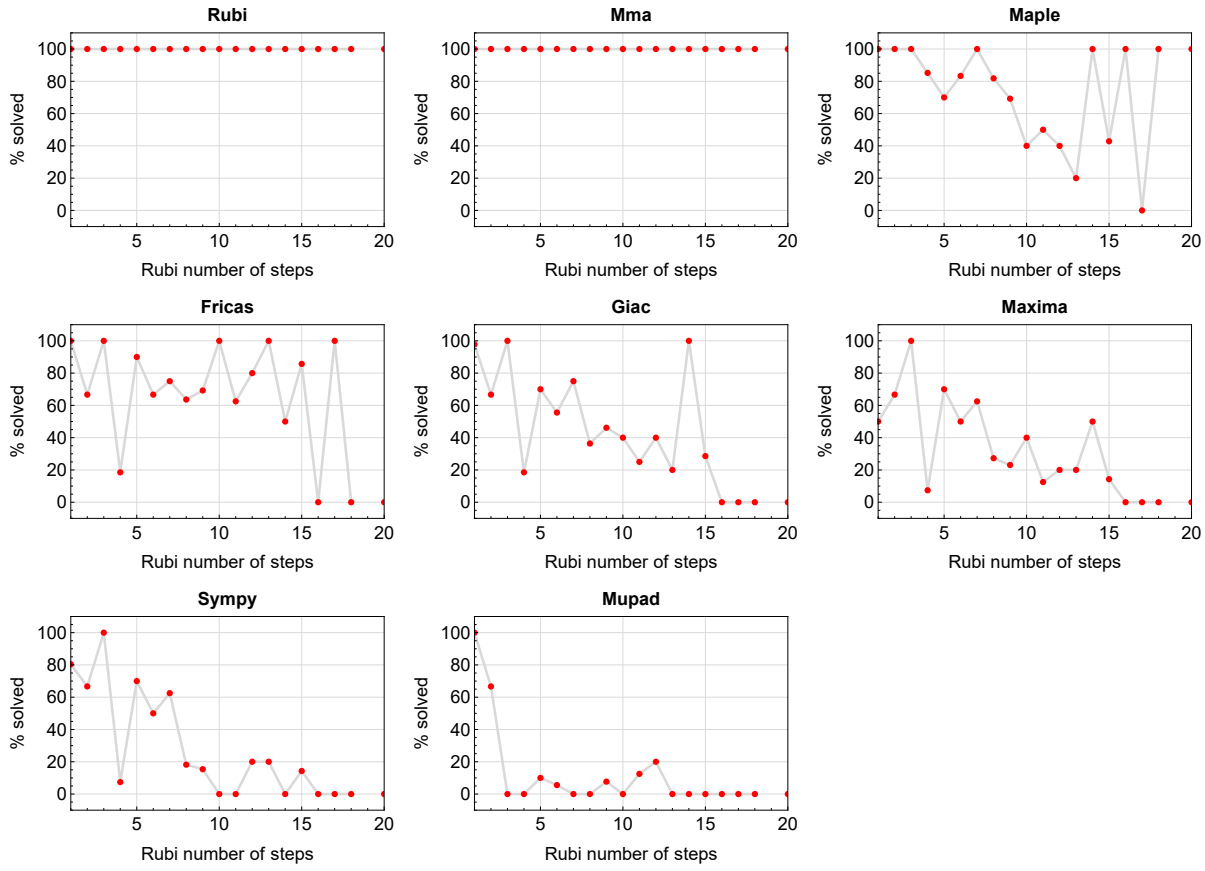


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

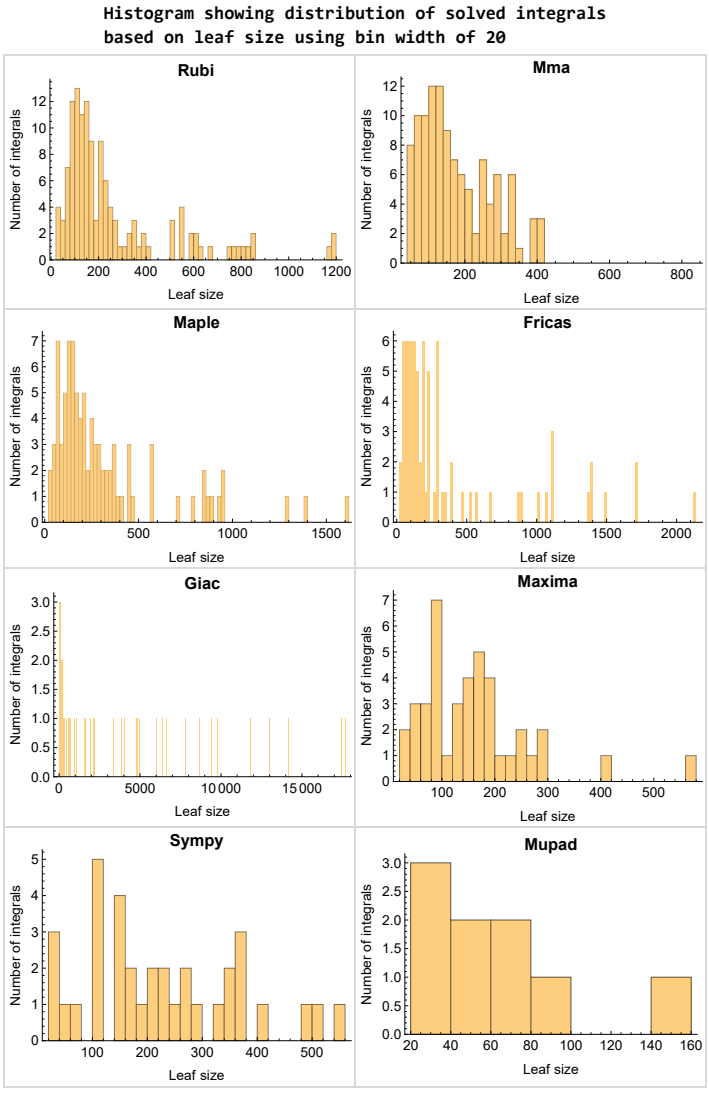


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

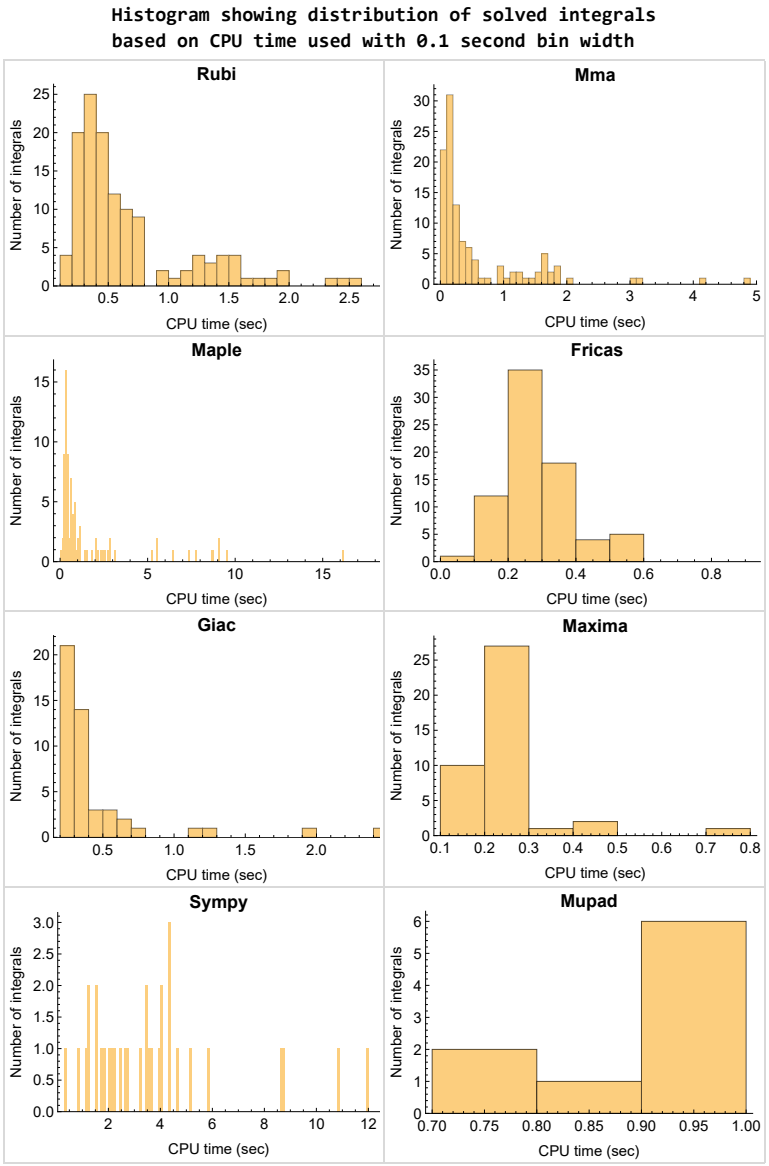


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

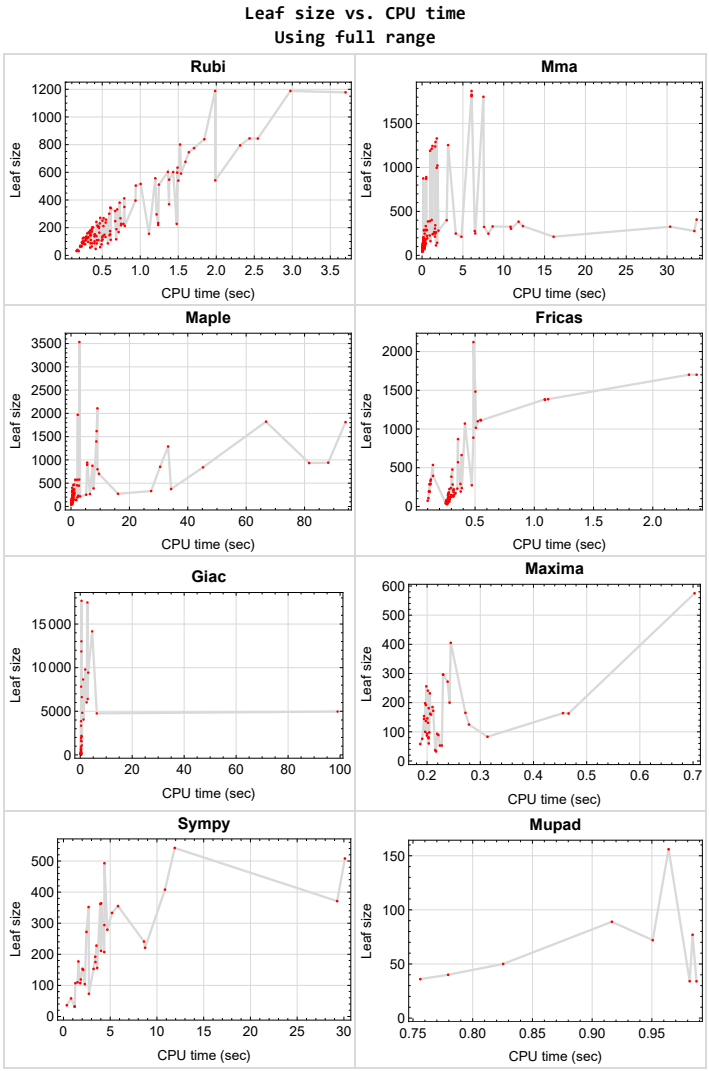


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{33, 34, 35, 39, 40, 41, 45, 46, 47, 51, 52, 54, 55, 114, 115, 116, 117, 118, 123, 124, 125, 126, 127, 128, 134, 135, 136, 137, 144, 145, 146, 147, 154, 155, 156, 157, 164, 165, 166, 167, 168, 169, 173, 174}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {171}

Mathematica {91, 96, 97, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110}

Maple {91, 92, 94, 96, 97, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

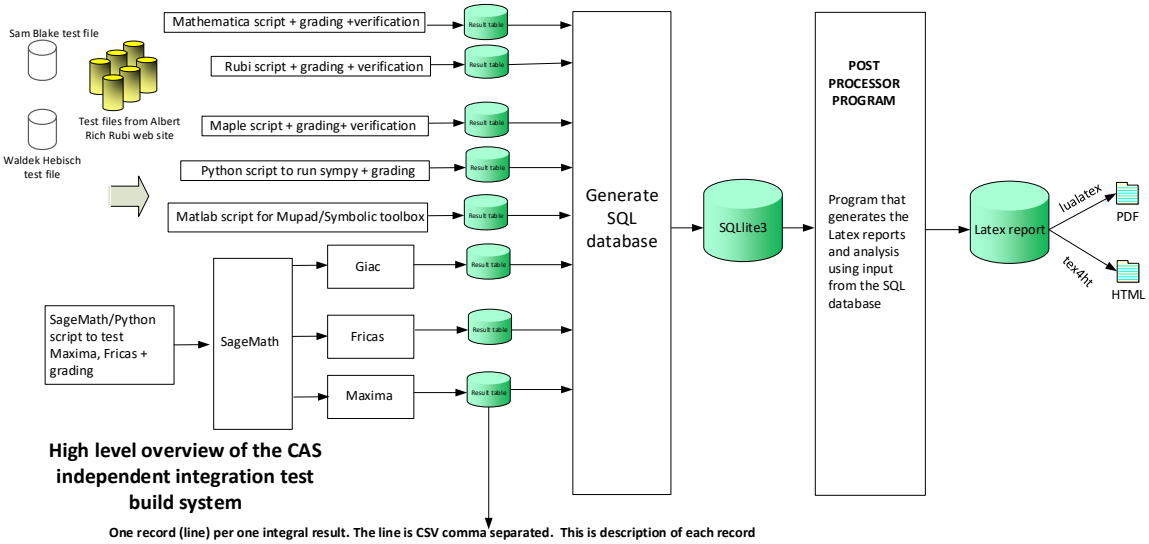
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

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2.1 List of integrals sorted by grade for each CAS

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2.1.5 Maxima 23
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2.1.8 Sympy 24

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 42, 43, 44, 48, 49, 50, 53, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 119, 120, 121, 122, 129, 130, 131, 132, 133, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 158, 159, 160, 161, 162, 163, 170, 171, 172 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 42, 43, 44, 48, 49, 50, 53, 56, 57, 58, 59, 60, 61, 62, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 100, 101, 102, 103, 108, 109, 110, 138, 148, 158, 161, 162, 163, 170, 171, 172 }

B grade { 96, 97, 99, 104, 107 }

C grade { 63, 64, 65, 67, 68, 98, 105, 106, 111, 112, 113, 119, 120, 121, 122, 129, 130, 131, 132, 133, 139, 140, 141, 142, 143, 149, 150, 151, 152, 153, 159, 160 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 14, 15, 16, 18, 19, 22, 24, 26, 36, 37, 38, 42, 43, 44, 48, 49, 50, 58, 59, 60, 61, 64, 65, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 83, 84, 85, 86, 87, 89, 90 }

B grade { 10, 12, 17, 20, 21, 23, 25, 28, 29, 30, 31, 32, 56, 57, 62, 63, 67, 68, 81, 82, 88, 98, 105, 106 }

C grade { 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110 }

F normal fail { 27, 53, 111, 112, 113, 119, 120, 121, 122, 129, 130, 131, 132, 133, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 158, 159, 160, 161, 162, 163, 170, 171, 172 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 20, 21, 22, 23, 29, 30, 31, 32, 56, 57, 58, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88, 98, 111, 112, 113, 119, 120, 121, 122, 129, 130, 131, 132, 133, 138, 139, 140, 141, 142, 143, 148, 149, 150, 158, 159, 160, 170, 171, 172 }

B grade { 7, 17, 59, 61, 62, 105, 106, 151, 152, 153 }

C grade { }

F normal fail { 8, 16, 18, 19, 24, 25, 26, 27, 28, 36, 37, 38, 42, 43, 44, 48, 49, 50, 53, 60, 64, 65, 66, 67, 68, 79, 80, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110, 161, 162, 163 }

F(-1) timedout fail { 63 }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 17, 20, 22, 29, 56, 57, 58, 59, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88 }

B grade { 31 }

C grade { }

F normal fail { 8, 16, 18, 19, 21, 23, 24, 25, 26, 27, 28, 30, 32, 36, 37, 38, 42, 43, 44, 48, 49, 50, 53, 60, 61, 62, 79, 80, 89, 90, 92, 94, 96, 97, 98, 99, 104, 105, 106, 107, 113, 122, 133, 143, 158, 159, 161, 162, 163, 170, 171, 172 }

F(-1) timedout fail { }

F(-2) exception fail { 63, 64, 65, 66, 67, 68, 91, 93, 95, 100, 101, 102, 103, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 160 }

2.1.6 Giac

A grade { 9, 10, 11, 12, 13, 14, 21, 22, 23, 36, 37, 38, 73, 74, 75, 85, 86 }

B grade { 1, 2, 3, 4, 5, 6, 7, 15, 17, 20, 29, 30, 31, 32, 42, 43, 44, 48, 49, 50, 56, 57, 58, 59, 69, 70, 71, 72, 76, 77, 78, 81, 82, 83, 84, 87, 88 }

C grade { }

F normal fail { 16, 18, 24, 25, 26, 27, 28, 53, 63, 64, 65, 66, 67, 68, 80, 90, 111, 112, 113, 119, 120, 121, 122, 129, 130, 131, 132, 133, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 158, 159, 160, 161, 162, 163, 172 }

F(-1) timedout fail { 96, 97, 100, 104 }

F(-2) exception fail { 8, 19, 45, 60, 61, 62, 79, 89, 91, 92, 93, 94, 95, 98, 99, 101, 102, 103, 105, 106, 107, 108, 109, 110, 170, 171 }

2.1.7 Mupad

A grade { }

B grade { 6, 7, 9, 10, 20, 29, 58, 59, 72 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 8, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 36, 37, 38, 42, 43, 44, 48, 49, 50, 53, 56, 57, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 119, 120, 121, 122, 129, 130, 131, 132, 133, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 158, 159, 160, 161, 162, 163, 170, 171, 172 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 56, 57, 58, 59, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88 }

B grade { }

C grade { }

F normal fail { 8, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 42, 43, 44, 48, 49, 50, 53, 60, 61, 62, 63, 64, 65, 66, 67, 79, 80, 89, 90, 91, 92, 93, 94, 95, 96, 98, 100, 101, 102, 103, 111, 112, 113, 119, 120, 122, 131, 132, 133, 138, 139, 140, 141, 142, 143, 148, 149, 151, 152, 153, 162, 163, 172 }

F(-1) timedout fail { 68, 97, 99, 104, 105, 106, 107, 108, 109, 110, 121, 125, 126, 129, 130, 145, 150, 154, 155, 156, 157, 158, 159, 160, 161, 165, 166, 170, 171 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	122	107	177	162	116	221	8644	0
N.S.	1	1.07	0.94	1.55	1.42	1.02	1.94	75.82	0.00
time (sec)	N/A	0.263	0.107	0.330	0.206	0.283	8.733	1.120	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	97	72	79	81	63	153	3862	0
N.S.	1	1.09	0.81	0.89	0.91	0.71	1.72	43.39	0.00
time (sec)	N/A	0.257	0.073	0.239	0.203	0.283	2.034	0.321	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	93	97	141	131	107	175	4828	0
N.S.	1	1.04	1.09	1.58	1.47	1.20	1.97	54.25	0.00
time (sec)	N/A	0.248	0.058	0.260	0.202	0.287	3.422	0.791	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	68	62	70	60	53	107	1926	0
N.S.	1	1.06	0.97	1.09	0.94	0.83	1.67	30.09	0.00
time (sec)	N/A	0.224	0.086	0.298	0.203	0.269	1.275	0.315	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	85	94	98	94	107	2101	0
N.S.	1	1.00	1.33	1.47	1.53	1.47	1.67	32.83	0.00
time (sec)	N/A	0.224	0.055	0.242	0.204	0.286	1.796	0.590	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	50	61	37	40	58	634	40
N.S.	1	1.00	1.28	1.56	0.95	1.03	1.49	16.26	1.03
time (sec)	N/A	0.189	0.025	0.247	0.215	0.269	0.810	0.287	0.779

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	59	38	53	63	32	63	34
N.S.	1	1.00	1.84	1.19	1.66	1.97	1.00	1.97	1.06
time (sec)	N/A	0.173	0.041	0.112	0.228	0.280	1.205	0.277	0.987

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	87	59	84	0	0	0	0	0
N.S.	1	1.36	0.92	1.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.425	0.020	0.618	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	40	58	33	27	36	43	36
N.S.	1	1.00	1.29	1.87	1.06	0.87	1.16	1.39	1.16
time (sec)	N/A	0.202	0.028	0.193	0.217	0.263	0.361	0.298	0.756

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	64	66	95	83	39	119	58	50
N.S.	1	1.25	1.29	1.86	1.63	0.76	2.33	1.14	0.98
time (sec)	N/A	0.228	0.036	0.243	0.314	0.254	1.854	0.280	0.825

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	65	59	71	58	40	110	65	0
N.S.	1	1.08	0.98	1.18	0.97	0.67	1.83	1.08	0.00
time (sec)	N/A	0.245	0.044	0.250	0.187	0.253	1.509	0.283	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	96	78	138	125	52	192	83	0
N.S.	1	1.26	1.03	1.82	1.64	0.68	2.53	1.09	0.00
time (sec)	N/A	0.249	0.056	0.240	0.279	0.258	3.407	0.284	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	86	69	79	76	51	156	87	0
N.S.	1	1.05	0.84	0.96	0.93	0.62	1.90	1.06	0.00
time (sec)	N/A	0.258	0.057	0.244	0.191	0.269	3.601	0.289	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	128	88	174	165	62	241	104	0
N.S.	1	1.27	0.87	1.72	1.63	0.61	2.39	1.03	0.00
time (sec)	N/A	0.277	0.068	0.313	0.272	0.254	8.615	0.287	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	114	124	170	163	146	0	6625	0
N.S.	1	1.07	1.16	1.59	1.52	1.36	0.00	61.92	0.00
time (sec)	N/A	0.531	0.151	0.710	0.466	0.298	0.000	0.608	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	140	226	285	0	0	0	0	0
N.S.	1	0.95	1.54	1.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.539	0.943	1.174	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	64	90	123	87	111	0	2181	0
N.S.	1	1.14	1.61	2.20	1.55	1.98	0.00	38.95	0.00
time (sec)	N/A	0.400	0.195	0.674	0.200	0.283	0.000	0.447	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	87	163	194	0	0	0	0	0
N.S.	1	0.95	1.77	2.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.396	0.179	0.500	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	107	129	206	0	0	0	0	0
N.S.	1	1.15	1.39	2.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.524	0.103	0.773	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	56	75	114	78	57	0	105	89
N.S.	1	1.12	1.50	2.28	1.56	1.14	0.00	2.10	1.78
time (sec)	N/A	0.371	0.097	0.555	0.203	0.274	0.000	0.293	0.917

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	97	102	177	0	82	0	147	0
N.S.	1	1.03	1.09	1.88	0.00	0.87	0.00	1.56	0.00
time (sec)	N/A	0.334	0.083	0.494	0.000	0.257	0.000	0.308	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	110	108	153	164	93	0	168	0
N.S.	1	1.08	1.06	1.50	1.61	0.91	0.00	1.65	0.00
time (sec)	N/A	0.486	0.126	0.903	0.455	0.276	0.000	0.299	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	139	148	265	0	120	0	215	0
N.S.	1	1.04	1.10	1.98	0.00	0.90	0.00	1.60	0.00
time (sec)	N/A	0.441	0.119	0.819	0.000	0.283	0.000	0.297	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	213	288	365	0	0	0	0	0
N.S.	1	1.03	1.39	1.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.853	0.595	1.158	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	227	403	574	0	0	0	0	0
N.S.	1	0.96	1.71	2.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.841	1.178	1.438	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	132	184	252	0	0	0	0	0
N.S.	1	1.05	1.46	2.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.593	0.478	1.096	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	158	149	289	0	0	0	0	0	0
N.S.	1	0.94	1.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.572	0.247	0.000	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	145	204	364	0	0	0	0	0
N.S.	1	1.13	1.59	2.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.645	0.141	0.859	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	86	141	196	146	98	0	196	156
N.S.	1	1.08	1.76	2.45	1.82	1.22	0.00	2.45	1.95
time (sec)	N/A	0.468	0.133	0.685	0.200	0.286	0.000	0.311	0.964

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	149	185	293	0	150	0	278	0
N.S.	1	1.09	1.35	2.14	0.00	1.09	0.00	2.03	0.00
time (sec)	N/A	0.430	0.144	0.806	0.000	0.271	0.000	0.304	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	189	204	299	575	172	0	336	0
N.S.	1	1.11	1.20	1.76	3.38	1.01	0.00	1.98	0.00
time (sec)	N/A	0.737	0.206	1.156	0.703	0.275	0.000	0.324	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	268	283	470	0	225	0	427	0
N.S.	1	1.29	1.36	2.26	0.00	1.08	0.00	2.05	0.00
time (sec)	N/A	0.772	0.229	1.095	0.000	0.283	0.000	0.314	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	10	14	18
N.S.	1	1.00	1.17	1.00	1.17	1.17	0.83	1.17	1.50
time (sec)	N/A	0.182	2.467	0.708	0.304	0.264	0.374	33.763	0.743

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	16
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.60
time (sec)	N/A	0.173	0.024	0.413	0.305	0.259	0.386	11.353	0.720

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	15	12	16	20
N.S.	1	1.00	1.14	1.00	1.14	1.07	0.86	1.14	1.43
time (sec)	N/A	0.192	0.240	0.342	0.327	0.251	0.901	1.689	0.788

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	47	0	0	0	55	0
N.S.	1	1.00	0.93	1.02	0.00	0.00	0.00	1.20	0.00
time (sec)	N/A	0.449	0.067	0.451	0.000	0.000	0.000	0.286	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	59	56	58	0	0	0	95	0
N.S.	1	0.94	0.89	0.92	0.00	0.00	0.00	1.51	0.00
time (sec)	N/A	0.505	0.059	0.372	0.000	0.000	0.000	0.275	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	109	91	102	0	0	0	199	0
N.S.	1	0.93	0.78	0.87	0.00	0.00	0.00	1.70	0.00
time (sec)	N/A	0.464	0.148	0.368	0.000	0.000	0.000	0.288	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	583	28	12	14	18
N.S.	1	1.00	1.17	1.00	48.58	2.33	1.00	1.17	1.50
time (sec)	N/A	0.187	11.915	0.561	0.954	0.265	0.927	105.842	0.781

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	577	26	12	12	16
N.S.	1	1.00	1.20	1.00	57.70	2.60	1.20	1.20	1.60
time (sec)	N/A	0.171	23.964	0.504	0.951	0.254	0.999	23.495	0.743

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	560	30	14	16	20
N.S.	1	1.00	1.14	1.00	40.00	2.14	1.00	1.14	1.43
time (sec)	N/A	0.190	3.274	0.400	0.826	0.277	1.839	4.042	0.822

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	79	69	78	0	0	0	226	0
N.S.	1	1.05	0.92	1.04	0.00	0.00	0.00	3.01	0.00
time (sec)	N/A	0.537	0.263	0.408	0.000	0.000	0.000	0.281	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	86	80	77	0	0	0	357	0
N.S.	1	1.02	0.95	0.92	0.00	0.00	0.00	4.25	0.00
time (sec)	N/A	0.617	0.326	0.371	0.000	0.000	0.000	0.272	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	164	223	153	0	0	0	694	0
N.S.	1	0.92	1.25	0.86	0.00	0.00	0.00	3.90	0.00
time (sec)	N/A	0.541	0.599	0.373	0.000	0.000	0.000	0.304	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	1790	42	12	0	18
N.S.	1	1.00	1.17	1.00	149.17	3.50	1.00	0.00	1.50
time (sec)	N/A	0.171	2.736	0.603	29.312	0.253	1.942	0.000	0.788

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	1744	40	12	12	16
N.S.	1	1.00	1.20	1.00	174.40	4.00	1.20	1.20	1.60
time (sec)	N/A	0.166	9.570	0.575	28.499	0.251	2.032	52.479	0.754

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	1568	45	14	16	20
N.S.	1	1.00	1.14	1.00	112.00	3.21	1.00	1.14	1.43
time (sec)	N/A	0.185	1.368	0.507	21.865	0.256	3.576	7.963	0.833

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	112	88	154	0	0	0	580	0
N.S.	1	1.09	0.85	1.50	0.00	0.00	0.00	5.63	0.00
time (sec)	N/A	0.658	0.299	0.468	0.000	0.000	0.000	0.311	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	116	114	157	0	0	0	929	0
N.S.	1	1.04	1.02	1.40	0.00	0.00	0.00	8.29	0.00
time (sec)	N/A	0.748	0.315	0.398	0.000	0.000	0.000	0.298	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	211	169	307	0	0	0	1640	0
N.S.	1	0.93	0.74	1.35	0.00	0.00	0.00	7.19	0.00
time (sec)	N/A	0.566	0.371	0.393	0.000	0.000	0.000	0.306	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	991	44	15	18	22
N.S.	1	1.00	1.12	1.00	61.94	2.75	0.94	1.12	1.38
time (sec)	N/A	0.180	4.374	0.760	14.782	0.287	22.090	0.803	1.069

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	512	30	15	18	22
N.S.	1	1.00	1.12	1.00	32.00	1.88	0.94	1.12	1.38
time (sec)	N/A	0.182	2.770	0.612	6.847	0.274	10.072	0.577	1.075

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	82	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.242	0.200	0.000	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	14	18	22
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.88	1.12	1.38
time (sec)	N/A	0.194	0.725	1.937	0.359	0.249	1.121	0.940	0.723

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	643	32	15	18	22
N.S.	1	1.00	1.12	1.00	40.19	2.00	0.94	1.12	1.38
time (sec)	N/A	0.195	1.430	1.370	1.900	0.258	5.312	1.654	0.716

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	168	166	402	272	291	362	9430	0
N.S.	1	1.01	0.99	2.41	1.63	1.74	2.17	56.47	0.00
time (sec)	N/A	0.757	0.181	0.391	0.239	0.376	3.941	3.068	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	130	124	306	200	209	228	6416	0
N.S.	1	1.05	1.00	2.47	1.61	1.69	1.84	51.74	0.00
time (sec)	N/A	0.531	0.113	0.385	0.242	0.316	3.528	2.894	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	87	114	110	93	130	104	1547	77
N.S.	1	1.04	1.36	1.31	1.11	1.55	1.24	18.42	0.92
time (sec)	N/A	0.368	0.146	0.316	0.219	0.315	2.286	0.528	0.984

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	59	38	53	63	32	63	34
N.S.	1	1.00	1.84	1.19	1.66	1.97	1.00	1.97	1.06
time (sec)	N/A	0.177	0.029	0.090	0.224	0.275	1.188	0.285	0.982

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	333	453	0	0	0	0	0
N.S.	1	1.00	1.35	1.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.718	0.453	2.082	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	111	142	193	0	477	0	0	0
N.S.	1	1.07	1.37	1.86	0.00	4.59	0.00	0.00	0.00
time (sec)	N/A	0.370	0.139	2.190	0.000	0.308	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	199	247	574	0	1117	0	0	0
N.S.	1	1.16	1.44	3.34	0.00	6.49	0.00	0.00	0.00
time (sec)	N/A	0.504	0.316	2.069	0.000	0.546	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	396	333	798	0	0	0	0	0
N.S.	1	1.06	0.90	2.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.028	12.350	9.077	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	350	277	386	0	0	0	0	0
N.S.	1	1.11	0.88	1.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.849	33.340	7.735	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	237	212	252	0	0	0	0	0
N.S.	1	1.12	1.00	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.593	16.108	5.207	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	137	124	215	0	0	0	0	0
N.S.	1	1.15	1.04	1.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.436	0.212	3.125	0.000	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	320	326	875	0	0	0	0	0
N.S.	1	1.07	1.09	2.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.721	30.378	7.329	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	540	516	407	1618	0	0	0	0	0
N.S.	1	0.96	0.75	3.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.074	33.621	8.780	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	167	141	328	296	192	408	17474	0
N.S.	1	0.81	0.68	1.59	1.44	0.93	1.98	84.83	0.00
time (sec)	N/A	0.341	0.176	0.658	0.230	0.379	10.861	2.764	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	136	123	254	232	170	294	9792	0
N.S.	1	0.84	0.76	1.58	1.44	1.06	1.83	60.82	0.00
time (sec)	N/A	0.316	0.131	0.603	0.205	0.335	4.376	1.928	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	101	150	132	154	141	153	4051	0
N.S.	1	0.93	1.38	1.21	1.41	1.29	1.40	37.17	0.00
time (sec)	N/A	0.264	0.180	0.313	0.194	0.309	3.220	1.280	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	83	104	116	89	123	73	1088	72
N.S.	1	0.95	1.20	1.33	1.02	1.41	0.84	12.51	0.83
time (sec)	N/A	0.278	0.100	0.326	0.221	0.298	2.723	0.631	0.951

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	101	69	108	94	67	150	113	0
N.S.	1	0.96	0.66	1.03	0.90	0.64	1.43	1.08	0.00
time (sec)	N/A	0.281	0.086	0.308	0.200	0.273	2.108	0.286	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	131	94	127	137	89	279	158	0
N.S.	1	0.86	0.62	0.84	0.90	0.59	1.84	1.04	0.00
time (sec)	N/A	0.304	0.101	0.310	0.198	0.281	4.696	0.288	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	159	110	145	172	110	371	202	0
N.S.	1	0.81	0.56	0.74	0.87	0.56	1.88	1.03	0.00
time (sec)	N/A	0.324	0.128	0.329	0.212	0.271	29.280	0.292	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	161	118	139	185	128	364	13018	0
N.S.	1	0.82	0.60	0.71	0.94	0.65	1.86	66.42	0.00
time (sec)	N/A	0.368	0.190	0.622	0.210	0.317	4.036	0.470	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	131	98	121	144	107	272	7820	0
N.S.	1	0.86	0.64	0.79	0.94	0.70	1.78	51.11	0.00
time (sec)	N/A	0.340	0.212	0.639	0.194	0.276	2.459	0.367	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	116	79	217	100	86	177	3346	0
N.S.	1	0.84	0.57	1.57	0.72	0.62	1.28	24.25	0.00
time (sec)	N/A	0.307	0.089	0.722	0.196	0.283	1.595	0.334	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	152	115	137	0	0	0	0	0
N.S.	1	1.23	0.93	1.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.654	0.074	1.855	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	175	132	143	0	0	0	0	0
N.S.	1	1.28	0.96	1.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.644	0.090	1.528	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	215	186	459	405	273	542	4760	0
N.S.	1	0.85	0.74	1.82	1.61	1.08	2.15	18.89	0.00
time (sec)	N/A	0.449	0.226	0.773	0.244	0.473	11.903	6.385	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	179	153	340	296	237	355	14166	0
N.S.	1	0.94	0.80	1.78	1.55	1.24	1.86	74.17	0.00
time (sec)	N/A	0.361	0.145	0.421	0.230	0.389	5.834	4.602	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	146	136	249	198	230	207	6018	0
N.S.	1	0.90	0.84	1.54	1.22	1.42	1.28	37.15	0.00
time (sec)	N/A	0.378	0.129	0.447	0.196	0.351	4.358	2.472	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	147	127	231	159	221	211	4968	0
N.S.	1	0.93	0.80	1.46	1.01	1.40	1.34	31.44	0.00
time (sec)	N/A	0.379	0.144	0.421	0.207	0.321	4.017	99.042	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	171	127	175	181	127	333	222	0
N.S.	1	0.93	0.69	0.96	0.99	0.69	1.82	1.21	0.00
time (sec)	N/A	0.396	0.157	0.456	0.203	0.295	5.191	0.306	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	201	153	207	241	159	508	293	0
N.S.	1	0.83	0.63	0.86	1.00	0.66	2.11	1.22	0.00
time (sec)	N/A	0.411	0.174	0.488	0.202	0.278	30.110	0.301	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	207	162	198	256	187	493	17666	0
N.S.	1	0.86	0.67	0.82	1.06	0.77	2.04	73.00	0.00
time (sec)	N/A	0.473	0.188	0.824	0.199	0.319	4.378	0.502	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	160	125	352	192	153	352	11858	0
N.S.	1	0.82	0.64	1.81	0.98	0.78	1.81	60.81	0.00
time (sec)	N/A	0.363	0.192	0.819	0.198	0.325	2.689	0.446	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	217	160	231	0	0	0	0	0
N.S.	1	1.17	0.86	1.24	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.790	0.254	2.524	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	228	194	216	0	0	0	0	0
N.S.	1	1.21	1.03	1.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.804	0.649	2.418	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	546	598	1023	374	0	0	0	0	0
N.S.	1	1.10	1.87	0.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.597	1.883	34.293	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	487	547	891	446	0	0	0	0	0
N.S.	1	1.12	1.83	0.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.509	0.508	2.713	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	509	557	871	272	0	0	0	0	0
N.S.	1	1.09	1.71	0.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.324	0.498	16.106	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	459	511	876	1970	0	0	0	0	0
N.S.	1	1.11	1.91	4.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.328	0.147	2.329	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	551	603	997	331	0	0	0	0	0
N.S.	1	1.09	1.81	0.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.523	1.792	27.443	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	608	676	1255	701	0	0	0	0	0
N.S.	1	1.11	2.06	1.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.768	3.185	9.589	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	570	634	1213	577	0	0	0	0	0
N.S.	1	1.11	2.13	1.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.663	1.169	2.806	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	118	286	265	0	384	0	0	0
N.S.	1	0.90	2.18	2.02	0.00	2.93	0.00	0.00	0.00
time (sec)	N/A	0.309	0.520	6.497	0.000	0.300	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	546	602	1190	2108	0	0	0	0	0
N.S.	1	1.10	2.18	3.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.567	0.985	9.061	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-1)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	784	844	1331	941	0	0	0	0	0
N.S.	1	1.08	1.70	1.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.938	1.804	88.106	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	745	801	1245	852	0	0	0	0	0
N.S.	1	1.08	1.67	1.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.762	1.270	30.533	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	739	795	1239	840	0	0	0	0	0
N.S.	1	1.08	1.68	1.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.625	1.601	45.227	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	785	845	1291	933	0	0	0	0	0
N.S.	1	1.08	1.64	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.723	1.649	81.579	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	707	775	1805	1396	0	0	0	0	0
N.S.	1	1.10	2.55	1.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.903	7.513	8.687	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	144	389	941	0	1015	0	0	0
N.S.	1	0.92	2.48	5.99	0.00	6.46	0.00	0.00	0.00
time (sec)	N/A	0.364	0.959	5.551	0.000	0.507	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	189	386	894	0	888	0	0	0
N.S.	1	0.98	2.00	4.63	0.00	4.60	0.00	0.00	0.00
time (sec)	N/A	0.389	0.723	5.549	0.000	0.485	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	685	745	1871	3533	0	0	0	0	0
N.S.	1	1.09	2.73	5.16	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.818	6.060	2.870	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1124	1188	1819	1822	0	0	0	0	0
N.S.	1	1.06	1.62	1.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.251	6.074	66.801	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1124	1188	1827	1288	0	0	0	0	0
N.S.	1	1.06	1.63	1.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.433	6.067	33.262	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1114	1178	1812	1812	0	0	0	0	0
N.S.	1	1.06	1.63	1.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.274	6.045	94.037	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	403	369	342	0	0	1701	0	0	0
N.S.	1	0.92	0.85	0.00	0.00	4.22	0.00	0.00	0.00
time (sec)	N/A	1.498	1.615	0.000	0.000	2.370	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	294	269	263	0	0	1383	0	0	0
N.S.	1	0.91	0.89	0.00	0.00	4.70	0.00	0.00	0.00
time (sec)	N/A	0.579	1.699	0.000	0.000	1.088	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	195	179	213	0	0	1100	0	0	0
N.S.	1	0.92	1.09	0.00	0.00	5.64	0.00	0.00	0.00
time (sec)	N/A	0.397	1.589	0.000	0.000	0.523	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	23	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.87	1.00	1.17
time (sec)	N/A	0.282	7.660	0.429	0.000	0.273	10.312	0.366	1.184

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	23	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.96	1.00	1.17
time (sec)	N/A	0.285	4.875	0.738	0.000	0.269	14.239	0.354	1.308

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	27	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.17	0.96	1.00	1.17
time (sec)	N/A	0.272	12.605	0.387	0.000	0.264	116.687	0.331	1.287

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	20	19	20	24
N.S.	1	1.00	1.10	0.90	0.00	1.00	0.95	1.00	1.20
time (sec)	N/A	0.215	16.513	0.414	0.000	0.256	47.320	0.355	1.169

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	23	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.96	1.00	1.17
time (sec)	N/A	0.261	1.554	0.344	0.000	0.265	7.258	0.360	1.447

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	328	301	247	0	0	194	0	0	0
N.S.	1	0.92	0.75	0.00	0.00	0.59	0.00	0.00	0.00
time (sec)	N/A	0.653	8.120	0.000	0.000	0.111	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	453	412	325	0	0	290	0	0	0
N.S.	1	0.91	0.72	0.00	0.00	0.64	0.00	0.00	0.00
time (sec)	N/A	0.850	10.831	0.000	0.000	0.118	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	374	341	305	0	0	1701	0	0	0
N.S.	1	0.91	0.82	0.00	0.00	4.55	0.00	0.00	0.00
time (sec)	N/A	0.675	1.579	0.000	0.000	2.305	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	262	242	247	0	0	1377	0	0	0
N.S.	1	0.92	0.94	0.00	0.00	5.26	0.00	0.00	0.00
time (sec)	N/A	0.469	1.625	0.000	0.000	1.090	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.87	1.00	1.17
time (sec)	N/A	0.293	7.849	2.136	0.000	0.279	71.460	0.493	1.139

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.96	1.00	1.17
time (sec)	N/A	0.301	6.268	2.031	0.000	0.268	61.958	0.376	1.427

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	43	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.87	0.00	1.00	1.17
time (sec)	N/A	0.286	12.552	1.086	0.000	0.257	0.000	0.361	1.403

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	37	0	20	24
N.S.	1	1.00	1.10	0.90	0.00	1.85	0.00	1.00	1.20
time (sec)	N/A	0.219	17.073	1.093	0.000	0.272	0.000	0.358	1.195

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.96	1.00	1.17
time (sec)	N/A	0.283	32.885	2.246	0.000	0.265	99.479	0.405	1.540

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.96	1.00	1.17
time (sec)	N/A	0.285	8.273	0.376	0.000	0.262	67.227	0.362	1.400

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	416	381	303	0	0	288	0	0	0
N.S.	1	0.92	0.73	0.00	0.00	0.69	0.00	0.00	0.00
time (sec)	N/A	0.799	10.892	0.000	0.000	0.121	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	554	504	383	0	0	395	0	0	0
N.S.	1	0.91	0.69	0.00	0.00	0.71	0.00	0.00	0.00
time (sec)	N/A	1.014	11.820	0.000	0.000	0.146	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	321	296	281	0	0	1385	0	0	0
N.S.	1	0.92	0.88	0.00	0.00	4.31	0.00	0.00	0.00
time (sec)	N/A	1.333	2.005	0.000	0.000	1.116	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	225	207	242	0	0	1111	0	0	0
N.S.	1	0.92	1.08	0.00	0.00	4.94	0.00	0.00	0.00
time (sec)	N/A	0.509	1.364	0.000	0.000	0.550	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	128	108	0	0	869	0	0	0
N.S.	1	0.97	0.82	0.00	0.00	6.58	0.00	0.00	0.00
time (sec)	N/A	0.348	0.364	0.000	0.000	0.356	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	31	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.35	0.87	1.00	1.17
time (sec)	N/A	0.267	1.144	0.427	0.000	0.253	7.816	0.349	1.263

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	33	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.43	0.96	1.00	1.17
time (sec)	N/A	0.282	4.734	0.817	0.000	0.292	27.725	0.361	1.361

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	27	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.17	0.96	1.00	1.17
time (sec)	N/A	0.264	32.378	0.452	0.000	0.261	54.463	0.350	1.475

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	20	19	20	24
N.S.	1	1.00	1.10	0.90	0.00	1.00	0.95	1.00	1.20
time (sec)	N/A	0.217	0.630	0.473	0.000	0.284	15.389	0.356	1.167

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	246	231	143	0	0	107	0	0	0
N.S.	1	0.94	0.58	0.00	0.00	0.43	0.00	0.00	0.00
time (sec)	N/A	0.512	1.849	0.000	0.000	0.106	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	362	332	249	0	0	196	0	0	0
N.S.	1	0.92	0.69	0.00	0.00	0.54	0.00	0.00	0.00
time (sec)	N/A	0.714	6.517	0.000	0.000	0.112	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1006	839	329	0	0	291	0	0	0
N.S.	1	0.83	0.33	0.00	0.00	0.29	0.00	0.00	0.00
time (sec)	N/A	1.989	8.637	0.000	0.000	0.125	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	252	234	265	0	0	1483	0	0	0
N.S.	1	0.93	1.05	0.00	0.00	5.88	0.00	0.00	0.00
time (sec)	N/A	1.337	1.480	0.000	0.000	0.503	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	157	152	161	0	0	1070	0	0	0
N.S.	1	0.97	1.03	0.00	0.00	6.82	0.00	0.00	0.00
time (sec)	N/A	0.440	1.030	0.000	0.000	0.414	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	79	0	0	283	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	3.54	0.00	0.00	0.00
time (sec)	N/A	0.293	0.239	0.000	0.000	0.310	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	42	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.83	0.87	1.00	1.17
time (sec)	N/A	0.287	8.180	1.748	0.000	0.278	82.808	0.363	1.343

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	44	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.91	0.00	1.00	1.17
time (sec)	N/A	0.306	10.616	2.420	0.000	0.268	0.000	0.360	1.571

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	47	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.04	0.96	1.00	1.17
time (sec)	N/A	0.288	14.410	1.352	0.000	0.260	144.523	0.394	1.502

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	47	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.04	0.96	1.00	1.17
time (sec)	N/A	0.285	5.264	1.476	0.000	0.261	35.071	0.428	1.242

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	113	0	0	74	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.301	1.705	0.000	0.000	0.100	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	274	258	212	0	0	190	0	0	0
N.S.	1	0.94	0.77	0.00	0.00	0.69	0.00	0.00	0.00
time (sec)	N/A	0.571	4.817	0.000	0.000	0.118	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	701	591	277	0	0	331	0	0	0
N.S.	1	0.84	0.40	0.00	0.00	0.47	0.00	0.00	0.00
time (sec)	N/A	1.669	6.453	0.000	0.000	0.124	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	244	219	242	0	0	2123	0	0	0
N.S.	1	0.90	0.99	0.00	0.00	8.70	0.00	0.00	0.00
time (sec)	N/A	1.374	1.300	0.000	0.000	0.486	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	148	137	0	0	664	0	0	0
N.S.	1	0.91	0.84	0.00	0.00	4.07	0.00	0.00	0.00
time (sec)	N/A	0.454	0.464	0.000	0.000	0.386	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	126	132	0	0	571	0	0	0
N.S.	1	0.91	0.96	0.00	0.00	4.14	0.00	0.00	0.00
time (sec)	N/A	0.327	0.446	0.000	0.000	0.356	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	53	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.30	0.00	1.00	1.17
time (sec)	N/A	0.300	14.073	1.816	0.000	0.267	0.000	0.379	1.284

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	55	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.39	0.00	1.00	1.17
time (sec)	N/A	0.318	15.460	2.326	0.000	0.264	0.000	0.382	1.578

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	58	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.52	0.00	1.00	1.17
time (sec)	N/A	0.300	14.359	1.362	0.000	0.271	0.000	0.390	1.560

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	58	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.52	0.00	1.00	1.17
time (sec)	N/A	0.293	13.525	1.423	0.000	0.271	0.000	0.436	1.390

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	276	251	186	0	0	286	0	0	0
N.S.	1	0.91	0.67	0.00	0.00	1.04	0.00	0.00	0.00
time (sec)	N/A	0.530	0.310	0.000	0.000	0.116	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	296	271	248	0	0	348	0	0	0
N.S.	1	0.92	0.84	0.00	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.535	4.127	0.000	0.000	0.127	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	631	540	323	0	0	536	0	0	0
N.S.	1	0.86	0.51	0.00	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	1.673	7.593	0.000	0.000	0.145	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	589	542	400	0	0	0	0	0	0
N.S.	1	0.92	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.147	3.012	0.000	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	374	345	291	0	0	0	0	0	0
N.S.	1	0.92	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.661	0.473	0.000	0.000	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	178	203	169	0	0	0	0	0	0
N.S.	1	1.14	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.402	0.328	0.000	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	20	25	29
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.87	1.09	1.26
time (sec)	N/A	0.253	2.293	2.823	0.548	0.271	39.070	0.323	0.910

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	36	0	25	29
N.S.	1	1.00	1.09	1.00	1.09	1.57	0.00	1.09	1.26
time (sec)	N/A	0.260	4.060	2.532	0.562	0.285	0.000	0.330	0.903

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	42	0	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.68	0.00	1.00	1.16
time (sec)	N/A	0.286	0.957	2.255	0.519	0.273	0.000	0.354	0.949

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.16
time (sec)	N/A	0.280	0.123	0.895	0.384	0.277	52.552	0.350	0.877

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.16
time (sec)	N/A	0.279	0.921	1.103	0.407	0.281	22.423	0.352	0.964

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	45	24	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.80	0.96	1.00	1.16
time (sec)	N/A	0.301	1.101	2.091	0.412	0.274	140.345	0.353	0.989

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	401	227	194	0	0	238	0	0	0
N.S.	1	0.57	0.48	0.00	0.00	0.59	0.00	0.00	0.00
time (sec)	N/A	1.617	0.239	0.000	0.000	0.297	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	268	156	159	0	0	181	0	0	0
N.S.	1	0.58	0.59	0.00	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	1.226	0.279	0.000	0.000	0.279	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	126	101	118	0	0	125	0	0	0
N.S.	1	0.80	0.94	0.00	0.00	0.99	0.00	0.00	0.00
time (sec)	N/A	0.424	0.222	0.000	0.000	0.275	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	87	37	34	26	30
N.S.	1	1.00	1.08	0.92	3.35	1.42	1.31	1.00	1.15
time (sec)	N/A	0.277	0.637	0.273	0.483	0.264	11.186	0.376	1.811

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	111	39	36	26	30
N.S.	1	1.00	1.08	0.92	4.27	1.50	1.38	1.00	1.15
time (sec)	N/A	0.294	8.065	3.075	0.493	0.259	82.450	0.354	1.446

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [68] had the largest ratio of [1.05556000000000005]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	7	1.07	12	0.583
2	A	4	4	1.09	12	0.333
3	A	7	6	1.04	12	0.500
4	A	3	3	1.06	12	0.250
5	A	6	5	1.00	12	0.417
6	A	2	2	1.00	10	0.200
7	A	1	1	1.00	8	0.125
8	A	8	7	1.36	12	0.583
9	A	2	2	1.00	12	0.167
10	A	5	4	1.25	12	0.333
11	A	5	4	1.08	12	0.333
12	A	6	5	1.26	12	0.417
13	A	5	4	1.05	12	0.333
14	A	7	6	1.27	12	0.500
15	A	10	9	1.07	14	0.643
16	A	9	8	0.95	14	0.571
17	A	8	7	1.14	12	0.583
18	A	7	6	0.95	10	0.600
19	A	8	7	1.15	14	0.500
20	A	9	8	1.12	14	0.571
21	A	6	5	1.03	14	0.357
22	A	10	9	1.08	14	0.643

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	8	7	1.04	14	0.500
24	A	15	14	1.03	14	1.000
25	A	11	10	0.96	14	0.714
26	A	11	10	1.05	12	0.833
27	A	8	7	0.94	10	0.700
28	A	9	8	1.13	14	0.571
29	A	11	10	1.08	14	0.714
30	A	9	8	1.09	14	0.571
31	A	14	13	1.11	14	0.929
32	A	15	14	1.29	14	1.000
33	N/A	1	0	1.00	12	0.000
34	N/A	1	0	1.00	10	0.000
35	N/A	1	0	1.00	14	0.000
36	A	7	6	1.00	14	0.429
37	A	9	8	0.94	14	0.571
38	A	4	3	0.93	14	0.214
39	N/A	1	0	1.00	12	0.000
40	N/A	1	0	1.00	10	0.000
41	N/A	1	0	1.00	14	0.000
42	A	9	8	1.05	14	0.571
43	A	11	10	1.02	14	0.714
44	A	4	3	0.92	14	0.214
45	N/A	1	0	1.00	12	0.000
46	N/A	1	0	1.00	10	0.000
47	N/A	1	0	1.00	14	0.000
48	A	12	11	1.09	14	0.786
49	A	14	13	1.04	14	0.929
50	A	4	3	0.93	14	0.214
51	N/A	1	0	1.00	16	0.000
52	N/A	1	0	1.00	16	0.000
53	A	4	3	1.00	14	0.214
54	N/A	1	0	1.00	16	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
55	N/A	1	0	1.00	16	0.000
56	A	15	14	1.01	16	0.875
57	A	13	12	1.05	16	0.750
58	A	12	11	1.04	14	0.786
59	A	1	1	1.00	8	0.125
60	A	2	2	1.00	16	0.125
61	A	8	7	1.07	16	0.438
62	A	9	8	1.16	16	0.500
63	A	18	17	1.06	18	0.944
64	A	16	15	1.11	18	0.833
65	A	4	4	1.12	18	0.222
66	A	8	7	1.15	18	0.389
67	A	16	15	1.07	18	0.833
68	A	20	19	0.96	18	1.056
69	A	8	7	0.81	19	0.368
70	A	7	6	0.84	19	0.316
71	A	6	5	0.93	16	0.312
72	A	6	5	0.95	19	0.263
73	A	4	4	0.96	19	0.211
74	A	5	5	0.86	19	0.263
75	A	6	6	0.81	19	0.316
76	A	6	5	0.82	19	0.263
77	A	6	5	0.86	19	0.263
78	A	5	4	0.84	17	0.235
79	A	6	5	1.23	19	0.263
80	A	6	5	1.28	19	0.263
81	A	9	8	0.85	21	0.381
82	A	7	6	0.94	18	0.333
83	A	9	8	0.90	21	0.381
84	A	7	6	0.93	21	0.286
85	A	5	5	0.93	21	0.238
86	A	6	6	0.83	21	0.286
87	A	6	5	0.86	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	5	4	0.82	19	0.211
89	A	6	5	1.17	21	0.238
90	A	6	5	1.21	21	0.238
91	A	4	3	1.10	21	0.143
92	A	4	3	1.12	19	0.158
93	A	4	3	1.09	18	0.167
94	A	4	3	1.11	21	0.143
95	A	4	3	1.09	21	0.143
96	A	4	3	1.11	21	0.143
97	A	4	3	1.11	21	0.143
98	A	6	5	0.90	19	0.263
99	A	4	3	1.10	21	0.143
100	A	4	3	1.08	21	0.143
101	A	4	3	1.08	21	0.143
102	A	4	3	1.08	18	0.167
103	A	4	3	1.08	21	0.143
104	A	4	3	1.10	21	0.143
105	A	7	6	0.92	21	0.286
106	A	8	7	0.98	19	0.368
107	A	4	3	1.09	21	0.143
108	A	4	3	1.06	21	0.143
109	A	4	3	1.06	21	0.143
110	A	4	3	1.06	18	0.167
111	A	15	14	0.92	23	0.609
112	A	13	12	0.91	23	0.522
113	A	10	9	0.92	21	0.429
114	N/A	1	0	1.00	23	0.000
115	N/A	1	0	1.00	23	0.000
116	N/A	1	0	1.00	23	0.000
117	N/A	1	0	1.00	20	0.000
118	N/A	1	0	1.00	23	0.000
119	A	13	13	0.92	23	0.565
120	A	15	15	0.91	23	0.652

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
121	A	15	14	0.91	23	0.609
122	A	12	11	0.92	21	0.524
123	N/A	1	0	1.00	23	0.000
124	N/A	1	0	1.00	23	0.000
125	N/A	1	0	1.00	23	0.000
126	N/A	1	0	1.00	20	0.000
127	N/A	1	0	1.00	23	0.000
128	N/A	1	0	1.00	23	0.000
129	A	15	15	0.92	23	0.652
130	A	17	17	0.91	23	0.739
131	A	13	12	0.92	23	0.522
132	A	11	10	0.92	23	0.435
133	A	9	8	0.97	21	0.381
134	N/A	1	0	1.00	23	0.000
135	N/A	1	0	1.00	23	0.000
136	N/A	1	0	1.00	23	0.000
137	N/A	1	0	1.00	20	0.000
138	A	12	12	0.94	23	0.522
139	A	13	13	0.92	23	0.565
140	A	4	4	0.83	23	0.174
141	A	11	10	0.93	23	0.435
142	A	9	8	0.97	23	0.348
143	A	5	4	1.00	21	0.190
144	N/A	1	0	1.00	23	0.000
145	N/A	1	0	1.00	23	0.000
146	N/A	1	0	1.00	23	0.000
147	N/A	1	0	1.00	23	0.000
148	A	5	5	1.00	20	0.250
149	A	12	12	0.94	23	0.522
150	A	4	4	0.84	23	0.174
151	A	11	10	0.90	23	0.435
152	A	9	8	0.91	23	0.348

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
153	A	6	5	0.91	21	0.238
154	N/A	1	0	1.00	23	0.000
155	N/A	1	0	1.00	23	0.000
156	N/A	1	0	1.00	23	0.000
157	N/A	1	0	1.00	23	0.000
158	A	10	10	0.91	23	0.435
159	A	11	11	0.92	20	0.550
160	A	4	4	0.86	23	0.174
161	A	6	6	0.92	23	0.261
162	A	6	6	0.92	23	0.261
163	A	5	5	1.14	21	0.238
164	N/A	1	0	1.00	23	0.000
165	N/A	1	0	1.00	23	0.000
166	N/A	1	0	1.00	25	0.000
167	N/A	1	0	1.00	25	0.000
168	N/A	1	0	1.00	25	0.000
169	N/A	1	0	1.00	25	0.000
170	A	8	7	0.57	26	0.269
171	A	10	9	0.58	26	0.346
172	A	9	8	0.80	26	0.308
173	N/A	1	0	1.00	26	0.000
174	N/A	1	0	1.00	26	0.000

CHAPTER 3

Listing of integrals

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3.170 $\int \frac{x^{11} \left(a+b \sec^{-1}(cx)\right)}{\sqrt{1-c^4 x^4}} \, dx$ 1189
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3.173 $\int \frac{a+b \sec^{-1}(cx)}{x \sqrt{1-c^4 x^4}} \, dx$ 1209
3.174 $\int \frac{a+b \sec^{-1}(cx)}{x^5 \sqrt{1-c^4 x^4}} \, dx$ 1213

3.1 $\int x^6(a+b\sec^{-1}(cx))\,dx$

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3.1.1 Optimal result

Integrand size = 12, antiderivative size = 114

$$\int x^6(a+b\sec^{-1}(cx))\,dx = -\frac{5b\sqrt{1-\frac{1}{c^2x^2}}x^2}{112c^5} - \frac{5b\sqrt{1-\frac{1}{c^2x^2}}x^4}{168c^3} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^6}{42c} + \frac{1}{7}x^7(a+b\sec^{-1}(cx)) - \frac{5b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{112c^7}$$

output
```
1/7*x^7*(a+b*arcsec(c*x))-5/112*b*arctanh((1-1/c^2/x^2)^(1/2))/c^7-5/112*b
*x^2*(1-1/c^2/x^2)^(1/2)/c^5-5/168*b*x^4*(1-1/c^2/x^2)^(1/2)/c^3-1/42*b*x^
6*(1-1/c^2/x^2)^(1/2)/c
```

3.1.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.94

$$\int x^6(a+b\sec^{-1}(cx))\,dx = \frac{ax^7}{7} + b\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\left(-\frac{5x^2}{112c^5} - \frac{5x^4}{168c^3} - \frac{x^6}{42c}\right) + \frac{1}{7}bx^7\sec^{-1}(cx) - \frac{5b\log\left(x\left(1+\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\right)\right)}{112c^7}$$

input
```
Integrate[x^6*(a + b*ArcSec[c*x]),x]
```

output
```
(a*x^7)/7 + b*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)]*((-5*x^2)/(112*c^5) - (5*x^4)
/(168*c^3) - x^6/(42*c)) + (b*x^7*ArcSec[c*x])/7 - (5*b*Log[x*(1 + Sqrt[(-
1 + c^2*x^2)/(c^2*x^2)])])/(112*c^7)
```

3.1.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5743, 798, 52, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6(a+b\sec^{-1}(cx))\,dx$$

$$\downarrow 5743$$

$$\frac{1}{7}x^7(a+b\sec^{-1}(cx)) - \frac{b\int \frac{x^5}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{7c}$$

$$\downarrow 798$$

$$\frac{b\int \frac{x^8}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{14c} + \frac{1}{7}x^7(a+b\sec^{-1}(cx))$$

$$\downarrow 52$$

$$\frac{b\left(\frac{5\int \frac{x^6}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{6c^2} - \frac{1}{3}x^6\sqrt{1-\frac{1}{c^2x^2}}\right)}{14c} + \frac{1}{7}x^7(a+b\sec^{-1}(cx))$$

$$\downarrow 52$$

$$\frac{b\left(\frac{5\left(\frac{3\int \frac{x^4}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{4c^2} - \frac{1}{2}x^4\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^2} - \frac{1}{3}x^6\sqrt{1-\frac{1}{c^2x^2}}\right)}{14c} + \frac{1}{7}x^7(a+b\sec^{-1}(cx))$$

$$\downarrow 52$$

$$\frac{b\left(\frac{5\left(\frac{3\left(\int \frac{x^2}{\sqrt{1-\frac{1}{c^2x^2}}}\,d\frac{1}{x^2}}{2c^2}-x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{4c^2}-\frac{1}{2}x^4\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^2}-\frac{1}{3}x^6\sqrt{1-\frac{1}{c^2x^2}}\right)}{14c}+\frac{1}{7}x^7\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 73$$

$$\frac{b\left(\frac{5\left(\frac{3\left(x^2\left(-\sqrt{1-\frac{1}{c^2x^2}}\right)-\int \frac{1}{c^2-\frac{c^2}{x^4}}\,d\sqrt{1-\frac{1}{c^2x^2}}\right)}{4c^2}-\frac{1}{2}x^4\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^2}-\frac{1}{3}x^6\sqrt{1-\frac{1}{c^2x^2}}\right)}{14c}+\frac{1}{7}x^7\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 221$$

$$\frac{1}{7}x^7\left(a+b\sec^{-1}(cx)\right)+\frac{b\left(\frac{5\left(\frac{3\left(x^2\left(-\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^2}\right)}{4c^2}-\frac{1}{2}x^4\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^2}-\frac{1}{3}x^6\sqrt{1-\frac{1}{c^2x^2}}\right)}{14c}$$

input
```
Int[x^6*(a + b*ArcSec[c*x]),x]
```

output
```
(x^7*(a + b*ArcSec[c*x]))/7 + (b*(-1/3*(Sqrt[1 - 1/(c^2*x^2)]*x^6) + (5*(-
1/2*(Sqrt[1 - 1/(c^2*x^2)]*x^4) + (3*(-(Sqrt[1 - 1/(c^2*x^2)]*x^2) - ArcTa
nh[Sqrt[1 - 1/(c^2*x^2)]]/c^2))/(4*c^2)))/(6*c^2)))/(14*c)
```

3.1.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 5743
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

3.1.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.55

method	result
parts	$\frac{a x^7}{7}+\frac{b x^7 \operatorname{arcsec}(cx)}{7}-\frac{b\left(c^2x^2-1\right)x^4}{42c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{5b\left(c^2x^2-1\right)x^2}{168c^5\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{5b\left(c^2x^2-1\right)}{112c^7\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{5b\sqrt{c^2x^2-1}\,\ln\left(cx+\sqrt{c^2x^2-1}\right)}{112c^8\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}$
derivativedivides	$\frac{\frac{a c^7x^7}{7}+\frac{b c^7x^7 \operatorname{arcsec}(cx)}{7}-\frac{b\left(c^2x^2-1\right)c^4x^4}{42\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{5b\left(c^2x^2-1\right)c^2x^2}{168\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{5b\left(c^2x^2-1\right)}{112\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{5b\sqrt{c^2x^2-1}\,\ln\left(cx+\sqrt{c^2x^2-1}\right)}{112\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{c^7}$
default	$\frac{\frac{a c^7x^7}{7}+\frac{b c^7x^7 \operatorname{arcsec}(cx)}{7}-\frac{b\left(c^2x^2-1\right)c^4x^4}{42\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{5b\left(c^2x^2-1\right)c^2x^2}{168\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{5b\left(c^2x^2-1\right)}{112\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{5b\sqrt{c^2x^2-1}\,\ln\left(cx+\sqrt{c^2x^2-1}\right)}{112\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{c^7}$

input
```
int(x^6*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/7*a*x^7+1/7*b*x^7*arcsec(c*x)-1/42*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^
2)^(1/2)*x^4-5/168*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^2-5/112
*b/c^7*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)-5/112*b/c^8*(c^2*x^2-1)^(1/
2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*ln(c*x+(c^2*x^2-1)^(1/2))
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02

$$\int x^6\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{48\,ac^7x^7+96\,bc^7\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+48\left(bc^7x^7-bc^7\right)\operatorname{arcsec}(cx)+15\,b\log\left(-cx+\sqrt{c^2x^2-1}\right)}{336\,c^7}$$

input
```
integrate(x^6*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/336*(48*a*c^7*x^7 + 96*b*c^7*arctan(-c*x + sqrt(c^2*x^2 - 1)) + 48*(b*c^
7*x^7 - b*c^7)*arcsec(c*x) + 15*b*log(-c*x + sqrt(c^2*x^2 - 1)) - (8*b*c^5
*x^5 + 10*b*c^3*x^3 + 15*b*c*x)*sqrt(c^2*x^2 - 1))/c^7
```

3.1.6 Sympy [A] (verification not implemented)

Time = 8.73 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.94

$$\int x^6\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{ax^7}{7}+\frac{bx^7\operatorname{asec}(cx)}{7}$$
$$-\frac{b\left(\begin{cases}\frac{cx^7}{6\sqrt{c^2x^2-1}}+\frac{x^5}{24c\sqrt{c^2x^2-1}}+\frac{5x^3}{48c^3\sqrt{c^2x^2-1}}-\frac{5x}{16c^5\sqrt{c^2x^2-1}}+\frac{5\operatorname{acosh}(cx)}{16c^6} & \text{for } |c^2x^2|>1\\ -\frac{icx^7}{6\sqrt{-c^2x^2+1}}-\frac{ix^5}{24c\sqrt{-c^2x^2+1}}-\frac{5ix^3}{48c^3\sqrt{-c^2x^2+1}}+\frac{5ix}{16c^5\sqrt{-c^2x^2+1}}-\frac{5i\operatorname{asin}(cx)}{16c^6} & \text{otherwise}\end{cases}\right)}{7c}$$

input
```
integrate(x**6*(a+b*asec(c*x)),x)
```

output
```
a*x**7/7 + b*x**7*asec(c*x)/7 - b*Piecewise((c*x**7/(6*sqrt(c**2*x**2 - 1)
) + x**5/(24*c*sqrt(c**2*x**2 - 1)) + 5*x**3/(48*c**3*sqrt(c**2*x**2 - 1))
 - 5*x/(16*c**5*sqrt(c**2*x**2 - 1)) + 5*acosh(c*x)/(16*c**6), Abs(c**2*x*
*2) > 1), (-I*c*x**7/(6*sqrt(-c**2*x**2 + 1)) - I*x**5/(24*c*sqrt(-c**2*x*
*2 + 1)) - 5*I*x**3/(48*c**3*sqrt(-c**2*x**2 + 1)) + 5*I*x/(16*c**5*sqrt(-
c**2*x**2 + 1)) - 5*I*asin(c*x)/(16*c**6), True))/(7*c)
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.42

$$\int x^6\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{1}{7}ax^7 + \frac{1}{672}\left(96\,x^7\operatorname{arcsec}(cx) - \frac{2\left(15\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} - 40\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 33\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^6\left(\frac{1}{c^2x^2}-1\right)^3 + 3c^6\left(\frac{1}{c^2x^2}-1\right)^2 + 3c^6\left(\frac{1}{c^2x^2}-1\right) + c^6} + \frac{15\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^6} - \frac{15\log\left(\sqrt{-\frac{1}{c^2}}\right.}{c^6}\right.$$
$$\left.\left.\vphantom{\frac{1}{1}}\right)\right)$$
$$\left.\vphantom{\frac{1}{1}}\right)$$
$$\left.\vphantom{\frac{1}{1}}\right)$$
$$\left.\vphantom{\frac{1}{1}}

output

```
1/336*c*(48*b*arccos(1/(c*x))/(c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)
^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)
^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(
1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) +
 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14) - 15*b*log(abs(sqrt(-1/
(c^2*x^2) + 1) + 1/(c*x) + 1))/(c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1
)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1
)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*
(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x)
+ 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14) + 15*b*log(abs(sqrt(-1
/(c^2*x^2) + 1) - 1/(c*x) - 1))/(c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) +
1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) -
1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8
*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x)
 + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14) + 48*a/(c^8 + 7*c^8*(
1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1
)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1
)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*
(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) +
1)^14) - 336*b*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^8 + 7*c^8*(1/(c^2*...
```

3.1.9 Mupad [F(-1)]

Timed out.

$$\int x^6 (a + b \sec^{-1}(cx))\, dx = \int x^6 \left(a + b \operatorname{acos}\left(\frac{1}{c\,x}\right)\right) dx$$

input

```
int(x^6*(a + b*acos(1/(c*x))),x)
```

output

```
int(x^6*(a + b*acos(1/(c*x))), x)
```

3.2 $\int x^5(a+b\sec^{-1}(cx))\,dx$

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3.2.1 Optimal result

Integrand size = 12, antiderivative size = 89

$$\int x^5(a+b\sec^{-1}(cx))\,dx = -\frac{4b\sqrt{1-\frac{1}{c^2x^2}}x}{45c^5} - \frac{2b\sqrt{1-\frac{1}{c^2x^2}}x^3}{45c^3} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^5}{30c} + \frac{1}{6}x^6(a+b\sec^{-1}(cx))$$

output
```
1/6*x^6*(a+b*arcsec(c*x))-4/45*b*x*(1-1/c^2/x^2)^(1/2)/c^5-2/45*b*x^3*(1-1
/c^2/x^2)^(1/2)/c^3-1/30*b*x^5*(1-1/c^2/x^2)^(1/2)/c
```

3.2.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.81

$$\int x^5(a+b\sec^{-1}(cx))\,dx = \frac{ax^6}{6} + b\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\left(-\frac{4x}{45c^5} - \frac{2x^3}{45c^3} - \frac{x^5}{30c}\right) + \frac{1}{6}bx^6\sec^{-1}(cx)$$

input
```
Integrate[x^5*(a + b*ArcSec[c*x]),x]
```

output
```
(a*x^6)/6 + b*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)]*((-4*x)/(45*c^5) - (2*x^3)/(4
5*c^3) - x^5/(30*c)) + (b*x^6*ArcSec[c*x])/6
```

3.2.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5743, 803, 803, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\downarrow 5743$$

$$\frac{1}{6}x^6\left(a+b\sec^{-1}(cx)\right)-\frac{b\int\frac{x^4}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{6c}$$

$$\downarrow 803$$

$$\frac{1}{6}x^6\left(a+b\sec^{-1}(cx)\right)-\frac{b\left(\frac{4\int\frac{x^2}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{5c^2}+\frac{1}{5}x^5\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c}$$

$$\downarrow 803$$

$$\frac{1}{6}x^6\left(a+b\sec^{-1}(cx)\right)-\frac{b\left(\frac{4\left(\frac{2\int\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{3c^2}+\frac{1}{3}x^3\sqrt{1-\frac{1}{c^2x^2}}\right)}{5c^2}+\frac{1}{5}x^5\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c}$$

$$\downarrow 746$$

$$\frac{1}{6}x^6\left(a+b\sec^{-1}(cx)\right)-\frac{b\left(\frac{1}{5}x^5\sqrt{1-\frac{1}{c^2x^2}}+\frac{4\left(\frac{2x\sqrt{1-\frac{1}{c^2x^2}}}{3c^2}+\frac{1}{3}x^3\sqrt{1-\frac{1}{c^2x^2}}\right)}{5c^2}\right)}{6c}$$

input
```
Int[x^5*(a + b*ArcSec[c*x]),x]
```

output
```
-1/6*(b*((Sqrt[1 - 1/(c^2*x^2)]*x^5)/5 + (4*((2*Sqrt[1 - 1/(c^2*x^2)]*x)/(
3*c^2) + (Sqrt[1 - 1/(c^2*x^2)]*x^3)/3))/(5*c^2)))/c + (x^6*(a + b*ArcSec[
c*x]))/6
```

3.2.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 5743
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

3.2.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.89

method	result	size
parts	$\frac{x^6 a}{6}+\frac{b\left(\frac{c^6 x^6 \operatorname{arcsec}(cx)}{6}-\frac{\left(c^2x^2-1\right)\left(3c^4x^4+4c^2x^2+8\right)}{90\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^6}$	79
derivativedivides	$\frac{\frac{a\,c^6x^6}{6}+b\left(\frac{c^6x^6\operatorname{arcsec}(cx)}{6}-\frac{\left(c^2x^2-1\right)\left(3c^4x^4+4c^2x^2+8\right)}{90\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^6}$	83
default	$\frac{\frac{a\,c^6x^6}{6}+b\left(\frac{c^6x^6\operatorname{arcsec}(cx)}{6}-\frac{\left(c^2x^2-1\right)\left(3c^4x^4+4c^2x^2+8\right)}{90\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^6}$	83

input
```
int(x^5*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*x^6*a+b/c^6*(1/6*c^6*x^6*arcsec(c*x)-1/90*(c^2*x^2-1)*(3*c^4*x^4+4*c^2
*x^2+8)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x)
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.71

$$\int x^5\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{15\,bc^6x^6\operatorname{arcsec}(cx)+15\,ac^6x^6-(3\,bc^4x^4+4\,bc^2x^2+8\,b)\sqrt{c^2x^2-1}}{90\,c^6}$$

input
```
integrate(x^5*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/90*(15*b*c^6*x^6*arcsec(c*x) + 15*a*c^6*x^6 - (3*b*c^4*x^4 + 4*b*c^2*x^2
 + 8*b)*sqrt(c^2*x^2 - 1))/c^6
```

3.2.6 Sympy [A] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.72

$$\int x^5\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{ax^6}{6}+\frac{bx^6\operatorname{asec}(cx)}{6}-\frac{b\left(\begin{cases}\frac{x^4\sqrt{c^2x^2-1}}{5c}+\frac{4x^2\sqrt{c^2x^2-1}}{15c^3}+\frac{8\sqrt{c^2x^2-1}}{15c^5} & \text{for } |c^2x^2|>1\\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c}+\frac{4ix^2\sqrt{-c^2x^2+1}}{15c^3}+\frac{8i\sqrt{-c^2x^2+1}}{15c^5} & \text{otherwise}\end{cases}\right)}{6c}$$

input
```
integrate(x**5*(a+b*asec(c*x)),x)
```

output
```
a*x**6/6 + b*x**6*asec(c*x)/6 - b*Piecewise((x**4*sqrt(c**2*x**2 - 1)/(5*c
) + 4*x**2*sqrt(c**2*x**2 - 1)/(15*c**3) + 8*sqrt(c**2*x**2 - 1)/(15*c**5)
, Abs(c**2*x**2) > 1), (I*x**4*sqrt(-c**2*x**2 + 1)/(5*c) + 4*I*x**2*sqrt(
-c**2*x**2 + 1)/(15*c**3) + 8*I*sqrt(-c**2*x**2 + 1)/(15*c**5), True))/(6*
c)
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int x^5\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{1}{6}ax^6 + \frac{1}{90}\left(15x^6\operatorname{arcsec}(cx) - \frac{3c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} + 10c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 15x\sqrt{-\frac{1}{c^2x^2}+1}}{c^5}\right)b$$

input
```
integrate(x^5*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*x^6 + 1/90*(15*x^6*arcsec(c*x) - (3*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2)
 + 10*c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(-1/(c^2*x^2) + 1))/c^5)
*b
```

3.2.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3862 vs. 2(75) = 150.

Time = 0.32 (sec) , antiderivative size = 3862, normalized size of antiderivative = 43.39

$$\int x^5\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output

```
1/90*c*(15*b*arccos(1/(c*x))/(c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^
2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^
3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/
(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^
12) + 15*a/(c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2
*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6
+ 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(
1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12) - 90*b*(1/(c^2
*x^2) - 1)*arccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3
/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(
c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^1
2)*(1/(c*x) + 1)^2) - 30*b*sqrt(-1/(c^2*x^2) + 1)/((c^7 + 6*c^7*(1/(c^2*x^
2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*
c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c
*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2)
 - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)) - 90*a*(1/(c^2*x^2) - 1)/((c^7 +
6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c
*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x
^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10...
```

3.2.9 Mupad [F(-1)]

Timed out.

$$\int x^5 (a + b \sec^{-1}(cx))\, dx = \int x^5 \left(a + b \operatorname{acos}\left(\frac{1}{c\,x}\right)\right) dx$$

input

```
int(x^5*(a + b*acos(1/(c*x))),x)
```

output

```
int(x^5*(a + b*acos(1/(c*x))), x)
```

3.3 $\int x^4(a+b\sec^{-1}(cx))\,dx$

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3.3.1 Optimal result

Integrand size = 12, antiderivative size = 89

$$\int x^4(a+b\sec^{-1}(cx))\,dx = -\frac{3b\sqrt{1-\frac{1}{c^2x^2}}x^2}{40c^3} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^4}{20c} + \frac{1}{5}x^5\left(a+b\sec^{-1}(cx)\right) - \frac{3b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{40c^5}$$

output

```
1/5*x^5*(a+b*arcsec(c*x))-3/40*b*arctanh((1-1/c^2/x^2)^(1/2))/c^5-3/40*b*x
^2*(1-1/c^2/x^2)^(1/2)/c^3-1/20*b*x^4*(1-1/c^2/x^2)^(1/2)/c
```

3.3.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09

$$\int x^4(a+b\sec^{-1}(cx))\,dx = \frac{ax^5}{5} + b\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\left(-\frac{3x^2}{40c^3} - \frac{x^4}{20c}\right) + \frac{1}{5}bx^5\sec^{-1}(cx) - \frac{3b\log\left(x\left(1+\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\right)\right)}{40c^5}$$

input

```
Integrate[x^4*(a + b*ArcSec[c*x]),x]
```

output
```
(a*x^5)/5 + b*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)]*((-3*x^2)/(40*c^3) - x^4/(20*
c)) + (b*x^5*ArcSec[c*x])/5 - (3*b*Log[x*(1 + Sqrt[(-1 + c^2*x^2)/(c^2*x^2
)])])/(40*c^5)
```

3.3.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5743, 798, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+b\sec^{-1}(cx))\,dx$$

$$\downarrow 5743$$

$$\frac{1}{5}x^5(a+b\sec^{-1}(cx)) - \frac{b\int \frac{x^3}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{5c}$$

$$\downarrow 798$$

$$\frac{b\int \frac{x^6}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{10c} + \frac{1}{5}x^5(a+b\sec^{-1}(cx))$$

$$\downarrow 52$$

$$\frac{b\left(\frac{3\int \frac{x^4}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{4c^2} - \frac{1}{2}x^4\sqrt{1-\frac{1}{c^2x^2}}\right)}{10c} + \frac{1}{5}x^5(a+b\sec^{-1}(cx))$$

$$\downarrow 52$$

$$\frac{b\left(\frac{3\left(\frac{\int \frac{x^2}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{2c^2} - x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{4c^2} - \frac{1}{2}x^4\sqrt{1-\frac{1}{c^2x^2}}\right)}{10c} + \frac{1}{5}x^5(a+b\sec^{-1}(cx))$$

$$\downarrow 73$$

$$\frac{b\left(\frac{3\left(x^2\left(-\sqrt{1-\frac{1}{c^2x^2}}\right)-\int\frac{1}{c^2-\frac{c^2}{x^4}}d\sqrt{1-\frac{1}{c^2x^2}}\right)}{4c^2}-\frac{1}{2}x^4\sqrt{1-\frac{1}{c^2x^2}}\right)}{10c}+\frac{1}{5}x^5(a+b\sec^{-1}(cx))$$

$$\downarrow 221$$

$$\frac{1}{5}x^5(a+b\sec^{-1}(cx))+\frac{b\left(\frac{3\left(x^2\left(-\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^2}\right)}{4c^2}-\frac{1}{2}x^4\sqrt{1-\frac{1}{c^2x^2}}\right)}{10c}$$

input
```
Int[x^4*(a + b*ArcSec[c*x]),x]
```

output
```
(x^5*(a + b*ArcSec[c*x]))/5 + (b*(-1/2*(Sqrt[1 - 1/(c^2*x^2)]*x^4) + (3*(-
(Sqrt[1 - 1/(c^2*x^2)]*x^2) - ArcTanh[Sqrt[1 - 1/(c^2*x^2)]]/c^2))/(4*c^2)
))/(10*c)
```

3.3.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 5743
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

3.3.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.58

method	result	size
parts	$\frac{a x^5}{5}+\frac{x^5 b \operatorname{arcsec}(cx)}{5}-\frac{b(c^2x^2-1)x^2}{20c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{3b(c^2x^2-1)}{40c^5\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{3b\sqrt{c^2x^2-1}\,\ln\left(cx+\sqrt{c^2x^2-1}\right)}{40c^6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}$	141
derivativedivides	$\frac{\frac{a c^5x^5}{5}+\frac{b c^5x^5 \operatorname{arcsec}(cx)}{5}-\frac{b\left(c^2x^2-1\right)c^2x^2}{20\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{3b\left(c^2x^2-1\right)}{40\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{3b\sqrt{c^2x^2-1}\,\ln\left(cx+\sqrt{c^2x^2-1}\right)}{40\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{c^5}$	148
default	$\frac{\frac{a c^5x^5}{5}+\frac{b c^5x^5 \operatorname{arcsec}(cx)}{5}-\frac{b\left(c^2x^2-1\right)c^2x^2}{20\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{3b\left(c^2x^2-1\right)}{40\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{3b\sqrt{c^2x^2-1}\,\ln\left(cx+\sqrt{c^2x^2-1}\right)}{40\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{c^5}$	148

input
```
int(x^4*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/5*a*x^5+1/5*x^5*b*arcsec(c*x)-1/20*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^
2)^(1/2)*x^2-3/40*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)-3/40*b/c^6
*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*ln(c*x+(c^2*x^2-1)^(1/2))
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.20

$$\int x^4\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{8\,ac^5x^5+16\,bc^5\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+8\left(bc^5x^5-bc^5\right)\operatorname{arcsec}\left(cx\right)+3\,b\log\left(-cx+\sqrt{c^2x^2-1}\right)-(2}{40\,c^5}$$

input
```
integrate(x^4*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/40*(8*a*c^5*x^5 + 16*b*c^5*arctan(-c*x + sqrt(c^2*x^2 - 1)) + 8*(b*c^5*x
^5 - b*c^5)*arcsec(c*x) + 3*b*log(-c*x + sqrt(c^2*x^2 - 1)) - (2*b*c^3*x^3
 + 3*b*c*x)*sqrt(c^2*x^2 - 1))/c^5
```

3.3.6 Sympy [A] (verification not implemented)

Time = 3.42 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.97

$$\int x^4\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{ax^5}{5}+\frac{bx^5\,\text{asec}\,(cx)}{5}$$
$$-\frac{b\left(\begin{cases}\frac{cx^5}{4\sqrt{c^2x^2-1}}+\frac{x^3}{8c\sqrt{c^2x^2-1}}-\frac{3x}{8c^3\sqrt{c^2x^2-1}}+\frac{3\,\text{acosh}\,(cx)}{8c^4} & \text{for } |c^2x^2|>1\\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}}-\frac{ix^3}{8c\sqrt{-c^2x^2+1}}+\frac{3ix}{8c^3\sqrt{-c^2x^2+1}}-\frac{3i\,\text{asin}\,(cx)}{8c^4} & \text{otherwise}\end{cases}\right)}{5c}$$

input
```
integrate(x**4*(a+b*asec(c*x)),x)
```

output
```
a*x**5/5 + b*x**5*asec(c*x)/5 - b*Piecewise((c*x**5/(4*sqrt(c**2*x**2 - 1)
) + x**3/(8*c*sqrt(c**2*x**2 - 1)) - 3*x/(8*c**3*sqrt(c**2*x**2 - 1)) + 3*
acosh(c*x)/(8*c**4), Abs(c**2*x**2) > 1), (-I*c*x**5/(4*sqrt(-c**2*x**2 +
1)) - I*x**3/(8*c*sqrt(-c**2*x**2 + 1)) + 3*I*x/(8*c**3*sqrt(-c**2*x**2 +
1)) - 3*I*asin(c*x)/(8*c**4), True))/(5*c)
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.47

$$\int x^4\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{1}{5}ax^5$$
$$+\frac{1}{80}\left(16\,x^5\,\text{arcsec}\,(cx)+\frac{\frac{2\left(3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2c^4\left(\frac{1}{c^2x^2}-1\right)+c^4}-\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)b$$

input
```
integrate(x^4*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*x^5 + 1/80*(16*x^5*arcsec(c*x) + (2*(3*(-1/(c^2*x^2) + 1)^(3/2) - 5*
sqrt(-1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1)
 + c^4) - 3*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(-1/(c^2*x^2)
+ 1) - 1)/c^4)/c)*b
```

3.3.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4828 vs. 2(75) = 150.

Time = 0.79 (sec) , antiderivative size = 4828, normalized size of antiderivative = 54.25

$$\int x^4(a+b\sec^{-1}(cx))\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
1/40*c*(8*b*arccos(1/(c*x))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3
/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2
*x^2) - 1)^5/(1/(c*x) + 1)^10) - 3*b*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c
*x) + 1))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*
x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 +
 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c
*x) + 1)^10) + 3*b*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(c^6 + 5
*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*
x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2
) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10) + 8*a
/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)
^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1
/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^
10) - 40*b*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^6 + 5*c^6*(1/(c^2*x^2) -
1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(
1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) +
1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^2) - 15*b*(
1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^6 + 5*
c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(...
```

3.3.9 Mupad [F(-1)]

Timed out.

$$\int x^4(a+b\sec^{-1}(cx))\,dx = \int x^4\left(a+b\,\text{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^4*(a + b*acos(1/(c*x))),x)
```

output
```
int(x^4*(a + b*acos(1/(c*x))), x)
```

3.4 $\int x^3(a+b\sec^{-1}(cx))\,dx$

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3.4.1 Optimal result

Integrand size = 12, antiderivative size = 64

$$\int x^3(a+b\sec^{-1}(cx))\,dx = -\frac{b\sqrt{1-\frac{1}{c^2x^2}}x}{6c^3} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^3}{12c} + \frac{1}{4}x^4(a+b\sec^{-1}(cx))$$

output
```
1/4*x^4*(a+b*arcsec(c*x))-1/6*b*x*(1-1/c^2/x^2)^(1/2)/c^3-1/12*b*x^3*(1-1/
c^2/x^2)^(1/2)/c
```

3.4.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int x^3(a+b\sec^{-1}(cx))\,dx = \frac{ax^4}{4} + b\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\left(-\frac{x}{6c^3} - \frac{x^3}{12c}\right) + \frac{1}{4}bx^4\sec^{-1}(cx)$$

input
```
Integrate[x^3*(a + b*ArcSec[c*x]),x]
```

output
```
(a*x^4)/4 + b*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)]*(-1/6*x/c^3 - x^3/(12*c)) + (
b*x^4*ArcSec[c*x])/4
```

3.4.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5743, 803, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+b\sec^{-1}(cx))\,dx$$

$$\downarrow 5743$$

$$\frac{1}{4}x^4(a+b\sec^{-1}(cx)) - \frac{b\int \frac{x^2}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{4c}$$

$$\downarrow 803$$

$$\frac{1}{4}x^4(a+b\sec^{-1}(cx)) - \frac{b\left(\frac{2\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{3c^2} + \frac{1}{3}x^3\sqrt{1-\frac{1}{c^2x^2}}\right)}{4c}$$

$$\downarrow 746$$

$$\frac{1}{4}x^4(a+b\sec^{-1}(cx)) - \frac{b\left(\frac{2x\sqrt{1-\frac{1}{c^2x^2}}}{3c^2} + \frac{1}{3}x^3\sqrt{1-\frac{1}{c^2x^2}}\right)}{4c}$$

input
```
Int[x^3*(a + b*ArcSec[c*x]),x]
```

output
```
-1/4*(b*((2*Sqrt[1 - 1/(c^2*x^2)]*x)/(3*c^2) + (Sqrt[1 - 1/(c^2*x^2)]*x^3)
/3))/c + (x^4*(a + b*ArcSec[c*x]))/4
```

3.4.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 5743
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

3.4.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

method	result	size
parts	$\frac{x^4 a}{4}+\frac{b\left(\frac{c^4 x^4 \operatorname{arcsec}(cx)}{4}-\frac{\left(c^2x^2-1\right)\left(c^2x^2+2\right)}{12\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^4}$	70
derivativedivides	$\frac{\frac{a\,c^4x^4}{4}+b\left(\frac{c^4x^4\operatorname{arcsec}(cx)}{4}-\frac{\left(c^2x^2-1\right)\left(c^2x^2+2\right)}{12\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^4}$	74
default	$\frac{\frac{a\,c^4x^4}{4}+b\left(\frac{c^4x^4\operatorname{arcsec}(cx)}{4}-\frac{\left(c^2x^2-1\right)\left(c^2x^2+2\right)}{12\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^4}$	74

input
```
int(x^3*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*a+b/c^4*(1/4*c^4*x^4*arcsec(c*x)-1/12*(c^2*x^2-1)*(c^2*x^2+2)/((c^
2*x^2-1)/c^2/x^2)^(1/2)/c/x)
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int x^3\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{3\,bc^4x^4\,\mathrm{arcsec}\,(cx)+3\,ac^4x^4-(bc^2x^2+2\,b)\sqrt{c^2x^2-1}}{12\,c^4}$$

input
```
integrate(x^3*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/12*(3*b*c^4*x^4*arcsec(c*x) + 3*a*c^4*x^4 - (b*c^2*x^2 + 2*b)*sqrt(c^2*x
^2 - 1))/c^4
```

3.4.6 Sympy [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.67

$$\int x^3\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{ax^4}{4}+\frac{bx^4\,\mathrm{asec}\,(cx)}{4} - \frac{b\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{4c}$$

input
```
integrate(x**3*(a+b*asec(c*x)),x)
```

output
```
a*x**4/4 + b*x**4*asec(c*x)/4 - b*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c
) + 2*sqrt(c**2*x**2 - 1)/(3*c**3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**
2*x**2 + 1)/(3*c) + 2*I*sqrt(-c**2*x**2 + 1)/(3*c**3), True))/(4*c)
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

$$\int x^3\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{1}{4}\,ax^4 + \frac{1}{12}\left(3\,x^4\,\mathrm{arcsec}\,(cx)-\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)b$$

input
```
integrate(x^3*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/4*a*x^4 + 1/12*(3*x^4*arcsec(c*x) - (c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) +
3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b
```

3.4.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1926 vs. $2(54) = 108$.

Time $= 0.31$ (sec) , antiderivative size $= 1926$, normalized size of antiderivative $= 30.09$

$$\int x^3(a + b\sec^{-1}(cx))\, dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
1/12*c*(3*b*arccos(1/(c*x))/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(
1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) + 3*a/(c^5 + 4*c
^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x)
+ 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)
^4/(1/(c*x) + 1)^8) - 12*b*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^5 + 4*c^5
*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) +
1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4
/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) - 6*b*sqrt(-1/(c^2*x^2) + 1)/((c^5 + 4*
c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x)
 + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1
)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)) - 12*a*(1/(c^2*x^2) - 1)/((c^5 + 4*c^5
*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) +
1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4
/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) + 18*b*(1/(c^2*x^2) - 1)^2*arccos(1/(c*
x))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) -
 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1
/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) + 10*b*(-1/(c^2*x^2) +
 1)^(3/2)/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*
x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6...
```

3.4.9 Mupad [F(-1)]

Timed out.

$$\int x^3(a+b\sec^{-1}(cx))\,dx = \int x^3\left(a+b\,\text{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^3*(a + b*acos(1/(c*x))),x)
```

output
```
int(x^3*(a + b*acos(1/(c*x))), x)
```

3.5 $\int x^2(a+b\sec^{-1}(cx))\,dx$

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3.5.1 Optimal result

Integrand size = 12, antiderivative size = 64

$$\int x^2(a+b\sec^{-1}(cx))\,dx = -\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^2}{6c}+\frac{1}{3}x^3(a+b\sec^{-1}(cx))-\frac{b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^3}$$

output
```
1/3*x^3*(a+b*arcsec(c*x))-1/6*b*arctanh((1-1/c^2/x^2)^(1/2))/c^3-1/6*b*x^2
*(1-1/c^2/x^2)^(1/2)/c
```

3.5.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.33

$$\int x^2(a+b\sec^{-1}(cx))\,dx = \frac{ax^3}{3}-\frac{bx^2\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{6c}+\frac{1}{3}bx^3\sec^{-1}(cx) - \frac{b\log\left(x\left(1+\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\right)\right)}{6c^3}$$

input
```
Integrate[x^2*(a + b*ArcSec[c*x]),x]
```

output
```
(a*x^3)/3 - (b*x^2*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(6*c) + (b*x^3*ArcSec[c
*x])/3 - (b*Log[x*(1 + Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])])/(6*c^3)
```

3.5.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5743, 798, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\sec^{-1}(cx))\,dx$$

$$\downarrow 5743$$

$$\frac{1}{3}x^3(a+b\sec^{-1}(cx)) - \frac{b\int \frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{3c}$$

$$\downarrow 798$$

$$\frac{b\int \frac{x^4}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{6c} + \frac{1}{3}x^3(a+b\sec^{-1}(cx))$$

$$\downarrow 52$$

$$\frac{b\left(\frac{\int \frac{x^2}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{2c^2} - x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c} + \frac{1}{3}x^3(a+b\sec^{-1}(cx))$$

$$\downarrow 73$$

$$\frac{b\left(x^2\left(-\sqrt{1-\frac{1}{c^2x^2}}\right) - \int \frac{1}{c^2-\frac{c^2}{x^4}}d\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c} + \frac{1}{3}x^3(a+b\sec^{-1}(cx))$$

$$\downarrow 221$$

$$\frac{1}{3}x^3(a+b\sec^{-1}(cx)) + \frac{b\left(x^2\left(-\sqrt{1-\frac{1}{c^2x^2}}\right) - \frac{\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^2}\right)}{6c}$$

input
```
Int[x^2*(a + b*ArcSec[c*x]),x]
```

output
```
(x^3*(a + b*ArcSec[c*x]))/3 + (b*(-(Sqrt[1 - 1/(c^2*x^2)]*x^2) - ArcTanh[S
qrt[1 - 1/(c^2*x^2)]]/c^2))/(6*c)
```

3.5.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 5743
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

3.5.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.47

method	result	size
parts	$\frac{x^3 a}{3}+\frac{b\left(\frac{c^3x^3\operatorname{arcsec}(cx)}{3}-\frac{\sqrt{c^2x^2-1}\left(cx\sqrt{c^2x^2-1}+\ln\left(cx+\sqrt{c^2x^2-1}\right)\right)}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^3}$	94
derivativedivides	$\frac{\frac{a\,c^3x^3}{3}+b\left(\frac{c^3x^3\operatorname{arcsec}(cx)}{3}-\frac{\sqrt{c^2x^2-1}\left(cx\sqrt{c^2x^2-1}+\ln\left(cx+\sqrt{c^2x^2-1}\right)\right)}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^3}$	98
default	$\frac{\frac{a\,c^3x^3}{3}+b\left(\frac{c^3x^3\operatorname{arcsec}(cx)}{3}-\frac{\sqrt{c^2x^2-1}\left(cx\sqrt{c^2x^2-1}+\ln\left(cx+\sqrt{c^2x^2-1}\right)\right)}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^3}$	98

input
```
int(x^2*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*a+b/c^3*(1/3*c^3*x^3*arcsec(c*x)-1/6*(c^2*x^2-1)^(1/2)*(c*x*(c^2*x
^2-1)^(1/2)+ln(c*x+(c^2*x^2-1)^(1/2)))/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x)
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.47

$$\int x^2\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{2\,ac^3x^3+4\,bc^3\arctan\left(-cx+\sqrt{c^2x^2-1}\right)-\sqrt{c^2x^2-1}bcx+2\left(bc^3x^3-bc^3\right)\operatorname{arcsec}\left(cx\right)+b\log\left(-cx+\sqrt{}\right.}{6\,c^3}$$

input
```
integrate(x^2*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/6*(2*a*c^3*x^3 + 4*b*c^3*arctan(-c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2
 - 1)*b*c*x + 2*(b*c^3*x^3 - b*c^3)*arcsec(c*x) + b*log(-c*x + sqrt(c^2*x^
2 - 1)))/c^3
```

3.5.6 Sympy [A] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.67

$$\int x^2\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{ax^3}{3}+\frac{bx^3\,\mathrm{asec}\,(cx)}{3} - \frac{b\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\mathrm{acosh}\,(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\,\mathrm{asin}\,(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}$$

input
```
integrate(x**2*(a+b*asec(c*x)),x)
```

output
```
a*x**3/3 + b*x**3*asec(c*x)/3 - b*Piecewise((x*sqrt(c**2*x**2 - 1)/(2*c) +
 acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-I*c*x**3/(2*sqrt(-c**2*x**2 +
 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin(c*x)/(2*c**2), True))/(3*c)
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.53

$$\int x^2\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{1}{3}ax^3+\frac{1}{12}\left(4\,x^3\,\mathrm{arcsec}\,(cx)-\frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}-\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)b$$

input
```
integrate(x^2*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*x^3 + 1/12*(4*x^3*arcsec(c*x) - (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c
^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c
^2*x^2) + 1) - 1)/c^2)/c)*b
```

3.5.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2101 vs. 2(54) = 108.

Time = 0.59 (sec) , antiderivative size = 2101, normalized size of antiderivative = 32.83

$$\int x^2(a+b\sec^{-1}(cx))\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
1/6*c*(2*b*arccos(1/(c*x))/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
+ 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(
c*x) + 1)^6) - b*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(c^4 + 3*c
^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x)
+ 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6) + b*log(abs(sqrt(-1/(c^2
*x^2) + 1) - 1/(c*x) - 1))/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
+ 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(
c*x) + 1)^6) + 2*a/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*
(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1
)^6) - 6*b*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^4 + 3*c^4*(1/(c^2*x^2) -
1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c
^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) - 3*b*(1/(c^2*x^2) - 1)*l
og(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^4 + 3*c^4*(1/(c^2*x^2) -
 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(
c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) + 3*b*(1/(c^2*x^2) - 1)*
log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^4 + 3*c^4*(1/(c^2*x^2)
- 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/
(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) - 2*b*sqrt(-1/(c^2*x^2)
 + 1)/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2)
 - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(...
```

3.5.9 Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\sec^{-1}(cx))\,dx = \int x^2\left(a+b\operatorname{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^2*(a + b*acos(1/(c*x))),x)
```

output
```
int(x^2*(a + b*acos(1/(c*x))), x)
```

3.6 $\int x(a+b\sec^{-1}(cx))\,dx$

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3.6.1 Optimal result

Integrand size = 10, antiderivative size = 39

$$\int x(a+b\sec^{-1}(cx))\,dx = -\frac{b\sqrt{1-\frac{1}{c^2x^2}}x}{2c} + \frac{1}{2}x^2(a+b\sec^{-1}(cx))$$

output
```
1/2*x^2*(a+b*arcsec(c*x))-1/2*b*x*(1-1/c^2/x^2)^(1/2)/c
```

3.6.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.28

$$\int x(a+b\sec^{-1}(cx))\,dx = \frac{ax^2}{2} - \frac{bx\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{2c} + \frac{1}{2}bx^2\sec^{-1}(cx)$$

input
```
Integrate[x*(a + b*ArcSec[c*x]),x]
```

output
```
(a*x^2)/2 - (b*x*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(2*c) + (b*x^2*ArcSec[c*x
])/2
```

3.6.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5743, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\downarrow 5743$$

$$\frac{1}{2}x^2\left(a+b\sec^{-1}(cx)\right)-\frac{b\int\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{2c}$$

$$\downarrow 746$$

$$\frac{1}{2}x^2\left(a+b\sec^{-1}(cx)\right)-\frac{bx\sqrt{1-\frac{1}{c^2x^2}}}{2c}$$

input
```
Int[x*(a + b*ArcSec[c*x]),x]
```

output
```
-1/2*(b*Sqrt[1 - 1/(c^2*x^2)]*x)/c + (x^2*(a + b*ArcSec[c*x]))/2
```

3.6.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 5743
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

3.6.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.56

method	result	size
parts	$\frac{a x^2}{2} + \frac{b\left(\frac{c^2 x^2 \operatorname{arcsec}(cx)}{2} - \frac{c^2x^2-1}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^2}$	61
derivativedivides	$\frac{\frac{a\,c^2x^2}{2} + b\left(\frac{c^2 x^2 \operatorname{arcsec}(cx)}{2} - \frac{c^2x^2-1}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^2}$	65
default	$\frac{\frac{a\,c^2x^2}{2} + b\left(\frac{c^2 x^2 \operatorname{arcsec}(cx)}{2} - \frac{c^2x^2-1}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^2}$	65

input
```
int(x*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+b/c^2*(1/2*c^2*x^2*arcsec(c*x)-1/2/((c^2*x^2-1)/c^2/x^2)^(1/2)/c
/x*(c^2*x^2-1))
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int x\left(a + b\sec^{-1}(cx)\right)\,dx = \frac{bc^2x^2\operatorname{arcsec}(cx) + ac^2x^2 - \sqrt{c^2x^2-1}b}{2\,c^2}$$

input
```
integrate(x*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/2*(b*c^2*x^2*arcsec(c*x) + a*c^2*x^2 - sqrt(c^2*x^2 - 1)*b)/c^2
```

3.6.6 Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.49

$$\int x\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{ax^2}{2}+\frac{bx^2\operatorname{asec}(cx)}{2}-\frac{b\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c}$$

input
```
integrate(x*(a+b*asec(c*x)),x)
```

output
```
a*x**2/2 + b*x**2*asec(c*x)/2 - b*Piecewise((sqrt(c**2*x**2 - 1)/c, Abs(c*
**2*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, True))/(2*c)
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int x\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{1}{2}ax^2+\frac{1}{2}\left(x^2\operatorname{arcsec}(cx)-\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)b$$

input
```
integrate(x*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/2*a*x^2 + 1/2*(x^2*arcsec(c*x) - x*sqrt(-1/(c^2*x^2) + 1)/c)*b
```

3.6.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 634 vs. $2(33)=66$.

Time = 0.29 (sec) , antiderivative size = 634, normalized size of antiderivative = 16.26

$$\int x\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$=\frac{1}{2}c\left(\frac{b\arccos\left(\frac{1}{cx}\right)}{c^3+\frac{2c^3\left(\frac{1}{c^2x^2}-1\right)}{\left(\frac{1}{cx}+1\right)^2}+\frac{c^3\left(\frac{1}{c^2x^2}-1\right)^2}{\left(\frac{1}{cx}+1\right)^4}}+\frac{a}{c^3+\frac{2c^3\left(\frac{1}{c^2x^2}-1\right)}{\left(\frac{1}{cx}+1\right)^2}+\frac{c^3\left(\frac{1}{c^2x^2}-1\right)^2}{\left(\frac{1}{cx}+1\right)^4}}-\frac{2b\left(\frac{1}{c^2x^2}-1\right)\arccos\left(\frac{1}{cx}\right)}{\left(c^3+\frac{2c^3\left(\frac{1}{c^2x^2}-1\right)}{\left(\frac{1}{cx}+1\right)^2}+\frac{c^3\left(\frac{1}{c^2x^2}-1\right)^2}{\left(\frac{1}{cx}+1\right)^4}\right)}\right.$$

input
```
integrate(x*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
1/2*c*(b*arccos(1/(c*x))/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 +
c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) + a/(c^3 + 2*c^3*(1/(c^2*x^2) - 1
)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) - 2*b*(1/(c^2
*x^2) - 1)*arccos(1/(c*x))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) - 2*b*sqrt(-1
/(c^2*x^2) + 1)/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(
c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)) - 2*a*(1/(c^2*x^2) - 1)/((
c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1
/(c*x) + 1)^4)*(1/(c*x) + 1)^2) + b*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((
c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1
/(c*x) + 1)^4)*(1/(c*x) + 1)^4) + 2*b*(-1/(c^2*x^2) + 1)^(3/2)/((c^3 + 2*c
^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) +
1)^4)*(1/(c*x) + 1)^3) + a*(1/(c^2*x^2) - 1)^2/((c^3 + 2*c^3*(1/(c^2*x^2)
- 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) +
 1)^4))
```

3.6.9 Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int x\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{a\,x^2}{2}+\frac{b\,x^2\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{2}-\frac{b\,x\,\sqrt{1-\frac{1}{c^2\,x^2}}}{2\,c}$$

input
```
int(x*(a + b*acos(1/(c*x))),x)
```

output
```
(a*x^2)/2 + (b*x^2*acos(1/(c*x)))/2 - (b*x*(1 - 1/(c^2*x^2))^(1/2))/(2*c)
```

3.7 $\int (a + b \sec^{-1}(cx))\, dx$

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3.7.1 Optimal result

Integrand size = 8, antiderivative size = 32

$$\int (a + b \sec^{-1}(cx))\, dx = ax + bx \sec^{-1}(cx) - \frac{b \operatorname{arctanh}\left(\sqrt{1 - \frac{1}{c^2 x^2}}\right)}{c}$$

output
```
a*x+b*x*arcsec(c*x)-b*arctanh((1-1/c^2/x^2)^(1/2))/c
```

3.7.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.84

$$\int (a + b \sec^{-1}(cx))\, dx = ax + bx \sec^{-1}(cx) - \frac{b\sqrt{1 - \frac{1}{c^2 x^2}}\, x \operatorname{arctanh}\left(\frac{cx}{\sqrt{-1 + c^2 x^2}}\right)}{\sqrt{-1 + c^2 x^2}}$$

input
```
Integrate[a + b*ArcSec[c*x],x]
```

output
```
a*x + b*x*ArcSec[c*x] - (b*Sqrt[1 - 1/(c^2*x^2)]*x*ArcTanh[(c*x)/Sqrt[-1 +
 c^2*x^2]])/Sqrt[-1 + c^2*x^2]
```

3.7.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b \sec^{-1}(cx))\, dx$$

$$\downarrow \text{2009}$$

$$ax - \frac{b \operatorname{arctanh}\left(\sqrt{1 - \frac{1}{c^2 x^2}}\right)}{c} + bx \sec^{-1}(cx)$$

input
```
Int[a + b*ArcSec[c*x],x]
```

output
```
a*x + b*x*ArcSec[c*x] - (b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/c
```

3.7.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.7.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

method	result	size
default	$ax + bx \operatorname{arcsec}(cx) - \frac{b \ln\left(cx + cx\sqrt{1 - \frac{1}{c^2 x^2}}\right)}{c}$	38
parts	$ax + bx \operatorname{arcsec}(cx) - \frac{b \ln\left(cx + cx\sqrt{1 - \frac{1}{c^2 x^2}}\right)}{c}$	38
derivativedivides	$\frac{acx + b\left(cx \operatorname{arcsec}(cx) - \ln\left(cx + cx\sqrt{1 - \frac{1}{c^2 x^2}}\right)\right)}{c}$	42

input
```
int(a+b*arcsec(c*x),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*x*arcsec(c*x)-b/c*ln(c*x+c*x*(1-1/c^2/x^2)^(1/2))
```

3.7.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(30) = 60$.

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.97

$$\int \left(a + b\sec^{-1}(cx)\right)\, dx$$
$$= \frac{acx + 2\,bc\arctan\left(-cx + \sqrt{c^2x^2 - 1}\right) + (bcx - bc)\operatorname{arcsec}(cx) + b\log\left(-cx + \sqrt{c^2x^2 - 1}\right)}{c}$$

input
```
integrate(a+b*arcsec(c*x),x, algorithm="fricas")
```

output
```
(a*c*x + 2*b*c*arctan(-c*x + sqrt(c^2*x^2 - 1)) + (b*c*x - b*c)*arcsec(c*x
) + b*log(-c*x + sqrt(c^2*x^2 - 1)))/c
```

3.7.6 Sympy [A] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \left(a + b\sec^{-1}(cx)\right)\, dx = ax + b\left(x\operatorname{asec}(cx) - \frac{\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2| > 1 \\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}}{c}\right)$$

input
```
integrate(a+b*asec(c*x),x)
```

output
```
a*x + b*(x*asec(c*x) - Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asi
n(c*x), True))/c)
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.66

$$\int \left(a + b \sec^{-1}(cx)\right)\, dx$$
$$= ax + \frac{\left(2\, cx \operatorname{arcsec}(cx) - \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) + \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)b}{2\, c}$$

input
```
integrate(a+b*arcsec(c*x),x, algorithm="maxima")
```

output
```
a*x + 1/2*(2*c*x*arcsec(c*x) - log(sqrt(-1/(c^2*x^2) + 1) + 1) + log(-sqrt
(-1/(c^2*x^2) + 1) + 1))*b/c
```

3.7.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(30) = 60.

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.97

$$\int \left(a + b \sec^{-1}(cx)\right)\, dx$$
$$= \frac{1}{2}\, bc \left(\frac{2\, x \arccos\left(\frac{1}{cx}\right)}{c} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) - \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} \right) + ax$$

input
```
integrate(a+b*arcsec(c*x),x, algorithm="giac")
```

output
```
1/2*b*c*(2*x*arccos(1/(c*x))/c - (log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-s
qrt(-1/(c^2*x^2) + 1) + 1))/c^2) + a*x
```

3.7.9 Mupad [B] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \left(a + b \sec^{-1}(cx)\right)\, dx = a\, x + b\, x \operatorname{acos}\left(\frac{1}{c\, x}\right) - \frac{b \operatorname{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{c^2\, x^2}}}\right)}{c}$$

input
```
int(a + b*acos(1/(c*x)),x)
```

output
```
a*x + b*x*acos(1/(c*x)) - (b*atanh(1/(1 - 1/(c^2*x^2))^(1/2)))/c
```

3.8 $\int \frac{a+b\sec^{-1}(cx)}{x}\,dx$

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3.8.1 Optimal result

Integrand size = 12, antiderivative size = 64

$$\int \frac{a+b\sec^{-1}(cx)}{x}\,dx = \frac{i(a+b\sec^{-1}(cx))^2}{2b} - (a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right) + \frac{1}{2}ib\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)$$

output
```
1/2*I*(a+b*arcsec(c*x))^2/b-(a+b*arcsec(c*x))*ln(1+(1/c/x+I*(1-1/c^2/x^2)^
(1/2))^2)+1/2*I*b*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)
```

3.8.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

$$\int \frac{a+b\sec^{-1}(cx)}{x}\,dx = \frac{1}{2}ib\sec^{-1}(cx)^2 - b\sec^{-1}(cx)\log\left(1+e^{2i\sec^{-1}(cx)}\right) + a\log(x) + \frac{1}{2}ib\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)$$

input
```
Integrate[(a + b*ArcSec[c*x])/x,x]
```

output
```
(I/2)*b*ArcSec[c*x]^2 - b*ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] + a*L
og[x] + (I/2)*b*PolyLog[2, -E^((2*I)*ArcSec[c*x])]
```

3.8.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.36, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5741, 5137, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \sec^{-1}(cx)}{x}\, dx$$

$$\downarrow 5741$$

$$-\int x\left(a + b \arccos\left(\frac{1}{cx}\right)\right) d\frac{1}{x}$$

$$\downarrow 5137$$

$$\int cx\sqrt{1 - \frac{1}{c^2x^2}}\left(a + b \arccos\left(\frac{1}{cx}\right)\right) d \arccos\left(\frac{1}{cx}\right)$$

$$\downarrow 3042$$

$$\int \tan\left(\arccos\left(\frac{1}{cx}\right)\right)\left(a + b \arccos\left(\frac{1}{cx}\right)\right) d \arccos\left(\frac{1}{cx}\right)$$

$$\downarrow 4202$$

$$\frac{i\left(a + b \arccos\left(\frac{1}{cx}\right)\right)^2}{2b} - 2i \int \frac{e^{2i \arccos\left(\frac{1}{cx}\right)}\left(a + b \arccos\left(\frac{1}{cx}\right)\right)}{1 + e^{2i \arccos\left(\frac{1}{cx}\right)}} d \arccos\left(\frac{1}{cx}\right)$$

$$\downarrow 2620$$

$$\frac{i\left(a + b \arccos\left(\frac{1}{cx}\right)\right)^2}{2b} -$$
$$2i\left(\frac{1}{2}ib \int \log\left(1 + e^{2i \arccos\left(\frac{1}{cx}\right)}\right) d \arccos\left(\frac{1}{cx}\right) - \frac{1}{2}i \log\left(1 + e^{2i \arccos\left(\frac{1}{cx}\right)}\right)\left(a + b \arccos\left(\frac{1}{cx}\right)\right)\right)$$

$$\downarrow 2715$$

$$\frac{i\left(a + b \arccos\left(\frac{1}{cx}\right)\right)^2}{2b} -$$
$$2i\left(\frac{1}{4}b \int x \log\left(1 + e^{2i \arccos\left(\frac{1}{cx}\right)}\right) de^{2i \arccos\left(\frac{1}{cx}\right)} - \frac{1}{2}i \log\left(1 + e^{2i \arccos\left(\frac{1}{cx}\right)}\right)\left(a + b \arccos\left(\frac{1}{cx}\right)\right)\right)$$

$$\downarrow 2838$$

$$\frac{i\left(a+b\arccos\left(\frac{1}{cx}\right)\right)^2}{2b} - 2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos\left(\frac{1}{cx}\right)}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right) - \frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2i\arccos\left(\frac{1}{cx}\right)}\right)\right)$$

input
```
Int[(a + b*ArcSec[c*x])/x,x]
```

output
```
((I/2)*(a + b*ArcCos[1/(c*x)])^2)/b - (2*I)*((-1/2*I)*(a + b*ArcCos[1/(c*x
)])*Log[1 + E^((2*I)*ArcCos[1/(c*x)])] - (b*PolyLog[2, -E^((2*I)*ArcCos[1/
(c*x)])])/4)
```

3.8.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))  Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])  Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I  Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5741
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a + b
*ArcCos[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

3.8.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.31

method	result
parts	$a\ln(x)+b\left(\frac{i\,\mathrm{arcsec}(cx)^2}{2}-\mathrm{arcsec}(cx)\ln\left(1+\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2\right)+\frac{i\,\mathrm{polylog}\left(2,-\left(\frac{1}{cx}+i\sqrt{}\right.\right.\right.$
derivativedivides	$a\ln(cx)+b\left(\frac{i\,\mathrm{arcsec}(cx)^2}{2}-\mathrm{arcsec}(cx)\ln\left(1+\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2\right)+\frac{i\,\mathrm{polylog}\left(2,-\left(\frac{1}{cx}+i\right.\right.\right.$
default	$a\ln(cx)+b\left(\frac{i\,\mathrm{arcsec}(cx)^2}{2}-\mathrm{arcsec}(cx)\ln\left(1+\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2\right)+\frac{i\,\mathrm{polylog}\left(2,-\left(\frac{1}{cx}+i\right.\right.\right.$

input
```
int((a+b*arcsec(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
a*ln(x)+b*(1/2*I*arcsec(c*x)^2-arcsec(c*x)*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/
2))^2)+1/2*I*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2))
```

3.8.5 Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x}\,dx=\int \frac{b\,\mathrm{arcsec}(cx)+a}{x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x,x, algorithm="fricas")
```

output
```
integral((b*arcsec(c*x) + a)/x, x)
```

3.8.6 Sympy [F]

$$\int \frac{a + b \sec^{-1}(cx)}{x}\, dx = \int \frac{a + b \operatorname{asec}(cx)}{x}\, dx$$

input
```
integrate((a+b*asec(c*x))/x,x)
```

output
```
Integral((a + b*asec(c*x))/x, x)
```

3.8.7 Maxima [F]

$$\int \frac{a + b \sec^{-1}(cx)}{x}\, dx = \int \frac{b \operatorname{arcsec}(cx) + a}{x}\, dx$$

input
```
integrate((a+b*arcsec(c*x))/x,x, algorithm="maxima")
```

output
```
-(c^2*integrate(sqrt(c*x + 1)*sqrt(c*x - 1)*log(x)/(c^4*x^3 - c^2*x), x) -
 arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(x))*b + a*log(x)
```

3.8.8 Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \sec^{-1}(cx)}{x}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsec(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Limit: Max order reached or unable to make series expan
sion Error: Bad Argument Value
```

3.8.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \sec^{-1}(cx)}{x}\, dx = \int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{x}\, dx$$

input
```
int((a + b*acos(1/(c*x)))/x,x)
```

output
```
int((a + b*acos(1/(c*x)))/x, x)
```

3.9 $\int \frac{a+b\sec^{-1}(cx)}{x^2}\,dx$

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3.9.1 Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \frac{a+b\sec^{-1}(cx)}{x^2}\,dx = bc\sqrt{1-\frac{1}{c^2x^2}} - \frac{a+b\sec^{-1}(cx)}{x}$$

output
```
(-a-b*arcsec(c*x))/x+b*c*(1-1/c^2/x^2)^(1/2)
```

3.9.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{a+b\sec^{-1}(cx)}{x^2}\,dx = -\frac{a}{x} + bc\sqrt{\frac{-1+c^2x^2}{c^2x^2}} - \frac{b\sec^{-1}(cx)}{x}$$

input
```
Integrate[(a + b*ArcSec[c*x])/x^2,x]
```

output
```
-(a/x) + b*c*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)] - (b*ArcSec[c*x])/x
```

3.9.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5743, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b\sec^{-1}(cx)}{x^2}\,dx$$

$$\downarrow 5743$$

$$\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^3}dx}{c} - \frac{a + b\sec^{-1}(cx)}{x}$$

$$\downarrow 793$$

$$bc\sqrt{1-\frac{1}{c^2x^2}} - \frac{a + b\sec^{-1}(cx)}{x}$$

input
```
Int[(a + b*ArcSec[c*x])/x^2,x]
```

output
```
b*c*Sqrt[1 - 1/(c^2*x^2)] - (a + b*ArcSec[c*x])/x
```

3.9.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 5743
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

3.9.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.87

method	result	size
parts	$-\frac{a}{x}+bc\left(-\frac{\operatorname{arcsec}(cx)}{cx}+\frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x^2}\right)$	58
derivativedivides	$c\left(-\frac{a}{cx}+b\left(-\frac{\operatorname{arcsec}(cx)}{cx}+\frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x^2}\right)\right)$	62
default	$c\left(-\frac{a}{cx}+b\left(-\frac{\operatorname{arcsec}(cx)}{cx}+\frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x^2}\right)\right)$	62

input
```
int((a+b*arcsec(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/x+b*c*(-1/c/x*arcsec(c*x)+1/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^2/x^2*(c^2*x^
2-1))
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{a+b\sec^{-1}(cx)}{x^2}\,dx=-\frac{b\operatorname{arcsec}(cx)-\sqrt{c^2x^2-1}b+a}{x}$$

input
```
integrate((a+b*arcsec(c*x))/x^2,x, algorithm="fricas")
```

output
```
-(b*arcsec(c*x) - sqrt(c^2*x^2 - 1)*b + a)/x
```

3.9.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{a+b\sec^{-1}(cx)}{x^2}\,dx=\begin{cases}-\frac{a}{x}+bc\sqrt{1-\frac{1}{c^2x^2}}-\frac{b\operatorname{asec}(cx)}{x} & \text{for } c\neq 0\\ -\frac{a+\tilde{\infty}b}{x} & \text{otherwise}\end{cases}$$

input
```
integrate((a+b*asec(c*x))/x**2,x)
```

output
```
Piecewise((-a/x + b*c*sqrt(1 - 1/(c**2*x**2)) - b*asec(c*x)/x, Ne(c, 0)),
(-(a + zoo*b)/x, True))
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{a+b\sec^{-1}(cx)}{x^2}\,dx = \left(c\sqrt{-\frac{1}{c^2x^2}+1} - \frac{\operatorname{arcsec}(cx)}{x}\right)b - \frac{a}{x}$$

input
```
integrate((a+b*arcsec(c*x))/x^2,x, algorithm="maxima")
```

output
```
(c*sqrt(-1/(c^2*x^2) + 1) - arcsec(c*x)/x)*b - a/x
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.39

$$\int \frac{a+b\sec^{-1}(cx)}{x^2}\,dx = \left(b\sqrt{-\frac{1}{c^2x^2}+1} - \frac{b\arccos\left(\frac{1}{cx}\right)}{cx} - \frac{a}{cx}\right)c$$

input
```
integrate((a+b*arcsec(c*x))/x^2,x, algorithm="giac")
```

output
```
(b*sqrt(-1/(c^2*x^2) + 1) - b*arccos(1/(c*x))/(c*x) - a/(c*x))*c
```

3.9.9 Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{a+b\sec^{-1}(cx)}{x^2}\,dx = b\,c\,\sqrt{1-\frac{1}{c^2\,x^2}} - \frac{a}{x} - \frac{b\,\operatorname{acos}\left(\frac{1}{c\,x}\right)}{x}$$

input
```
int((a + b*acos(1/(c*x)))/x^2,x)
```

output
```
b*c*(1 - 1/(c^2*x^2))^(1/2) - a/x - (b*acos(1/(c*x)))/x
```

3.10 $\int \frac{a+b\sec^{-1}(cx)}{x^3}\,dx$

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3.10.1 Optimal result

Integrand size = 12, antiderivative size = 51

$$\int \frac{a+b\sec^{-1}(cx)}{x^3}\,dx = \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2\csc^{-1}(cx) - \frac{a+b\sec^{-1}(cx)}{2x^2}$$

output
```
-1/4*b*c^2*arccsc(c*x)+1/2*(-a-b*arcsec(c*x))/x^2+1/4*b*c*(1-1/c^2/x^2)^(1
/2)/x
```

3.10.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int \frac{a+b\sec^{-1}(cx)}{x^3}\,dx = -\frac{a}{2x^2} + \frac{bc\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{4x} - \frac{b\sec^{-1}(cx)}{2x^2} - \frac{1}{4}bc^2\arcsin\left(\frac{1}{cx}\right)$$

input
```
Integrate[(a + b*ArcSec[c*x])/x^3,x]
```

output
```
-1/2*a/x^2 + (b*c*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(4*x) - (b*ArcSec[c*x])/
(2*x^2) - (b*c^2*ArcSin[1/(c*x)])/4
```

3.10.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5743, 858, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x^3}\,dx$$

$$\downarrow 5743$$

$$\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^4}dx}{2c} - \frac{a+b\sec^{-1}(cx)}{2x^2}$$

$$\downarrow 858$$

$$-\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2}d\frac{1}{x}}{2c} - \frac{a+b\sec^{-1}(cx)}{2x^2}$$

$$\downarrow 262$$

$$-\frac{b\left(\frac{1}{2}c^2\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x} - \frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{2x}\right)}{2c} - \frac{a+b\sec^{-1}(cx)}{2x^2}$$

$$\downarrow 223$$

$$-\frac{a+b\sec^{-1}(cx)}{2x^2} - \frac{b\left(\frac{1}{2}c^3\arcsin\left(\frac{1}{cx}\right) - \frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{2x}\right)}{2c}$$

input
```
Int[(a + b*ArcSec[c*x])/x^3,x]
```

output
```
-1/2*(a + b*ArcSec[c*x])/x^2 - (b*(-1/2*(c^2*Sqrt[1 - 1/(c^2*x^2)])/x + (c
^3*ArcSin[1/(c*x)])/2))/(2*c)
```

3.10.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 5743
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

3.10.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(46) = 92$.

Time $= 0.24$ (sec) , antiderivative size $= 95$, normalized size of antiderivative $= 1.86$

method	result	size
parts	$-\frac{a}{2x^2} + b\,c^2\left(-\frac{\operatorname{arcsec}(cx)}{2c^2x^2} + \frac{\sqrt{c^2x^2-1}\left(-\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)c^2x^2+\sqrt{c^2x^2-1}\right)}{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}\right)$	95
derivativedivides	$c^2\left(-\frac{a}{2c^2x^2} + b\left(-\frac{\operatorname{arcsec}(cx)}{2c^2x^2} + \frac{\sqrt{c^2x^2-1}\left(-\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)c^2x^2+\sqrt{c^2x^2-1}\right)}{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}\right)\right)$	99
default	$c^2\left(-\frac{a}{2c^2x^2} + b\left(-\frac{\operatorname{arcsec}(cx)}{2c^2x^2} + \frac{\sqrt{c^2x^2-1}\left(-\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)c^2x^2+\sqrt{c^2x^2-1}\right)}{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}\right)\right)$	99

input
```
int((a+b*arcsec(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/x^2+b*c^2*(-1/2/c^2/x^2*arcsec(c*x)+1/4*(c^2*x^2-1)^(1/2)*(-arctan(
1/(c^2*x^2-1)^(1/2))*c^2*x^2+(c^2*x^2-1)^(1/2))/((c^2*x^2-1)/c^2/x^2)^(1/2
)/c^3/x^3)
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{a + b \sec^{-1}(cx)}{x^3}\,dx = \frac{(bc^2x^2 - 2\,b)\operatorname{arcsec}(cx) + \sqrt{c^2x^2 - 1}b - 2\,a}{4\,x^2}$$

input
```
integrate((a+b*arcsec(c*x))/x^3,x, algorithm="fricas")
```

output
```
1/4*((b*c^2*x^2 - 2*b)*arcsec(c*x) + sqrt(c^2*x^2 - 1)*b - 2*a)/x^2
```

3.10.6 Sympy [A] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.33

$$\int \frac{a + b \sec^{-1}(cx)}{x^3}\,dx = -\frac{a}{2x^2} - \frac{b\operatorname{asec}(cx)}{2x^2} + \frac{b\left(\begin{cases} \frac{ic^3 \operatorname{acosh}\left(\frac{1}{cx}\right)}{2} - \frac{ic^2}{2x\sqrt{-1+\frac{1}{c^2x^2}}} + \frac{i}{2x^3\sqrt{-1+\frac{1}{c^2x^2}}} & \text{for } \frac{1}{|c^2x^2|} > 1 \\ -\frac{c^3 \operatorname{asin}\left(\frac{1}{cx}\right)}{2} + \frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{2x} & \text{otherwise} \end{cases}\right)}{2c}$$

input
```
integrate((a+b*asec(c*x))/x**3,x)
```

output
```
-a/(2*x**2) - b*asec(c*x)/(2*x**2) + b*Piecewise((I*c**3*acosh(1/(c*x))/2
- I*c**2/(2*x*sqrt(-1 + 1/(c**2*x**2))) + I/(2*x**3*sqrt(-1 + 1/(c**2*x**2
))), 1/Abs(c**2*x**2) > 1), (-c**3*asin(1/(c*x))/2 + c**2*sqrt(1 - 1/(c**2
*x**2))/(2*x), True))/(2*c)
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.63

$$\int \frac{a+b\sec^{-1}(cx)}{x^3}\,dx = -\frac{1}{4}b\left(\frac{\frac{c^4x\sqrt{-\frac{1}{c^2x^2}+1}}{c^2x^2\left(\frac{1}{c^2x^2}-1\right)-1}-c^3\arctan\left(cx\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c}+\frac{2\,\mathrm{arcsec}\,(cx)}{x^2}\right) - \frac{a}{2\,x^2}$$

input
```
integrate((a+b*arcsec(c*x))/x^3,x, algorithm="maxima")
```

output
```
-1/4*b*((c^4*x*sqrt(-1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) - c^
3*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)))/c + 2*arcsec(c*x)/x^2) - 1/2*a/x^2
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14

$$\int \frac{a+b\sec^{-1}(cx)}{x^3}\,dx = \frac{1}{4}\left(bc\arccos\left(\frac{1}{cx}\right)+\frac{b\sqrt{-\frac{1}{c^2x^2}+1}}{x}-\frac{2\,b\arccos\left(\frac{1}{cx}\right)}{cx^2}-\frac{2\,a}{cx^2}\right)c$$

input
```
integrate((a+b*arcsec(c*x))/x^3,x, algorithm="giac")
```

output
```
1/4*(b*c*arccos(1/(c*x)) + b*sqrt(-1/(c^2*x^2) + 1)/x - 2*b*arccos(1/(c*x)
)/(c*x^2) - 2*a/(c*x^2))*c
```

3.10.9 Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{a+b\sec^{-1}(cx)}{x^3}\,dx = \frac{b\,c\,\sqrt{1-\frac{1}{c^2\,x^2}}}{4\,x} - \frac{b\,c^2\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\,\left(\frac{2}{c^2\,x^2}-1\right)}{4} - \frac{a}{2\,x^2}$$

input
```
int((a + b*acos(1/(c*x)))/x^3,x)
```

output
```
(b*c*(1 - 1/(c^2*x^2))^(1/2))/(4*x) - (b*c^2*acos(1/(c*x))*(2/(c^2*x^2) -
1))/4 - a/(2*x^2)
```

3.11 $\int \frac{a+b\sec^{-1}(cx)}{x^4}\,dx$

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3.11.1 Optimal result

Integrand size = 12, antiderivative size = 60

$$\int \frac{a+b\sec^{-1}(cx)}{x^4}\,dx = \frac{1}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}} - \frac{1}{9}bc^3\left(1-\frac{1}{c^2x^2}\right)^{3/2} - \frac{a+b\sec^{-1}(cx)}{3x^3}$$

output
```
-1/9*b*c^3*(1-1/c^2/x^2)^(3/2)+1/3*(-a-b*arcsec(c*x))/x^3+1/3*b*c^3*(1-1/c
^2/x^2)^(1/2)
```

3.11.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

$$\int \frac{a+b\sec^{-1}(cx)}{x^4}\,dx = -\frac{a}{3x^3} + b\left(\frac{2c^3}{9}+\frac{c}{9x^2}\right)\sqrt{\frac{-1+c^2x^2}{c^2x^2}} - \frac{b\sec^{-1}(cx)}{3x^3}$$

input
```
Integrate[(a + b*ArcSec[c*x])/x^4,x]
```

output
```
-1/3*a/x^3 + b*((2*c^3)/9 + c/(9*x^2))*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)] - (b
*ArcSec[c*x])/(3*x^3)
```

3.11.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5743, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x^4}\,dx$$

$$\downarrow 5743$$

$$\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^5}dx}{3c} - \frac{a+b\sec^{-1}(cx)}{3x^3}$$

$$\downarrow 798$$

$$-\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2}d\frac{1}{x^2}}{6c} - \frac{a+b\sec^{-1}(cx)}{3x^3}$$

$$\downarrow 53$$

$$-\frac{b\int \left(\frac{c^2}{\sqrt{1-\frac{1}{c^2x^2}}} - c^2\sqrt{1-\frac{1}{c^2x^2}}\right)d\frac{1}{x^2}}{6c} - \frac{a+b\sec^{-1}(cx)}{3x^3}$$

$$\downarrow 2009$$

$$-\frac{a+b\sec^{-1}(cx)}{3x^3} - \frac{b\left(\frac{2}{3}c^4\left(1-\frac{1}{c^2x^2}\right)^{3/2} - 2c^4\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c}$$

input
```
Int[(a + b*ArcSec[c*x])/x^4,x]
```

output
```
-1/6*(b*(-2*c^4*Sqrt[1 - 1/(c^2*x^2)] + (2*c^4*(1 - 1/(c^2*x^2))^(3/2))/3)
)/c - (a + b*ArcSec[c*x])/(3*x^3)
```

3.11.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5743
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

3.11.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

method	result	size
parts	$-\frac{a}{3x^3}+b\,c^3\left(-\frac{\operatorname{arcsec}(cx)}{3c^3x^3}+\frac{(c^2x^2-1)(2c^2x^2+1)}{9\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^4x^4}\right)$	71
derivativedivides	$c^3\left(-\frac{a}{3c^3x^3}+b\left(-\frac{\operatorname{arcsec}(cx)}{3c^3x^3}+\frac{(c^2x^2-1)(2c^2x^2+1)}{9\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^4x^4}\right)\right)$	75
default	$c^3\left(-\frac{a}{3c^3x^3}+b\left(-\frac{\operatorname{arcsec}(cx)}{3c^3x^3}+\frac{(c^2x^2-1)(2c^2x^2+1)}{9\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^4x^4}\right)\right)$	75

input
```
int((a+b*arcsec(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/x^3+b*c^3*(-1/3/c^3/x^3*arcsec(c*x)+1/9*(c^2*x^2-1)*(2*c^2*x^2+1)/(
(c^2*x^2-1)/c^2/x^2)^(1/2)/c^4/x^4)
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.67

$$\int \frac{a+b\sec^{-1}(cx)}{x^4}\,dx = -\frac{3\,b\,\text{arcsec}\,(cx) - (2\,bc^2x^2+b)\sqrt{c^2x^2-1}+3\,a}{9\,x^3}$$

input
```
integrate((a+b*arcsec(c*x))/x^4,x, algorithm="fricas")
```

output
```
-1/9*(3*b*arcsec(c*x) - (2*b*c^2*x^2 + b)*sqrt(c^2*x^2 - 1) + 3*a)/x^3
```

3.11.6 Sympy [A] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.83

$$\int \frac{a+b\sec^{-1}(cx)}{x^4}\,dx = -\frac{a}{3x^3} - \frac{b\,\text{asec}\,(cx)}{3x^3} + \frac{b\left(\begin{cases} \frac{2c^3\sqrt{c^2x^2-1}}{3x} + \frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2|>1 \\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x} + \frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise} \end{cases}\right)}{3c}$$

input
```
integrate((a+b*asec(c*x))/x**4,x)
```

output
```
-a/(3*x**3) - b*asec(c*x)/(3*x**3) + b*Piecewise((2*c**3*sqrt(c**2*x**2 -
1)/(3*x) + c*sqrt(c**2*x**2 - 1)/(3*x**3), Abs(c**2*x**2) > 1), (2*I*c**3*
sqrt(-c**2*x**2 + 1)/(3*x) + I*c*sqrt(-c**2*x**2 + 1)/(3*x**3), True))/(3*
c)
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int \frac{a+b\sec^{-1}(cx)}{x^4}\,dx = -\frac{1}{9}\,b\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c} + \frac{3\,\text{arcsec}\,(cx)}{x^3}\right) - \frac{a}{3\,x^3}$$

input
```
integrate((a+b*arcsec(c*x))/x^4,x, algorithm="maxima")
```

output
```
-1/9*b*((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c +
3*arcsec(c*x)/x^3) - 1/3*a/x^3
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int \frac{a+b\sec^{-1}(cx)}{x^4}\,dx = \frac{1}{9}\left(2bc^2\sqrt{-\frac{1}{c^2x^2}+1}+\frac{b\sqrt{-\frac{1}{c^2x^2}+1}}{x^2}-\frac{3\,b\arccos\left(\frac{1}{cx}\right)}{cx^3}-\frac{3\,a}{cx^3}\right)c$$

input
```
integrate((a+b*arcsec(c*x))/x^4,x, algorithm="giac")
```

output
```
1/9*(2*b*c^2*sqrt(-1/(c^2*x^2) + 1) + b*sqrt(-1/(c^2*x^2) + 1)/x^2 - 3*b*a
rccos(1/(c*x))/(c*x^3) - 3*a/(c*x^3))*c
```

3.11.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^4}\,dx = \int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x^4}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/x^4,x)
```

output
```
int((a + b*acos(1/(c*x)))/x^4, x)
```

3.12 $\int \frac{a+b\sec^{-1}(cx)}{x^5}\,dx$

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3.12.1 Optimal result

Integrand size = 12, antiderivative size = 76

$$\int \frac{a+b\sec^{-1}(cx)}{x^5}\,dx = \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16x^3} + \frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}}{32x} - \frac{3}{32}bc^4\csc^{-1}(cx) - \frac{a+b\sec^{-1}(cx)}{4x^4}$$

output
```
-3/32*b*c^4*arccsc(c*x)+1/4*(-a-b*arcsec(c*x))/x^4+1/16*b*c*(1-1/c^2/x^2)^
(1/2)/x^3+3/32*b*c^3*(1-1/c^2/x^2)^(1/2)/x
```

3.12.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int \frac{a+b\sec^{-1}(cx)}{x^5}\,dx = -\frac{a}{4x^4} + b\left(\frac{c}{16x^3} + \frac{3c^3}{32x}\right)\sqrt{\frac{-1+c^2x^2}{c^2x^2}} - \frac{b\sec^{-1}(cx)}{4x^4} - \frac{3}{32}bc^4\arcsin\left(\frac{1}{cx}\right)$$

input
```
Integrate[(a + b*ArcSec[c*x])/x^5,x]
```

output
```
-1/4*a/x^4 + b*(c/(16*x^3) + (3*c^3)/(32*x))*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)
] - (b*ArcSec[c*x])/(4*x^4) - (3*b*c^4*ArcSin[1/(c*x)])/32
```

3.12.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5743, 858, 262, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x^5}\,dx$$

$$\downarrow 5743$$

$$\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^6}dx}{4c} - \frac{a+b\sec^{-1}(cx)}{4x^4}$$

$$\downarrow 858$$

$$-\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^4}d\frac{1}{x}}{4c} - \frac{a+b\sec^{-1}(cx)}{4x^4}$$

$$\downarrow 262$$

$$-\frac{b\left(\frac{3}{4}c^2\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2}d\frac{1}{x} - \frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{4x^3}\right)}{4c} - \frac{a+b\sec^{-1}(cx)}{4x^4}$$

$$\downarrow 262$$

$$-\frac{b\left(\frac{3}{4}c^2\left(\frac{1}{2}c^2\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x} - \frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{2x}\right) - \frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{4x^3}\right)}{4c} - \frac{a+b\sec^{-1}(cx)}{4x^4}$$

$$\downarrow 223$$

$$-\frac{a+b\sec^{-1}(cx)}{4x^4} - \frac{b\left(\frac{3}{4}c^2\left(\frac{1}{2}c^3\arcsin\left(\frac{1}{cx}\right) - \frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{2x}\right) - \frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{4x^3}\right)}{4c}$$

input
```
Int[(a + b*ArcSec[c*x])/x^5,x]
```

output
```
-1/4*(a + b*ArcSec[c*x])/x^4 - (b*(-1/4*(c^2*Sqrt[1 - 1/(c^2*x^2)])/x^3 +
(3*c^2*(-1/2*(c^2*Sqrt[1 - 1/(c^2*x^2)])/x + (c^3*ArcSin[1/(c*x)])/2))/4))
/(4*c)
```

3.12.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 5743
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

3.12.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(67) = 134.

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.82

method	result	s
parts	$-\frac{a}{4x^4}-\frac{b\,\mathrm{arcsec}(cx)}{4x^4}-\frac{3bc^3\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}+\frac{3bc(c^2x^2-1)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^3}+\frac{b(c^2x^2-1)}{16c\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^5}$	1
derivativedivides	$c^4\left(-\frac{a}{4c^4x^4}-\frac{b\,\mathrm{arcsec}(cx)}{4c^4x^4}-\frac{3b\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}+\frac{3b(c^2x^2-1)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}+\frac{b(c^2x^2-1)}{16\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^5x^5}\right)$	1
default	$c^4\left(-\frac{a}{4c^4x^4}-\frac{b\,\mathrm{arcsec}(cx)}{4c^4x^4}-\frac{3b\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}+\frac{3b(c^2x^2-1)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}+\frac{b(c^2x^2-1)}{16\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^5x^5}\right)$	1

input
```
int((a+b*arcsec(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/x^4-1/4*b/x^4*arcsec(c*x)-3/32*b*c^3*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)
/c^2/x^2)^(1/2)/x*arctan(1/(c^2*x^2-1)^(1/2))+3/32*b*c*(c^2*x^2-1)/((c^2*x
^2-1)/c^2/x^2)^(1/2)/x^3+1/16*b/c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/
x^5
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.68

$$\int \frac{a+b\sec^{-1}(cx)}{x^5}\,dx = \frac{(3\,bc^4x^4-8\,b)\operatorname{arcsec}(cx)+(3\,bc^2x^2+2\,b)\sqrt{c^2x^2-1}-8\,a}{32\,x^4}$$

input
```
integrate((a+b*arcsec(c*x))/x^5,x, algorithm="fricas")
```

output
```
1/32*((3*b*c^4*x^4 - 8*b)*arcsec(c*x) + (3*b*c^2*x^2 + 2*b)*sqrt(c^2*x^2 -
 1) - 8*a)/x^4
```

3.12.6 Sympy [A] (verification not implemented)

Time = 3.41 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.53

$$\begin{aligned}
&\int \frac{a+b\sec^{-1}(cx)}{x^5}\,dx \\
&= -\frac{a}{4x^4} - \frac{b\operatorname{asec}(cx)}{4x^4} \\
&\quad + \frac{b\left(\begin{cases} \frac{3ic^5\operatorname{acosh}\left(\frac{1}{cx}\right)}{8} - \frac{3ic^4}{8x\sqrt{-1+\frac{1}{c^2x^2}}} + \frac{ic^2}{8x^3\sqrt{-1+\frac{1}{c^2x^2}}} + \frac{i}{4x^5\sqrt{-1+\frac{1}{c^2x^2}}} & \text{for } \frac{1}{|c^2x^2|} > 1 \\ -\frac{3c^5\operatorname{asin}\left(\frac{1}{cx}\right)}{8} + \frac{3c^4}{8x\sqrt{1-\frac{1}{c^2x^2}}} - \frac{c^2}{8x^3\sqrt{1-\frac{1}{c^2x^2}}} - \frac{1}{4x^5\sqrt{1-\frac{1}{c^2x^2}}} & \text{otherwise} \end{cases}\right)}{4c}
\end{aligned}$$

input
```
integrate((a+b*asec(c*x))/x**5,x)
```

output
```
-a/(4*x**4) - b*asec(c*x)/(4*x**4) + b*Piecewise((3*I*c**5*acosh(1/(c*x))/
8 - 3*I*c**4/(8*x*sqrt(-1 + 1/(c**2*x**2))) + I*c**2/(8*x**3*sqrt(-1 + 1/(
c**2*x**2))) + I/(4*x**5*sqrt(-1 + 1/(c**2*x**2))), 1/Abs(c**2*x**2) > 1),
 (-3*c**5*asin(1/(c*x))/8 + 3*c**4/(8*x*sqrt(1 - 1/(c**2*x**2))) - c**2/(8
*x**3*sqrt(1 - 1/(c**2*x**2))) - 1/(4*x**5*sqrt(1 - 1/(c**2*x**2))), True)
)/(4*c)
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.64

$$\int \frac{a+b\sec^{-1}(cx)}{x^5}\,dx$$
$$= \frac{1}{32}b\left(\frac{3c^5\arctan\left(cx\sqrt{-\frac{1}{c^2x^2}+1}\right)+\frac{3c^8x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+5c^6x\sqrt{-\frac{1}{c^2x^2}+1}}{c^4x^4\left(\frac{1}{c^2x^2}-1\right)^2-2c^2x^2\left(\frac{1}{c^2x^2}-1\right)+1}}{c}-\frac{8\,\mathrm{arcsec}\,(cx)}{x^4}\right)$$
$$-\frac{a}{4x^4}$$

input
```
integrate((a+b*arcsec(c*x))/x^5,x, algorithm="maxima")
```

output
```
1/32*b*((3*c^5*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)) + (3*c^8*x^3*(-1/(c^2*x^
2) + 1)^(3/2) + 5*c^6*x*sqrt(-1/(c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2) - 1)
^2 - 2*c^2*x^2*(1/(c^2*x^2) - 1) + 1))/c - 8*arcsec(c*x)/x^4) - 1/4*a/x^4
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09

$$\int \frac{a+b\sec^{-1}(cx)}{x^5}\,dx$$
$$= \frac{1}{32}\left(3bc^3\arccos\left(\frac{1}{cx}\right)+\frac{3bc^2\sqrt{-\frac{1}{c^2x^2}+1}}{x}+\frac{2b\sqrt{-\frac{1}{c^2x^2}+1}}{x^3}-\frac{8b\arccos\left(\frac{1}{cx}\right)}{cx^4}-\frac{8a}{cx^4}\right)c$$

input
```
integrate((a+b*arcsec(c*x))/x^5,x, algorithm="giac")
```

output
```
1/32*(3*b*c^3*arccos(1/(c*x)) + 3*b*c^2*sqrt(-1/(c^2*x^2) + 1)/x + 2*b*sqr
t(-1/(c^2*x^2) + 1)/x^3 - 8*b*arccos(1/(c*x))/(c*x^4) - 8*a/(c*x^4))*c
```

3.12.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \sec^{-1}(cx)}{x^5}\, dx = \int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{x^5}\, dx$$

input
```
int((a + b*acos(1/(c*x)))/x^5,x)
```

output
```
int((a + b*acos(1/(c*x)))/x^5, x)
```

3.13 $\int \frac{a+b\sec^{-1}(cx)}{x^6}\,dx$

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3.13.1 Optimal result

Integrand size = 12, antiderivative size = 82

$$\int \frac{a+b\sec^{-1}(cx)}{x^6}\,dx = \frac{1}{5}bc^5\sqrt{1-\frac{1}{c^2x^2}} - \frac{2}{15}bc^5\left(1-\frac{1}{c^2x^2}\right)^{3/2} + \frac{1}{25}bc^5\left(1-\frac{1}{c^2x^2}\right)^{5/2} - \frac{a+b\sec^{-1}(cx)}{5x^5}$$

output
```
-2/15*b*c^5*(1-1/c^2/x^2)^(3/2)+1/25*b*c^5*(1-1/c^2/x^2)^(5/2)+1/5*(-a-b*a
rcsec(c*x))/x^5+1/5*b*c^5*(1-1/c^2/x^2)^(1/2)
```

3.13.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

$$\int \frac{a+b\sec^{-1}(cx)}{x^6}\,dx = -\frac{a}{5x^5} + b\left(\frac{8c^5}{75} + \frac{c}{25x^4} + \frac{4c^3}{75x^2}\right)\sqrt{\frac{-1+c^2x^2}{c^2x^2}} - \frac{b\sec^{-1}(cx)}{5x^5}$$

input
```
Integrate[(a + b*ArcSec[c*x])/x^6,x]
```

output
```
-1/5*a/x^5 + b*((8*c^5)/75 + c/(25*x^4) + (4*c^3)/(75*x^2))*Sqrt[(-1 + c^2
*x^2)/(c^2*x^2)] - (b*ArcSec[c*x])/(5*x^5)
```

3.13.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5743, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x^6}\,dx$$

$$\downarrow 5743$$

$$\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^7}dx}{5c} - \frac{a+b\sec^{-1}(cx)}{5x^5}$$

$$\downarrow 798$$

$$-\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^4}d\frac{1}{x^2}}{10c} - \frac{a+b\sec^{-1}(cx)}{5x^5}$$

$$\downarrow 53$$

$$-\frac{b\int \left(\left(1-\frac{1}{c^2x^2}\right)^{3/2}c^4 - 2\sqrt{1-\frac{1}{c^2x^2}}c^4 + \frac{c^4}{\sqrt{1-\frac{1}{c^2x^2}}}\right)d\frac{1}{x^2}}{10c} - \frac{a+b\sec^{-1}(cx)}{5x^5}$$

$$\downarrow 2009$$

$$-\frac{a+b\sec^{-1}(cx)}{5x^5} - \frac{b\left(-\frac{2}{5}c^6\left(1-\frac{1}{c^2x^2}\right)^{5/2} + \frac{4}{3}c^6\left(1-\frac{1}{c^2x^2}\right)^{3/2} - 2c^6\sqrt{1-\frac{1}{c^2x^2}}\right)}{10c}$$

input
```
Int[(a + b*ArcSec[c*x])/x^6,x]
```

output
```
-1/10*(b*(-2*c^6*Sqrt[1 - 1/(c^2*x^2)] + (4*c^6*(1 - 1/(c^2*x^2))^(3/2))/3
 - (2*c^6*(1 - 1/(c^2*x^2))^(5/2))/5))/c - (a + b*ArcSec[c*x])/(5*x^5)
```

3.13.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5743
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

3.13.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

method	result	size
parts	$-\frac{a}{5x^5}+b\,c^5\left(-\frac{\operatorname{arcsec}(cx)}{5c^5x^5}+\frac{(c^2x^2-1)\left(8c^4x^4+4c^2x^2+3\right)}{75\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right)$	79
derivativedivides	$c^5\left(-\frac{a}{5c^5x^5}+b\left(-\frac{\operatorname{arcsec}(cx)}{5c^5x^5}+\frac{(c^2x^2-1)\left(8c^4x^4+4c^2x^2+3\right)}{75\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right)\right)$	83
default	$c^5\left(-\frac{a}{5c^5x^5}+b\left(-\frac{\operatorname{arcsec}(cx)}{5c^5x^5}+\frac{(c^2x^2-1)\left(8c^4x^4+4c^2x^2+3\right)}{75\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right)\right)$	83

input
```
int((a+b*arcsec(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a/x^5+b*c^5*(-1/5/c^5/x^5*arcsec(c*x)+1/75*(c^2*x^2-1)*(8*c^4*x^4+4*c
^2*x^2+3)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^6/x^6)
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.62

$$\int \frac{a+b\sec^{-1}(cx)}{x^6}\,dx = -\frac{15\,b\,\text{arcsec}\,(cx) - (8\,bc^4x^4+4\,bc^2x^2+3\,b)\sqrt{c^2x^2-1}+15\,a}{75\,x^5}$$

input
```
integrate((a+b*arcsec(c*x))/x^6,x, algorithm="fricas")
```

output
```
-1/75*(15*b*arcsec(c*x) - (8*b*c^4*x^4 + 4*b*c^2*x^2 + 3*b)*sqrt(c^2*x^2 -
 1) + 15*a)/x^5
```

3.13.6 Sympy [A] (verification not implemented)

Time = 3.60 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.90

$$\int \frac{a+b\sec^{-1}(cx)}{x^6}\,dx = -\frac{a}{5x^5} - \frac{b\,\text{asec}\,(cx)}{5x^5} + \frac{b\left(\begin{cases} \frac{8c^5\sqrt{c^2x^2-1}}{15x} + \frac{4c^3\sqrt{c^2x^2-1}}{15x^3} + \frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for } |c^2x^2|>1 \\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x} + \frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3} + \frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise} \end{cases}\right)}{5c}$$

input
```
integrate((a+b*asec(c*x))/x**6,x)
```

output
```
-a/(5*x**5) - b*asec(c*x)/(5*x**5) + b*Piecewise((8*c**5*sqrt(c**2*x**2 -
1)/(15*x) + 4*c**3*sqrt(c**2*x**2 - 1)/(15*x**3) + c*sqrt(c**2*x**2 - 1)/(
5*x**5), Abs(c**2*x**2) > 1), (8*I*c**5*sqrt(-c**2*x**2 + 1)/(15*x) + 4*I*
c**3*sqrt(-c**2*x**2 + 1)/(15*x**3) + I*c*sqrt(-c**2*x**2 + 1)/(5*x**5), T
rue))/(5*c)
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

$$\int \frac{a+b\sec^{-1}(cx)}{x^6}\,dx$$
$$= \frac{1}{75}b\left(\frac{3c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} - 10c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 15c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c} - \frac{15\ \mathrm{arcsec}\,(cx)}{x^5}\right)$$
$$- \frac{a}{5x^5}$$

input
```
integrate((a+b*arcsec(c*x))/x^6,x, algorithm="maxima")
```

output
```
1/75*b*((3*c^6*(-1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(-1/(c^2*x^2) + 1)^(3/2)
+ 15*c^6*sqrt(-1/(c^2*x^2) + 1))/c - 15*arcsec(c*x)/x^5) - 1/5*a/x^5
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.06

$$\int \frac{a+b\sec^{-1}(cx)}{x^6}\,dx$$
$$= \frac{1}{75}\left(8bc^4\sqrt{-\frac{1}{c^2x^2}+1} + \frac{4bc^2\sqrt{-\frac{1}{c^2x^2}+1}}{x^2} + \frac{3b\sqrt{-\frac{1}{c^2x^2}+1}}{x^4} - \frac{15b\arccos\left(\frac{1}{cx}\right)}{cx^5} - \frac{15a}{cx^5}\right)c$$

input
```
integrate((a+b*arcsec(c*x))/x^6,x, algorithm="giac")
```

output
```
1/75*(8*b*c^4*sqrt(-1/(c^2*x^2) + 1) + 4*b*c^2*sqrt(-1/(c^2*x^2) + 1)/x^2
+ 3*b*sqrt(-1/(c^2*x^2) + 1)/x^4 - 15*b*arccos(1/(c*x))/(c*x^5) - 15*a/(c*
x^5))*c
```

3.13.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \sec^{-1}(cx)}{x^6}\, dx = \int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{x^6}\, dx$$

input
```
int((a + b*acos(1/(c*x)))/x^6,x)
```

output
```
int((a + b*acos(1/(c*x)))/x^6, x)
```

3.14 $\int \frac{a+b\sec^{-1}(cx)}{x^7}\,dx$

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3.14.1 Optimal result

Integrand size = 12, antiderivative size = 101

$$\int \frac{a+b\sec^{-1}(cx)}{x^7}\,dx = \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{36x^5} + \frac{5bc^3\sqrt{1-\frac{1}{c^2x^2}}}{144x^3} + \frac{5bc^5\sqrt{1-\frac{1}{c^2x^2}}}{96x} - \frac{5}{96}bc^6\csc^{-1}(cx) - \frac{a+b\sec^{-1}(cx)}{6x^6}$$

output
```
-5/96*b*c^6*arccsc(c*x)+1/6*(-a-b*arcsec(c*x))/x^6+1/36*b*c*(1-1/c^2/x^2)^
(1/2)/x^5+5/144*b*c^3*(1-1/c^2/x^2)^(1/2)/x^3+5/96*b*c^5*(1-1/c^2/x^2)^(1/
2)/x
```

3.14.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87

$$\int \frac{a+b\sec^{-1}(cx)}{x^7}\,dx = -\frac{a}{6x^6} + b\left(\frac{c}{36x^5} + \frac{5c^3}{144x^3} + \frac{5c^5}{96x}\right)\sqrt{\frac{-1+c^2x^2}{c^2x^2}} - \frac{b\sec^{-1}(cx)}{6x^6} - \frac{5}{96}bc^6\arcsin\left(\frac{1}{cx}\right)$$

input
```
Integrate[(a + b*ArcSec[c*x])/x^7,x]
```

output
```
-1/6*a/x^6 + b*(c/(36*x^5) + (5*c^3)/(144*x^3) + (5*c^5)/(96*x))*Sqrt[(-1
+ c^2*x^2)/(c^2*x^2)] - (b*ArcSec[c*x])/(6*x^6) - (5*b*c^6*ArcSin[1/(c*x)]
)/96
```

3.14.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.27, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5743, 858, 262, 262, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x^7}\,dx$$

$$\downarrow 5743$$

$$\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^8}dx}{6c} - \frac{a+b\sec^{-1}(cx)}{6x^6}$$

$$\downarrow 858$$

$$-\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^6}d\frac{1}{x}}{6c} - \frac{a+b\sec^{-1}(cx)}{6x^6}$$

$$\downarrow 262$$

$$-\frac{b\left(\frac{5}{6}c^2\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^4}d\frac{1}{x} - \frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{6x^5}\right)}{6c} - \frac{a+b\sec^{-1}(cx)}{6x^6}$$

$$\downarrow 262$$

$$-\frac{b\left(\frac{5}{6}c^2\left(\frac{3}{4}c^2\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2}d\frac{1}{x} - \frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{4x^3}\right) - \frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{6x^5}\right)}{6c} - \frac{a+b\sec^{-1}(cx)}{6x^6}$$

$$\downarrow 262$$

$$-\frac{b\left(\frac{5}{6}c^2\left(\frac{3}{4}c^2\left(\frac{1}{2}c^2\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x} - \frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{2x}\right) - \frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{4x^3}\right) - \frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{6x^5}\right)}{6c} - \frac{a+b\sec^{-1}(cx)}{6x^6}$$

$$\downarrow 223$$

$$-\frac{a+b\sec^{-1}(cx)}{6x^6}-\frac{b\left(\frac{5}{6}c^2\left(\frac{3}{4}c^2\left(\frac{1}{2}c^3\arcsin\left(\frac{1}{cx}\right)-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{2x}\right)-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{4x^3}\right)-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{6x^5}\right)}{6c}$$

input
```
Int[(a + b*ArcSec[c*x])/x^7,x]
```

output
```
-1/6*(a + b*ArcSec[c*x])/x^6 - (b*(-1/6*(c^2*Sqrt[1 - 1/(c^2*x^2)])/x^5 +
(5*c^2*(-1/4*(c^2*Sqrt[1 - 1/(c^2*x^2)])/x^3 + (3*c^2*(-1/2*(c^2*Sqrt[1 -
1/(c^2*x^2)])/x + (c^3*ArcSin[1/(c*x)])/2))/4))/6))/(6*c)
```

3.14.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 5743
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

3.14.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.72

method	result
parts	$-\frac{a}{6x^6}-\frac{b\,\mathrm{arcsec}(cx)}{6x^6}-\frac{5bc^5\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}+\frac{5bc^3(c^2x^2-1)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^3}+\frac{5bc(c^2x^2-1)}{144\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^5}+\frac{b(c^2x^2-$
derivativedivides	$c^6\left(-\frac{a}{6c^6x^6}-\frac{b\,\mathrm{arcsec}(cx)}{6c^6x^6}-\frac{5b\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}+\frac{5b(c^2x^2-1)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}+\frac{5b(c^2x^2-1)}{144\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^5x^5}+$
default	$c^6\left(-\frac{a}{6c^6x^6}-\frac{b\,\mathrm{arcsec}(cx)}{6c^6x^6}-\frac{5b\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}+\frac{5b(c^2x^2-1)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}+\frac{5b(c^2x^2-1)}{144\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^5x^5}+$

input
```
int((a+b*arcsec(c*x))/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a/x^6-1/6*b/x^6*arcsec(c*x)-5/96*b*c^5*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)
/c^2/x^2)^(1/2)/x*arctan(1/(c^2*x^2-1)^(1/2))+5/96*b*c^3*(c^2*x^2-1)/((c^2
*x^2-1)/c^2/x^2)^(1/2)/x^3+5/144*b*c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/
2)/x^5+1/36*b/c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^7
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.61

$$\int \frac{a+b\sec^{-1}(cx)}{x^7}\,dx$$
$$=\frac{3\left(5\,bc^6x^6-16\,b\right)\mathrm{arcsec}\left(cx\right)+\left(15\,bc^4x^4+10\,bc^2x^2+8\,b\right)\sqrt{c^2x^2-1}-48\,a}{288\,x^6}$$

input
```
integrate((a+b*arcsec(c*x))/x^7,x, algorithm="fricas")
```

output
```
1/288*(3*(5*b*c^6*x^6 - 16*b)*arcsec(c*x) + (15*b*c^4*x^4 + 10*b*c^2*x^2 +
 8*b)*sqrt(c^2*x^2 - 1) - 48*a)/x^6
```

3.14.6 Sympy [A] (verification not implemented)

Time = 8.61 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.39

$$\int \frac{a+b\sec^{-1}(cx)}{x^7}\,dx = -\frac{a}{6x^6} - \frac{b\,\text{asec}\,(cx)}{6x^6} + \frac{b\left(\begin{cases} \frac{5ic^7\,\text{acosh}\left(\frac{1}{cx}\right)}{16} - \frac{5ic^6}{16x\sqrt{-1+\frac{1}{c^2x^2}}} + \frac{5ic^4}{48x^3\sqrt{-1+\frac{1}{c^2x^2}}} + \frac{ic^2}{24x^5\sqrt{-1+\frac{1}{c^2x^2}}} + \frac{i}{6x^7\sqrt{-1+\frac{1}{c^2x^2}}} & \text{for } \frac{1}{|c^2x^2|} > 1 \\ -\frac{5c^7\,\text{asin}\left(\frac{1}{cx}\right)}{16} + \frac{5c^6}{16x\sqrt{1-\frac{1}{c^2x^2}}} - \frac{5c^4}{48x^3\sqrt{1-\frac{1}{c^2x^2}}} - \frac{c^2}{24x^5\sqrt{1-\frac{1}{c^2x^2}}} - \frac{1}{6x^7\sqrt{1-\frac{1}{c^2x^2}}} & \text{otherwise} \end{cases}\right)}{6c}$$

input
```
integrate((a+b*asec(c*x))/x**7,x)
```

output
```
-a/(6*x**6) - b*asec(c*x)/(6*x**6) + b*Piecewise((5*I*c**7*acosh(1/(c*x))/
16 - 5*I*c**6/(16*x*sqrt(-1 + 1/(c**2*x**2))) + 5*I*c**4/(48*x**3*sqrt(-1
+ 1/(c**2*x**2))) + I*c**2/(24*x**5*sqrt(-1 + 1/(c**2*x**2))) + I/(6*x**7*
sqrt(-1 + 1/(c**2*x**2))), 1/Abs(c**2*x**2) > 1), (-5*c**7*asin(1/(c*x))/1
6 + 5*c**6/(16*x*sqrt(1 - 1/(c**2*x**2))) - 5*c**4/(48*x**3*sqrt(1 - 1/(c*
*2*x**2))) - c**2/(24*x**5*sqrt(1 - 1/(c**2*x**2))) - 1/(6*x**7*sqrt(1 - 1
/(c**2*x**2))), True))/(6*c)
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.63

$$\int \frac{a+b\sec^{-1}(cx)}{x^7}\,dx = \frac{1}{288}\,b\left(\frac{15\,c^7\arctan\left(cx\sqrt{-\frac{1}{c^2x^2}+1}\right) - \frac{15\,c^{12}x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} + 40\,c^{10}x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 33\,c^8x\sqrt{-\frac{1}{c^2x^2}+1}}{c^6x^6\left(\frac{1}{c^2x^2}-1\right)^3 - 3\,c^4x^4\left(\frac{1}{c^2x^2}-1\right)^2 + 3\,c^2x^2\left(\frac{1}{c^2x^2}-1\right) - 1}}{c} - \frac{48\,\text{arcsec}\,(c}{x^6}\right.$$
$$\left. - \frac{a}{6\,x^6}\right.$$

input
```
integrate((a+b*arcsec(c*x))/x^7,x, algorithm="maxima")
```

output
```
1/288*b*((15*c^7*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)) - (15*c^12*x^5*(-1/(c^
2*x^2) + 1)^(5/2) + 40*c^10*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 33*c^8*x*sqrt(-
1/(c^2*x^2) + 1))/(c^6*x^6*(1/(c^2*x^2) - 1)^3 - 3*c^4*x^4*(1/(c^2*x^2) -
1)^2 + 3*c^2*x^2*(1/(c^2*x^2) - 1) - 1))/c - 48*arcsec(c*x)/x^6) - 1/6*a/x
^6
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03

$$\int \frac{a+b\sec^{-1}(cx)}{x^7}\,dx$$
$$= \frac{1}{288}\left(15\,bc^5\arccos\left(\frac{1}{cx}\right) + \frac{15\,bc^4\sqrt{-\frac{1}{c^2x^2}+1}}{x} + \frac{10\,bc^2\sqrt{-\frac{1}{c^2x^2}+1}}{x^3} + \frac{8\,b\sqrt{-\frac{1}{c^2x^2}+1}}{x^5} - \frac{48\,b\arccos\left(\frac{1}{cx}\right)}{cx^6}\right.$$

input
```
integrate((a+b*arcsec(c*x))/x^7,x, algorithm="giac")
```

output
```
1/288*(15*b*c^5*arccos(1/(c*x)) + 15*b*c^4*sqrt(-1/(c^2*x^2) + 1)/x + 10*b
*c^2*sqrt(-1/(c^2*x^2) + 1)/x^3 + 8*b*sqrt(-1/(c^2*x^2) + 1)/x^5 - 48*b*ar
ccos(1/(c*x))/(c*x^6) - 48*a/(c*x^6))*c
```

3.14.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^7}\,dx = \int \frac{a+b\operatorname{acos}\left(\frac{1}{cx}\right)}{x^7}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/x^7,x)
```

output
```
int((a + b*acos(1/(c*x)))/x^7, x)
```

3.15 $\int x^3(a+b\sec^{-1}(cx))^2\,dx$

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3.15.1 Optimal result

Integrand size = 14, antiderivative size = 107

$$\int x^3\left(a+b\sec^{-1}(cx)\right)^2\,dx = \frac{b^2x^2}{12c^2} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(a+b\sec^{-1}(cx)\right)}{3c^3} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^3\left(a+b\sec^{-1}(cx)\right)}{6c} + \frac{1}{4}x^4\left(a+b\sec^{-1}(cx)\right)^2 + \frac{b^2\log(x)}{3c^4}$$

output
```
1/12*b^2*x^2/c^2+1/4*x^4*(a+b*arcsec(c*x))^2+1/3*b^2*ln(x)/c^4-1/3*b*x*(a+
b*arcsec(c*x))*(1-1/c^2/x^2)^(1/2)/c^3-1/6*b*x^3*(a+b*arcsec(c*x))*(1-1/c^
2/x^2)^(1/2)/c
```

3.15.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.16

$$\int x^3\left(a+b\sec^{-1}(cx)\right)^2\,dx = \frac{cx\left(b^2cx+3a^2c^3x^3-2ab\sqrt{1-\frac{1}{c^2x^2}}\left(2+c^2x^2\right)\right)-2bcx\left(-3ac^3x^3+b\sqrt{1-\frac{1}{c^2x^2}}\left(2+c^2x^2\right)\right)\sec^{-1}(cx)+3b$$

input
```
Integrate[x^3*(a + b*ArcSec[c*x])^2,x]
```

output
```
(c*x*(b^2*c*x + 3*a^2*c^3*x^3 - 2*a*b*Sqrt[1 - 1/(c^2*x^2)]*(2 + c^2*x^2))
 - 2*b*c*x*(-3*a*c^3*x^3 + b*Sqrt[1 - 1/(c^2*x^2)]*(2 + c^2*x^2))*ArcSec[c
*x] + 3*b^2*c^4*x^4*ArcSec[c*x]^2 + 4*b^2*Log[x])/(12*c^4)
```

3.15.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {5745, 4909, 3042, 4673, 3042, 4672, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a + b \sec^{-1}(cx)\right)^2 dx$$

$$\downarrow 5745$$

$$\frac{\int c^5 \sqrt{1 - \frac{1}{c^2 x^2}} x^5 \left(a + b \sec^{-1}(cx)\right)^2 d \sec^{-1}(cx)}{c^4}$$

$$\downarrow 4909$$

$$\frac{\frac{1}{4} c^4 x^4 \left(a + b \sec^{-1}(cx)\right)^2 - \frac{1}{2} b \int c^4 x^4 \left(a + b \sec^{-1}(cx)\right) d \sec^{-1}(cx)}{c^4}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{4} c^4 x^4 \left(a + b \sec^{-1}(cx)\right)^2 - \frac{1}{2} b \int \left(a + b \sec^{-1}(cx)\right) \csc\left(\sec^{-1}(cx) + \frac{\pi}{2}\right)^4 d \sec^{-1}(cx)}{c^4}$$

$$\downarrow 4673$$

$$\frac{\frac{1}{4} c^4 x^4 \left(a + b \sec^{-1}(cx)\right)^2 - \frac{1}{2} b \left(\frac{2}{3} \int c^2 x^2 \left(a + b \sec^{-1}(cx)\right) d \sec^{-1}(cx) + \frac{1}{3} c^3 x^3 \sqrt{1 - \frac{1}{c^2 x^2}} \left(a + b \sec^{-1}(cx)\right) - \frac{1}{6} b c^2 x \right.}{c^4}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{4} c^4 x^4 \left(a + b \sec^{-1}(cx)\right)^2 - \frac{1}{2} b \left(\frac{2}{3} \int \left(a + b \sec^{-1}(cx)\right) \csc\left(\sec^{-1}(cx) + \frac{\pi}{2}\right)^2 d \sec^{-1}(cx) + \frac{1}{3} c^3 x^3 \sqrt{1 - \frac{1}{c^2 x^2}} (a + b \sec \right.}{c^4}$$

$$\downarrow 4672$$

$$\frac{\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^2-\frac{1}{2}b\left(\frac{2}{3}\left(b\int -c\sqrt{1-\frac{1}{c^2x^2}}xd\sec^{-1}(cx)+cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)\right)+\frac{1}{3}c^3x^3\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^4}$$

$$\downarrow 25$$

$$\frac{\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^2-\frac{1}{2}b\left(\frac{2}{3}\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)-b\int c\sqrt{1-\frac{1}{c^2x^2}}xd\sec^{-1}(cx)\right)+\frac{1}{3}c^3x^3\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^4}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^2-\frac{1}{2}b\left(\frac{2}{3}\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)-b\int \tan\left(\sec^{-1}(cx)\right)d\sec^{-1}(cx)\right)+\frac{1}{3}c^3x^3\sqrt{1-\right.}}{c^4}$$

$$\downarrow 3956$$

$$\frac{\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^2-\frac{1}{2}b\left(\frac{2}{3}\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)+b\log\left(\frac{1}{cx}\right)\right)+\frac{1}{3}c^3x^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)\right.}{c^4}$$

input
```
Int[x^3*(a + b*ArcSec[c*x])^2,x]
```

output
```
((c^4*x^4*(a + b*ArcSec[c*x])^2)/4 - (b*(-1/6*(b*c^2*x^2) + (c^3*Sqrt[1 -
1/(c^2*x^2)]*x^3*(a + b*ArcSec[c*x]))/3 + (2*(c*Sqrt[1 - 1/(c^2*x^2)]*x*(a
 + b*ArcSec[c*x]) + b*Log[1/(c*x)]))/3))/2)/c^4
```

3.15.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

3.15.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.59

method	result
parts	$\frac{a^2x^4}{4}+\frac{b^2\left(\frac{\operatorname{arcsec}(cx)^2c^4x^4}{4}-\frac{\operatorname{arcsec}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}{6}+\frac{c^2x^2}{12}-\frac{\operatorname{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3}-\frac{\ln\left(\frac{1}{cx}\right)}{3}\right)}{c^4}+\frac{2ab\left(\frac{c^4x^4\operatorname{arcsec}(c$
derivativedivides	$\frac{\frac{a^2c^4x^4}{4}+b^2\left(\frac{\operatorname{arcsec}(cx)^2c^4x^4}{4}-\frac{\operatorname{arcsec}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}{6}+\frac{c^2x^2}{12}-\frac{\operatorname{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3}-\frac{\ln\left(\frac{1}{cx}\right)}{3}\right)+2ab\left(\frac{c^4x^4\operatorname{arcsec}(cx)}{4}-$
default	$\frac{\frac{a^2c^4x^4}{4}+b^2\left(\frac{\operatorname{arcsec}(cx)^2c^4x^4}{4}-\frac{\operatorname{arcsec}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}{6}+\frac{c^2x^2}{12}-\frac{\operatorname{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3}-\frac{\ln\left(\frac{1}{cx}\right)}{3}\right)+2ab\left(\frac{c^4x^4\operatorname{arcsec}(cx)}{4}-$

input
```
int(x^3*(a+b*arcsec(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*x^4+b^2/c^4*(1/4*arcsec(c*x)^2*c^4*x^4-1/6*arcsec(c*x)*((c^2*x^2-1
)/c^2/x^2)^(1/2)*c^3*x^3+1/12*c^2*x^2-1/3*arcsec(c*x)*c*x*((c^2*x^2-1)/c^2
/x^2)^(1/2)-1/3*ln(1/c/x))+2*a*b/c^4*(1/4*c^4*x^4*arcsec(c*x)-1/12*(c^2*x^
2-1)*(c^2*x^2+2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x)
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.36

$$\int x^3 \left(a + b \sec^{-1}(cx)\right)^2 \, dx$$
$$= \frac{3\,b^2c^4x^4 \operatorname{arcsec}(cx)^2 + 3\,a^2c^4x^4 + 12\,abc^4 \arctan\left(-cx + \sqrt{c^2x^2 - 1}\right) + b^2c^2x^2 + 4\,b^2 \log(x) + 6\,(abc^4x^4 -}{12\,c^4}$$

input
```
integrate(x^3*(a+b*arcsec(c*x))^2,x, algorithm="fricas")
```

output
```
1/12*(3*b^2*c^4*x^4*arcsec(c*x)^2 + 3*a^2*c^4*x^4 + 12*a*b*c^4*arctan(-c*x
 + sqrt(c^2*x^2 - 1)) + b^2*c^2*x^2 + 4*b^2*log(x) + 6*(a*b*c^4*x^4 - a*b*
c^4)*arcsec(c*x) - 2*(a*b*c^2*x^2 + 2*a*b + (b^2*c^2*x^2 + 2*b^2)*arcsec(c
*x))*sqrt(c^2*x^2 - 1))/c^4
```

3.15.6 Sympy [F]

$$\int x^3 \left(a + b \sec^{-1}(cx)\right)^2 \, dx = \int x^3 (a + b \operatorname{asec}(cx))^2 \, dx$$

input
```
integrate(x**3*(a+b*asec(c*x))**2,x)
```

output
```
Integral(x**3*(a + b*asec(c*x))**2, x)
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.52

$$\int x^3\left(a+b\sec^{-1}(cx)\right)^2\,dx = \frac{1}{4}b^2x^4\operatorname{arcsec}(cx)^2+\frac{1}{4}a^2x^4 + \frac{1}{6}\left(3\,x^4\operatorname{arcsec}(cx)-\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)ab + \frac{\left(\left(c^2x^2+2\log\left(x^2\right)\right)\sqrt{cx+1}\sqrt{cx-1}-2\left(c^4x^4+c^2x^2-2\right)\arctan\left(\sqrt{cx+1}\sqrt{cx-1}\right)\right)b^2}{12\sqrt{cx+1}\sqrt{cx-1}c^4}$$

input
```
integrate(x^3*(a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

output
```
1/4*b^2*x^4*arcsec(c*x)^2 + 1/4*a^2*x^4 + 1/6*(3*x^4*arcsec(c*x) - (c^2*x^
3*(-1/(c^2*x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*a*b + 1/12*(
(c^2*x^2 + 2*log(x^2))*sqrt(c*x + 1)*sqrt(c*x - 1) - 2*(c^4*x^4 + c^2*x^2
- 2)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)))*b^2/(sqrt(c*x + 1)*sqrt(c*x - 1)
*c^4)
```

3.15.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6625 vs. 2(93) = 186.

Time = 0.61 (sec) , antiderivative size = 6625, normalized size of antiderivative = 61.92

$$\int x^3\left(a+b\sec^{-1}(cx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

output
```
1/12*(3*b^2*arccos(1/(c*x))^2/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)
^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3
/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) + 6*a*b*arccos
(1/(c*x))/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x
^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c
^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) - 12*b^2*(1/(c^2*x^2) - 1)*arccos(
1/(c*x))^2/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2
*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 +
 c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) - 4*b^2*log(2)/
(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2
/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2
*x^2) - 1)^4/(1/(c*x) + 1)^8) + 4*b^2*log(2/(c*x) + 2)/(c^5 + 4*c^5*(1/(c^
2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +
4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*
x) + 1)^8) - 4*b^2*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(c^5 + 4
*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x
) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) -
1)^4/(1/(c*x) + 1)^8) - 4*b^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1
))/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1
)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(...
```

3.15.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a + b \sec^{-1}(cx)\right)^2 \, dx = \int x^3 \left(a + b \operatorname{acos}\left(\frac{1}{c\,x}\right)\right)^2 dx$$

input
```
int(x^3*(a + b*acos(1/(c*x)))^2,x)
```

output
```
int(x^3*(a + b*acos(1/(c*x)))^2, x)
```

3.16 $\int x^2(a+b\sec^{-1}(cx))^2\,dx$

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3.16.1 Optimal result

Integrand size = 14, antiderivative size = 147

$$\int x^2(a+b\sec^{-1}(cx))^2\,dx = \frac{b^2x}{3c^2} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^2(a+b\sec^{-1}(cx))}{3c} + \frac{1}{3}x^3(a+b\sec^{-1}(cx))^2 + \frac{2ib(a+b\sec^{-1}(cx))\arctan\left(e^{i\sec^{-1}(cx)}\right)}{3c^3} - \frac{ib^2\,\mathrm{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)}{3c^3} + \frac{ib^2\,\mathrm{PolyLog}\left(2,ie^{i\sec^{-1}(cx)}\right)}{3c^3}$$

output

```
1/3*b^2*x/c^2+1/3*x^3*(a+b*arcsec(c*x))^2+2/3*I*b*(a+b*arcsec(c*x))*arctan
(1/c/x+I*(1-1/c^2/x^2)^(1/2))/c^3-1/3*I*b^2*polylog(2,-I*(1/c/x+I*(1-1/c^2
/x^2)^(1/2)))/c^3+1/3*I*b^2*polylog(2,I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))/c^3
-1/3*b*x^2*(a+b*arcsec(c*x))*(1-1/c^2/x^2)^(1/2)/c
```

3.16.2 Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.54

$$\int x^2 \left(a + b \sec^{-1}(cx)\right)^2 \, dx$$

$$= \frac{1}{3}\left(a^2 x^3 + \frac{ab\left(2x^4 \sec^{-1}(cx) + \frac{cx - c^3x^3 + \sqrt{-1+c^2x^2}\log\left(-cx+\sqrt{-1+c^2x^2}\right)}{c^4\sqrt{1-\frac{1}{c^2x^2}}}\right)}{x} \right.$$

$$\left. + \frac{b^2\left(cx - c^2\sqrt{1-\frac{1}{c^2x^2}}x^2 \sec^{-1}(cx) + c^3x^3 \sec^{-1}(cx)^2 - \sec^{-1}(cx)\log\left(1 - ie^{i\sec^{-1}(cx)}\right) + \sec^{-1}(cx)\log\left(\right.\right.\right.}{c^3}$$

input
```
Integrate[x^2*(a + b*ArcSec[c*x])^2,x]
```

output
```
(a^2*x^3 + (a*b*(2*x^4*ArcSec[c*x] + (c*x - c^3*x^3 + Sqrt[-1 + c^2*x^2]*L
og[-(c*x) + Sqrt[-1 + c^2*x^2]])/(c^4*Sqrt[1 - 1/(c^2*x^2)])))/x + (b^2*(c
*x - c^2*Sqrt[1 - 1/(c^2*x^2)]*x^2*ArcSec[c*x] + c^3*x^3*ArcSec[c*x]^2 - A
rcSec[c*x]*Log[1 - I*E^(I*ArcSec[c*x])] + ArcSec[c*x]*Log[1 + I*E^(I*ArcSe
c[c*x])] - I*PolyLog[2, (-I)*E^(I*ArcSec[c*x])] + I*PolyLog[2, I*E^(I*ArcS
ec[c*x])]))/c^3)/3
```

3.16.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5745, 4909, 3042, 4673, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a + b \sec^{-1}(cx)\right)^2 \, dx$$

$$\downarrow 5745$$

$$\frac{\int c^4 \sqrt{1-\frac{1}{c^2x^2}} x^4 \left(a + b \sec^{-1}(cx)\right)^2 d\sec^{-1}(cx)}{c^3}$$

$$\downarrow 4909$$

$$\frac{\frac{1}{3}c^3x^3\left(a+b\sec^{-1}(cx)\right)^2-\frac{2}{3}b\int c^3x^3\left(a+b\sec^{-1}(cx)\right)d\sec^{-1}(cx)}{c^3}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{3}c^3x^3\left(a+b\sec^{-1}(cx)\right)^2-\frac{2}{3}b\int \left(a+b\sec^{-1}(cx)\right)\csc\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^3 d\sec^{-1}(cx)}{c^3}$$

$$\downarrow 4673$$

$$\frac{\frac{1}{3}c^3x^3\left(a+b\sec^{-1}(cx)\right)^2-\frac{2}{3}b\left(\frac{1}{2}\int cx\left(a+b\sec^{-1}(cx)\right)d\sec^{-1}(cx)+\frac{1}{2}c^2x^2\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)-\frac{bcx}{2}\right)}{c^3}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{3}c^3x^3\left(a+b\sec^{-1}(cx)\right)^2-\frac{2}{3}b\left(\frac{1}{2}\int \left(a+b\sec^{-1}(cx)\right)\csc\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)d\sec^{-1}(cx)+\frac{1}{2}c^2x^2\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}\right.\right.}{c^3}$$

$$\downarrow 4669$$

$$\frac{\frac{1}{3}c^3x^3\left(a+b\sec^{-1}(cx)\right)^2-\frac{2}{3}b\left(\frac{1}{2}\left(-b\int\log\left(1-ie^{i\sec^{-1}(cx)}\right)d\sec^{-1}(cx)+b\int\log\left(1+ie^{i\sec^{-1}(cx)}\right)d\sec^{-1}(cx)-\right.\right.}{c^3}$$

$$\downarrow 2715$$

$$\frac{\frac{1}{3}c^3x^3\left(a+b\sec^{-1}(cx)\right)^2-\frac{2}{3}b\left(\frac{1}{2}\left(ib\int e^{-i\sec^{-1}(cx)}\log\left(1-ie^{i\sec^{-1}(cx)}\right)de^{i\sec^{-1}(cx)}-ib\int e^{-i\sec^{-1}(cx)}\log\left(1+ie^{i\sec^{-1}}\right.\right.\right.}{c^3}$$

$$\downarrow 2838$$

$$\frac{\frac{1}{3}c^3x^3\left(a+b\sec^{-1}(cx)\right)^2-\frac{2}{3}b\left(\frac{1}{2}\left(-2i\arctan\left(e^{i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)+ib\,\mathrm{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)-ib\,\mathrm{P}\right.\right.\right.}{c^3}$$

input
```
Int[x^2*(a + b*ArcSec[c*x])^2,x]
```

output
```
((c^3*x^3*(a + b*ArcSec[c*x])^2)/3 - (2*b*(-1/2*(b*c*x) + (c^2*Sqrt[1 - 1/
(c^2*x^2)]*x^2*(a + b*ArcSec[c*x]))/2 + ((-2*I)*(a + b*ArcSec[c*x])*ArcTan
[E^(I*ArcSec[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcSec[c*x])] - I*b*PolyLog
[2, I*E^(I*ArcSec[c*x])])/2))/3)/c^3
```

3.16. $\int x^2\left(a+b\sec^{-1}(cx)\right)^2\,dx$

3.16.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

3.16.4 Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.94

method	result
parts	$\frac{a^2x^3}{3}+\frac{b^2\left(\frac{\left(c^2x^2\operatorname{arcsec}(cx)^2-\operatorname{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}+1\right)cx}{3}+\frac{\operatorname{arcsec}(cx)\ln\left(1+i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{3}-\frac{\operatorname{arcsec}(cx)\ln\left(1-i\left(\frac{1}{cx}\right.\right.\right.}{3}}{c^3}$
derivativedivides	$\frac{a^2c^3x^3}{3}+b^2\left(\frac{\left(c^2x^2\operatorname{arcsec}(cx)^2-\operatorname{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}+1\right)cx}{3}+\frac{\operatorname{arcsec}(cx)\ln\left(1+i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{3}-\frac{\operatorname{arcsec}(cx)\ln\left(1-i\left(\frac{1}{cx}\right.\right.\right.}{3}\right)$
default	$\frac{a^2c^3x^3}{3}+b^2\left(\frac{\left(c^2x^2\operatorname{arcsec}(cx)^2-\operatorname{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}+1\right)cx}{3}+\frac{\operatorname{arcsec}(cx)\ln\left(1+i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{3}-\frac{\operatorname{arcsec}(cx)\ln\left(1-i\left(\frac{1}{cx}\right.\right.\right.}{3}\right)$

input
```
int(x^2*(a+b*arcsec(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*x^3+b^2/c^3*(1/3*(c^2*x^2*arcsec(c*x)^2-arcsec(c*x)*c*x*((c^2*x^2-
1)/c^2/x^2)^(1/2)+1)*c*x+1/3*arcsec(c*x)*ln(1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/
2)))-1/3*arcsec(c*x)*ln(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-1/3*I*dilog(1+I
*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))+1/3*I*dilog(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/
2))))+2*a*b/c^3*(1/3*c^3*x^3*arcsec(c*x)-1/6*(c^2*x^2-1)^(1/2)*(c*x*(c^2*x
^2-1)^(1/2)+ln(c*x+(c^2*x^2-1)^(1/2)))/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x)
```

3.16.5 Fricas [F]

$$\int x^2\left(a+b\sec^{-1}(cx)\right)^2\,dx=\int\left(b\operatorname{arcsec}(cx)+a\right)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arcsec(c*x))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^2*arcsec(c*x)^2 + 2*a*b*x^2*arcsec(c*x) + a^2*x^2, x)
```

3.16.6 Sympy [F]

$$\int x^2(a+b\sec^{-1}(cx))^2\,dx = \int x^2(a+b\,\text{asec}\,(cx))^2\,dx$$

input
```
integrate(x**2*(a+b*asec(c*x))**2,x)
```

output
```
Integral(x**2*(a + b*asec(c*x))**2, x)
```

3.16.7 Maxima [F]

$$\int x^2(a+b\sec^{-1}(cx))^2\,dx = \int (b\,\text{arcsec}\,(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*x^3 + 1/6*(4*x^3*arcsec(c*x) - (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(
c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(
c^2*x^2) + 1) - 1)/c^2)/c)*a*b + 1/12*(4*x^3*arctan(sqrt(c*x + 1)*sqrt(c*x
 - 1))^2 - x^3*log(c^2*x^2)^2 - 2*c^2*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x +
 1)/c^5 + 3*log(c*x - 1)/c^5)*log(c)^2 + 36*c^2*integrate(1/3*x^4*log(c^2*
x^2)/(c^2*x^2 - 1), x)*log(c) - 72*c^2*integrate(1/3*x^4*log(x)/(c^2*x^2 -
 1), x)*log(c) + 36*c^2*integrate(1/3*x^4*log(c^2*x^2)*log(x)/(c^2*x^2 - 1
), x) - 36*c^2*integrate(1/3*x^4*log(x)^2/(c^2*x^2 - 1), x) + 12*c^2*integ
rate(1/3*x^4*log(c^2*x^2)/(c^2*x^2 - 1), x) + 6*(2*x/c^2 - log(c*x + 1)/c^
3 + log(c*x - 1)/c^3)*log(c)^2 - 36*integrate(1/3*x^2*log(c^2*x^2)/(c^2*x^
2 - 1), x)*log(c) + 72*integrate(1/3*x^2*log(x)/(c^2*x^2 - 1), x)*log(c) -
 24*integrate(1/3*sqrt(c*x + 1)*sqrt(c*x - 1)*x^2*arctan(sqrt(c*x + 1)*sqr
t(c*x - 1))/(c^2*x^2 - 1), x) - 36*integrate(1/3*x^2*log(c^2*x^2)*log(x)/(
c^2*x^2 - 1), x) + 36*integrate(1/3*x^2*log(x)^2/(c^2*x^2 - 1), x) - 12*in
tegrate(1/3*x^2*log(c^2*x^2)/(c^2*x^2 - 1), x))*b^2
```

3.16.8 Giac [F]

$$\int x^2 \left(a + b \sec^{-1}(cx)\right)^2 \, dx = \int (b \operatorname{arcsec}(cx) + a)^2 x^2 \, dx$$

input
```
integrate(x^2*(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)^2*x^2, x)
```

3.16.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a + b \sec^{-1}(cx)\right)^2 \, dx = \int x^2 \left(a + b \operatorname{acos}\left(\frac{1}{c\,x}\right)\right)^2 dx$$

input
```
int(x^2*(a + b*acos(1/(c*x)))^2,x)
```

output
```
int(x^2*(a + b*acos(1/(c*x)))^2, x)
```

3.17 $\int x(a+b\sec^{-1}(cx))^2\,dx$

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3.17.1 Optimal result

Integrand size = 12, antiderivative size = 56

$$\int x\left(a+b\sec^{-1}(cx)\right)^2\,dx = -\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(a+b\sec^{-1}(cx)\right)}{c} + \frac{1}{2}x^2\left(a+b\sec^{-1}(cx)\right)^2 + \frac{b^2\log(x)}{c^2}$$

output
```
1/2*x^2*(a+b*arcsec(c*x))^2+b^2*ln(x)/c^2-b*x*(a+b*arcsec(c*x))*(1-1/c^2/x
^2)^(1/2)/c
```

3.17.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.61

$$\int x\left(a+b\sec^{-1}(cx)\right)^2\,dx = \frac{acx\left(-2b\sqrt{1-\frac{1}{c^2x^2}}+acx\right)+2bcx\left(-b\sqrt{1-\frac{1}{c^2x^2}}+acx\right)\sec^{-1}(cx)+b^2c^2x^2\sec^{-1}(cx)^2+2b^2\log(cx)}{2c^2}$$

input
```
Integrate[x*(a + b*ArcSec[c*x])^2,x]
```

output
```
(a*c*x*(-2*b*Sqrt[1 - 1/(c^2*x^2)] + a*c*x) + 2*b*c*x*(-(b*Sqrt[1 - 1/(c^2
*x^2)]) + a*c*x)*ArcSec[c*x] + b^2*c^2*x^2*ArcSec[c*x]^2 + 2*b^2*Log[c*x])
/(2*c^2)
```

3.17.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5745, 4909, 3042, 4672, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\sec^{-1}(cx)\right)^2\,dx$$

$$\downarrow 5745$$

$$\frac{\int c^3\sqrt{1-\frac{1}{c^2x^2}}x^3\left(a+b\sec^{-1}(cx)\right)^2 d\sec^{-1}(cx)}{c^2}$$

$$\downarrow 4909$$

$$\frac{\frac{1}{2}c^2x^2\left(a+b\sec^{-1}(cx)\right)^2 - b\int c^2x^2\left(a+b\sec^{-1}(cx)\right)d\sec^{-1}(cx)}{c^2}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{2}c^2x^2\left(a+b\sec^{-1}(cx)\right)^2 - b\int \left(a+b\sec^{-1}(cx)\right)\csc\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^2 d\sec^{-1}(cx)}{c^2}$$

$$\downarrow 4672$$

$$\frac{\frac{1}{2}c^2x^2\left(a+b\sec^{-1}(cx)\right)^2 - b\left(b\int -c\sqrt{1-\frac{1}{c^2x^2}}x d\sec^{-1}(cx) + cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)\right)}{c^2}$$

$$\downarrow 25$$

$$\frac{\frac{1}{2}c^2x^2\left(a+b\sec^{-1}(cx)\right)^2 - b\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right) - b\int c\sqrt{1-\frac{1}{c^2x^2}}x d\sec^{-1}(cx)\right)}{c^2}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{2}c^2x^2\left(a+b\sec^{-1}(cx)\right)^2 - b\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right) - b\int \tan\left(\sec^{-1}(cx)\right)d\sec^{-1}(cx)\right)}{c^2}$$

$$\downarrow 3956$$

$$\frac{\frac{1}{2}c^2x^2\left(a+b\sec^{-1}(cx)\right)^2 - b\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right) + b\log\left(\frac{1}{cx}\right)\right)}{c^2}$$

input
```
Int[x*(a + b*ArcSec[c*x])^2,x]
```

output
```
((c^2*x^2*(a + b*ArcSec[c*x])^2)/2 - b*(c*Sqrt[1 - 1/(c^2*x^2)]*x*(a + b*A
rcSec[c*x]) + b*Log[1/(c*x)]))/c^2
```

3.17.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

3.17.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(52) = 104$.

Time = 0.67 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.20

method	result	size
parts	$\frac{a^2x^2}{2} + \frac{b^2\left(\frac{c^2x^2\,\mathrm{arcsec}(cx)^2}{2} - \mathrm{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}} - \ln\left(\frac{1}{cx}\right)\right)}{c^2} + \frac{2ab\left(\frac{c^2x^2\,\mathrm{arcsec}(cx)}{2} - \frac{c^2x^2-1}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^2}$	123
derivativedivides	$\frac{\frac{a^2c^2x^2}{2} + b^2\left(\frac{c^2x^2\,\mathrm{arcsec}(cx)^2}{2} - \mathrm{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}} - \ln\left(\frac{1}{cx}\right)\right) + 2ab\left(\frac{c^2x^2\,\mathrm{arcsec}(cx)}{2} - \frac{c^2x^2-1}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^2}$	124
default	$\frac{\frac{a^2c^2x^2}{2} + b^2\left(\frac{c^2x^2\,\mathrm{arcsec}(cx)^2}{2} - \mathrm{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}} - \ln\left(\frac{1}{cx}\right)\right) + 2ab\left(\frac{c^2x^2\,\mathrm{arcsec}(cx)}{2} - \frac{c^2x^2-1}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^2}$	124

input
```
int(x*(a+b*arcsec(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x^2+b^2/c^2*(1/2*c^2*x^2*arcsec(c*x)^2-arcsec(c*x)*c*x*((c^2*x^2-1
)/c^2/x^2)^(1/2)-ln(1/c/x))+2*a*b/c^2*(1/2*c^2*x^2*arcsec(c*x)-1/2/((c^2*x
^2-1)/c^2/x^2)^(1/2)/c/x*(c^2*x^2-1))
```

3.17.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(52) = 104$.

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.98

$$\int x\left(a + b\sec^{-1}(cx)\right)^2\,dx$$

$$= \frac{b^2c^2x^2\,\mathrm{arcsec}\left(cx\right)^2 + a^2c^2x^2 + 4\,abc^2\arctan\left(-cx + \sqrt{c^2x^2-1}\right) + 2\,b^2\log\left(x\right) + 2\left(abc^2x^2 - abc^2\right)\mathrm{arcsec}\left(\right.}{2\,c^2}$$

input
```
integrate(x*(a+b*arcsec(c*x))^2,x, algorithm="fricas")
```

output
```
1/2*(b^2*c^2*x^2*arcsec(c*x)^2 + a^2*c^2*x^2 + 4*a*b*c^2*arctan(-c*x + sqr
t(c^2*x^2 - 1)) + 2*b^2*log(x) + 2*(a*b*c^2*x^2 - a*b*c^2)*arcsec(c*x) - 2
*sqrt(c^2*x^2 - 1)*(b^2*arcsec(c*x) + a*b))/c^2
```

3.17.6 Sympy [F]

$$\int x\left(a+b\sec^{-1}(cx)\right)^2\,dx=\int x(a+b\operatorname{asec}(cx))^2\,dx$$

input
```
integrate(x*(a+b*asec(c*x))**2,x)
```

output
```
Integral(x*(a + b*asec(c*x))**2, x)
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.55

$$\begin{aligned}\int x\left(a+b\sec^{-1}(cx)\right)^2\,dx=&\frac{1}{2}\,b^2x^2\operatorname{arcsec}(cx)^2+\frac{1}{2}\,a^2x^2\\&+\left(x^2\operatorname{arcsec}(cx)-\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)ab\\&-\left(\frac{x\sqrt{-\frac{1}{c^2x^2}+1}\operatorname{arcsec}(cx)}{c}-\frac{\log(x)}{c^2}\right)b^2\end{aligned}$$

input
```
integrate(x*(a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

output
```
1/2*b^2*x^2*arcsec(c*x)^2 + 1/2*a^2*x^2 + (x^2*arcsec(c*x) - x*sqrt(-1/(c^
2*x^2) + 1)/c)*a*b - (x*sqrt(-1/(c^2*x^2) + 1)*arcsec(c*x)/c - log(x)/c^2)
*b^2
```

3.17.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2181 vs. 2(52) = 104.

Time = 0.45 (sec) , antiderivative size = 2181, normalized size of antiderivative = 38.95

$$\int x\left(a+b\sec^{-1}(cx)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate(x*(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

output
```
1/2*(b^2*arccos(1/(c*x))^2/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
+ c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) + 2*a*b*arccos(1/(c*x))/(c^3 +
2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x)
 + 1)^4) - 2*b^2*(1/(c^2*x^2) - 1)*arccos(1/(c*x))^2/((c^3 + 2*c^3*(1/(c^2
*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(
c*x) + 1)^2) - 2*b^2*log(2)/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) + 2*b^2*log(2/(c*x) + 2)/(c^3
+ 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*
x) + 1)^4) - 2*b^2*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(c^3 + 2
*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x)
+ 1)^4) - 2*b^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(c^3 + 2*c^
3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1
)^4) - 4*b^2*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x))/((c^3 + 2*c^3*(1/(c^2*
x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c
*x) + 1)) + a^2/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c
^2*x^2) - 1)^2/(1/(c*x) + 1)^4) - 4*a*b*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/
((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/
(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) + b^2*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x)
)^2/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1
)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4) - 4*b^2*(1/(c^2*x^2) - 1)*log(2)/...
```

3.17.9 Mupad [F(-1)]

Timed out.

$$\int x(a+b\sec^{-1}(cx))^2\,dx = \int x\left(a+b\operatorname{acos}\left(\frac{1}{c\,x}\right)\right)^2 dx$$

input
```
int(x*(a + b*acos(1/(c*x)))^2,x)
```

output
```
int(x*(a + b*acos(1/(c*x)))^2, x)
```

3.18 $\int (a + b \sec^{-1}(cx))^2 \, dx$

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3.18.1 Optimal result

Integrand size = 10, antiderivative size = 92

$$\int (a + b \sec^{-1}(cx))^2 \, dx = x(a + b \sec^{-1}(cx))^2 + \frac{4ib(a + b \sec^{-1}(cx)) \arctan\left(e^{i \sec^{-1}(cx)}\right)}{c} - \frac{2ib^2 \operatorname{PolyLog}\left(2, -ie^{i \sec^{-1}(cx)}\right)}{c} + \frac{2ib^2 \operatorname{PolyLog}\left(2, ie^{i \sec^{-1}(cx)}\right)}{c}$$

output

```
x*(a+b*arcsec(c*x))^2+4*I*b*(a+b*arcsec(c*x))*arctan(1/c/x+I*(1-1/c^2/x^2)
^(1/2))/c-2*I*b^2*polylog(2,-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))/c+2*I*b^2*po
lylog(2,I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))/c
```

3.18.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.77

$$\int (a + b \sec^{-1}(cx))^2 \, dx$$
$$= \frac{a^2 cx + 2ab\left(cx \sec^{-1}(cx) + \log\left(\cos\left(\frac{1}{2}\sec^{-1}(cx)\right) - \sin\left(\frac{1}{2}\sec^{-1}(cx)\right)\right) - \log\left(\cos\left(\frac{1}{2}\sec^{-1}(cx)\right) + \sin\left(\frac{1}{2}\sec^{-1}(cx)\right)\right)\right) + b^2\left(cx \sec^{-1}(cx)^2 - 2\sec^{-1}(cx) \log\left(1 - e^{i \sec^{-1}(cx)}\right) + 2\sec^{-1}(cx) \log\left(1 + e^{i \sec^{-1}(cx)}\right) + 2i \operatorname{PolyLog}\left(2, e^{i \sec^{-1}(cx)}\right) - 2i \operatorname{PolyLog}\left(2, -e^{i \sec^{-1}(cx)}\right)\right)}{c}$$

input

```
Integrate[(a + b*ArcSec[c*x])^2,x]
```

output
```
(a^2*c*x + 2*a*b*(c*x*ArcSec[c*x] + Log[Cos[ArcSec[c*x]/2] - Sin[ArcSec[c*
x]/2]] - Log[Cos[ArcSec[c*x]/2] + Sin[ArcSec[c*x]/2]]) + b^2*(ArcSec[c*x]*
(c*x*ArcSec[c*x] - 2*Log[1 - I*E^(I*ArcSec[c*x])] + 2*Log[1 + I*E^(I*ArcSe
c[c*x])]) - (2*I)*PolyLog[2, (-I)*E^(I*ArcSec[c*x])] + (2*I)*PolyLog[2, I*
E^(I*ArcSec[c*x])]))/c
```

3.18.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5739, 4909, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sec^{-1}(cx)\right)^2 dx$$

$$\downarrow 5739$$

$$\frac{\int c^2\sqrt{1-\frac{1}{c^2x^2}}x^2\left(a+b\sec^{-1}(cx)\right)^2 d\sec^{-1}(cx)}{c}$$

$$\downarrow 4909$$

$$\frac{cx\left(a+b\sec^{-1}(cx)\right)^2-2b\int cx\left(a+b\sec^{-1}(cx)\right)d\sec^{-1}(cx)}{c}$$

$$\downarrow 3042$$

$$\frac{cx\left(a+b\sec^{-1}(cx)\right)^2-2b\int \left(a+b\sec^{-1}(cx)\right)\csc\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)d\sec^{-1}(cx)}{c}$$

$$\downarrow 4669$$

$$\frac{cx\left(a+b\sec^{-1}(cx)\right)^2-2b\left(-b\int \log\left(1-ie^{i\sec^{-1}(cx)}\right)d\sec^{-1}(cx)+b\int \log\left(1+ie^{i\sec^{-1}(cx)}\right)d\sec^{-1}(cx)-2i\arct\right.}{c}$$

$$\downarrow 2715$$

$$\frac{cx\left(a+b\sec^{-1}(cx)\right)^2-2b\left(ib\int e^{-i\sec^{-1}(cx)}\log\left(1-ie^{i\sec^{-1}(cx)}\right)de^{i\sec^{-1}(cx)}-ib\int e^{-i\sec^{-1}(cx)}\log\left(1+ie^{i\sec^{-1}(cx)}\right.\right.}{c}$$

$$\downarrow 2838$$

$$\frac{cx\left(a+b\sec^{-1}(cx)\right)^2-2b\Big(-2i\arctan\left(e^{i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)+ib\,\mathrm{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)-ib\,\mathrm{PolyLog}}{c}$$

input
```
Int[(a + b*ArcSec[c*x])^2,x]
```

output
```
(c*x*(a + b*ArcSec[c*x])^2 - 2*b*((-2*I)*(a + b*ArcSec[c*x])*ArcTan[E^(I*A
rcSec[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcSec[c*x])] - I*b*PolyLog[2, I*E
^(I*ArcSec[c*x])]))/c
```

3.18.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5739
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/c   Subst[
Int[(a + b*x)^n*Sec[x]*Tan[x], x], x, ArcSec[c*x]], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[n, 0]
```

3.18.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.11

method	result
derivativedivides	$\frac{a^2cx+b^2\left(\operatorname{arcsec}(cx)^2cx+2\operatorname{arcsec}(cx)\ln\left(1+i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)-2\operatorname{arcsec}(cx)\ln\left(1-i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)-2i\operatorname{dilog}\left(1\right.\right.}{c}$
default	$\frac{a^2cx+b^2\left(\operatorname{arcsec}(cx)^2cx+2\operatorname{arcsec}(cx)\ln\left(1+i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)-2\operatorname{arcsec}(cx)\ln\left(1-i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)-2i\operatorname{dilog}\left(1\right.\right.}{c}$
parts	$a^2x+\frac{b^2\left(\operatorname{arcsec}(cx)^2cx+2\operatorname{arcsec}(cx)\ln\left(1+i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)-2\operatorname{arcsec}(cx)\ln\left(1-i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)-2i\operatorname{dilog}\right.}{c}$

input
```
int((a+b*arcsec(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(a^2*c*x+b^2*(arcsec(c*x)^2*c*x+2*arcsec(c*x)*ln(1+I*(1/c/x+I*(1-1/c^2
/x^2)^(1/2)))-2*arcsec(c*x)*ln(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-2*I*dilo
g(1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))+2*I*dilog(1-I*(1/c/x+I*(1-1/c^2/x^2)^
(1/2))))+2*a*b*(c*x*arcsec(c*x)-ln(c*x+c*x*(1-1/c^2/x^2)^(1/2))))
```

3.18.5 Fricas [F]

$$\int \left(a+b\sec^{-1}(cx)\right)^2\,dx = \int \left(b\operatorname{arcsec}(cx)+a\right)^2\,dx$$

input
```
integrate((a+b*arcsec(c*x))^2,x, algorithm="fricas")
```

output
```
integral(b^2*arcsec(c*x)^2 + 2*a*b*arcsec(c*x) + a^2, x)
```

3.18.6 Sympy [F]

$$\int \left(a + b \sec^{-1}(cx)\right)^2 \, dx = \int \left(a + b \operatorname{asec}(cx)\right)^2 \, dx$$

input
```
integrate((a+b*asec(c*x))**2,x)
```

output
```
Integral((a + b*asec(c*x))**2, x)
```

3.18.7 Maxima [F]

$$\int \left(a + b \sec^{-1}(cx)\right)^2 \, dx = \int \left(b \operatorname{arcsec}(cx) + a\right)^2 dx$$

input
```
integrate((a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

output
```
-1/4*(2*c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*log(c)^2 - 4*c
^2*integrate(x^2*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) + 8*c^2*integrate(x
^2*log(x)/(c^2*x^2 - 1), x)*log(c) - 4*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1
))^2 - 4*c^2*integrate(x^2*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) + 4*c^2*i
ntegrate(x^2*log(x)^2/(c^2*x^2 - 1), x) - 4*c^2*integrate(x^2*log(c^2*x^2)
/(c^2*x^2 - 1), x) + x*log(c^2*x^2)^2 + 2*(log(c*x + 1)/c - log(c*x - 1)/c
)*log(c)^2 + 4*integrate(log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) - 8*integra
te(log(x)/(c^2*x^2 - 1), x)*log(c) + 8*integrate(sqrt(c*x + 1)*sqrt(c*x -
1)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^2 - 1), x) + 4*integrate(log
(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) - 4*integrate(log(x)^2/(c^2*x^2 - 1), x
) + 4*integrate(log(c^2*x^2)/(c^2*x^2 - 1), x))*b^2 + a^2*x + (2*c*x*arcse
c(c*x) - log(sqrt(-1/(c^2*x^2) + 1) + 1) + log(-sqrt(-1/(c^2*x^2) + 1) + 1
))*a*b/c
```

3.18.8 Giac [F]

$$\int \left(a + b \sec^{-1}(cx)\right)^2 \, dx = \int \left(b \operatorname{arcsec}(cx) + a\right)^2 \, dx$$

input
```
integrate((a+b*arcsec(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)^2, x)
```

3.18.9 Mupad [F(-1)]

Timed out.

$$\int \left(a + b \sec^{-1}(cx)\right)^2 \, dx = \int \left(a + b \operatorname{acos}\left(\frac{1}{c\,x}\right)\right)^2 \, dx$$

input
```
int((a + b*acos(1/(c*x)))^2,x)
```

output
```
int((a + b*acos(1/(c*x)))^2, x)
```

3.19 $\int \frac{(a+b\sec^{-1}(cx))^2}{x}\,dx$

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3.19.1 Optimal result

Integrand size = 14, antiderivative size = 93

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x}\,dx = \frac{i(a+b\sec^{-1}(cx))^3}{3b} - (a+b\sec^{-1}(cx))^2\log\left(1+e^{2i\sec^{-1}(cx)}\right) + ib(a+b\sec^{-1}(cx))\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right) - \frac{1}{2}b^2\,\text{PolyLog}\left(3,-e^{2i\sec^{-1}(cx)}\right)$$

output
```
1/3*I*(a+b*arcsec(c*x))^3/b-(a+b*arcsec(c*x))^2*ln(1+(1/c/x+I*(1-1/c^2/x^2
)^(1/2))^2)+I*b*(a+b*arcsec(c*x))*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))
^2)-1/2*b^2*polylog(3,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)
```

3.19.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.39

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x}\,dx = iab\sec^{-1}(cx)^2 + \frac{1}{3}ib^2\sec^{-1}(cx)^3 - 2ab\sec^{-1}(cx)\log\left(1+e^{2i\sec^{-1}(cx)}\right) - b^2\sec^{-1}(cx)^2\log\left(1+e^{2i\sec^{-1}(cx)}\right) + a^2\log(cx) + ib(a+b\sec^{-1}(cx))\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right) - \frac{1}{2}b^2\,\text{PolyLog}\left(3,-e^{2i\sec^{-1}(cx)}\right)$$

input
```
Integrate[(a + b*ArcSec[c*x])^2/x,x]
```

output
```
I*a*b*ArcSec[c*x]^2 + (I/3)*b^2*ArcSec[c*x]^3 - 2*a*b*ArcSec[c*x]*Log[1 +
E^((2*I)*ArcSec[c*x])] - b^2*ArcSec[c*x]^2*Log[1 + E^((2*I)*ArcSec[c*x])]
+ a^2*Log[c*x] + I*b*(a + b*ArcSec[c*x])*PolyLog[2, -E^((2*I)*ArcSec[c*x])
] - (b^2*PolyLog[3, -E^((2*I)*ArcSec[c*x])])/2
```

3.19.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5745, 3042, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x}\,dx$$

$$\downarrow 5745$$

$$\int cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2 d\sec^{-1}(cx)$$

$$\downarrow 3042$$

$$\int \tan\left(\sec^{-1}(cx)\right)\left(a+b\sec^{-1}(cx)\right)^2 d\sec^{-1}(cx)$$

$$\downarrow 4202$$

$$\frac{i\left(a+b\sec^{-1}(cx)\right)^3}{3b} - 2i\int \frac{e^{2i\sec^{-1}(cx)}\left(a+b\sec^{-1}(cx)\right)^2}{1+e^{2i\sec^{-1}(cx)}}d\sec^{-1}(cx)$$

$$\downarrow 2620$$

$$\frac{i\left(a+b\sec^{-1}(cx)\right)^3}{3b} -$$
$$2i\left(ib\int \left(a+b\sec^{-1}(cx)\right)\log\left(1+e^{2i\sec^{-1}(cx)}\right)d\sec^{-1}(cx) - \frac{1}{2}i\log\left(1+e^{2i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)^2\right)$$

$$\downarrow 3011$$

$$\frac{i\left(a+b\sec^{-1}(cx)\right)^3}{3b} -$$
$$2i\left(ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right) - \frac{1}{2}ib\int \text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)d\sec^{-1}(cx)\right) - \frac{1}{2}i\log($$

$$\downarrow 2720$$

$$\frac{i\left(a+b\sec^{-1}(cx)\right)^3}{3b} -$$

$$2i\left(ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)-\frac{1}{4}b\int e^{-2i\sec^{-1}(cx)}\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)de^{2i\sec^{-1}(cx}\right.\right.$$

$$\downarrow 7143$$

$$\frac{i\left(a+b\sec^{-1}(cx)\right)^3}{3b} -$$

$$2i\left(ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)-\frac{1}{4}b\,\text{PolyLog}\left(3,-e^{2i\sec^{-1}(cx)}\right)\right)-\frac{1}{2}i\log\left(1+e^{2i\sec^{-1}(cx)}\right.\right.$$

input
```
Int[(a + b*ArcSec[c*x])^2/x,x]
```

output
```
((I/3)*(a + b*ArcSec[c*x])^3)/b - (2*I)*((-1/2*I)*(a + b*ArcSec[c*x])^2*Lo
g[1 + E^((2*I)*ArcSec[c*x])] + I*b*((I/2)*(a + b*ArcSec[c*x])*PolyLog[2, -
E^((2*I)*ArcSec[c*x])] - (b*PolyLog[3, -E^((2*I)*ArcSec[c*x])])/4))
```

3.19.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.19.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.22

method	result
parts	$a^2 \ln(x) + b^2 \left(\frac{i \operatorname{arcsec}(cx)^3}{3} - \operatorname{arcsec}(cx)^2 \ln\left(1 + \left(\frac{1}{cx} + i\sqrt{1 - \frac{1}{c^2x^2}}\right)^2\right) + i \operatorname{arcsec}(cx)\,\mathrm{p}$
derivativedivides	$a^2 \ln(cx) + b^2 \left(\frac{i \operatorname{arcsec}(cx)^3}{3} - \operatorname{arcsec}(cx)^2 \ln\left(1 + \left(\frac{1}{cx} + i\sqrt{1 - \frac{1}{c^2x^2}}\right)^2\right) + i \operatorname{arcsec}(cx)\,\mathrm{p}$
default	$a^2 \ln(cx) + b^2 \left(\frac{i \operatorname{arcsec}(cx)^3}{3} - \operatorname{arcsec}(cx)^2 \ln\left(1 + \left(\frac{1}{cx} + i\sqrt{1 - \frac{1}{c^2x^2}}\right)^2\right) + i \operatorname{arcsec}(cx)\,\mathrm{p}$

input
```
int((a+b*arcsec(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
a^2*ln(x)+b^2*(1/3*I*arcsec(c*x)^3-arcsec(c*x)^2*ln(1+(1/c/x+I*(1-1/c^2/x^
2)^(1/2))^2)+I*arcsec(c*x)*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)-1/2
*polylog(3,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2))+I*a*b*arcsec(c*x)^2-2*a*b*ar
csec(c*x)*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)+I*a*b*polylog(2,-(1/c/x+I*
(1-1/c^2/x^2)^(1/2))^2)
```

3.19.5 Fricas [F]

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)^2}{x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*arcsec(c*x)^2 + 2*a*b*arcsec(c*x) + a^2)/x, x)
```

3.19.6 Sympy [F]

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x}\,dx = \int \frac{(a+b\operatorname{asec}(cx))^2}{x}\,dx$$

input
```
integrate((a+b*asec(c*x))**2/x,x)
```

output
```
Integral((a + b*asec(c*x))**2/x, x)
```

3.19.7 Maxima [F]

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)^2}{x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))^2/x,x, algorithm="maxima")
```

output
```
-1/2*b^2*c^2*(log(c*x + 1)/c^2 + log(c*x - 1)/c^2)*log(c)^2 + b^2*c^2*inte
grate(x^2*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) - 2*b^2*c^2*integrate(x^2*
log(x)/(c^2*x^3 - x), x)*log(c) + 2*b^2*c^2*integrate(x^2*log(c^2*x^2)*log
(x)/(c^2*x^3 - x), x) - b^2*c^2*integrate(x^2*log(x)^2/(c^2*x^3 - x), x) +
 2*a*b*c^2*integrate(x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^3 - x)
, x) + 1/2*b^2*(log(c*x + 1) + log(c*x - 1) - 2*log(x))*log(c)^2 + b^2*arc
tan(sqrt(c*x + 1)*sqrt(c*x - 1))^2*log(x) - 1/4*b^2*log(c^2*x^2)^2*log(x)
- b^2*integrate(log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) + 2*b^2*integrate(lo
g(x)/(c^2*x^3 - x), x)*log(c) - 2*b^2*integrate(sqrt(c*x + 1)*sqrt(c*x - 1
)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(x)/(c^2*x^3 - x), x) - 2*b^2*int
egrate(log(c^2*x^2)*log(x)/(c^2*x^3 - x), x) + b^2*integrate(log(x)^2/(c^2
*x^3 - x), x) - 2*a*b*integrate(arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x
^3 - x), x) + a^2*log(x)
```

3.19.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\left(a + b \sec^{-1}(cx)\right)^2}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsec(c*x))^2/x,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:ln of unsigned or minus infinity Error: Bad Argument Va
lue
```

3.19.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a + b \sec^{-1}(cx)\right)^2}{x}\,dx = \int \frac{\left(a + b \operatorname{acos}\left(\frac{1}{cx}\right)\right)^2}{x}\,dx$$

input
```
int((a + b*acos(1/(c*x)))^2/x,x)
```

output
```
int((a + b*acos(1/(c*x)))^2/x, x)
```

3.20 $\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^2}\,dx$

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3.20.1 Optimal result

Integrand size = 14, antiderivative size = 50

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^2}\,dx = \frac{2b^2}{x} + 2bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right) - \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x}$$

output
```
2*b^2/x-(a+b*arcsec(c*x))^2/x+2*b*c*(a+b*arcsec(c*x))*(1-1/c^2/x^2)^(1/2)
```

3.20.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.50

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^2}\,dx = \frac{-a^2+2b^2+2abc\sqrt{1-\frac{1}{c^2x^2}}\,x + 2b\left(-a+bc\sqrt{1-\frac{1}{c^2x^2}}\,x\right)\sec^{-1}(cx) - b^2\sec^{-1}(cx)^2}{x}$$

input
```
Integrate[(a + b*ArcSec[c*x])^2/x^2,x]
```

output
```
(-a^2 + 2*b^2 + 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x + 2*b*(-a + b*c*Sqrt[1 - 1
/(c^2*x^2)]*x)*ArcSec[c*x] - b^2*ArcSec[c*x]^2)/x
```

3.20.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5745, 3042, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^2}\,dx$$

$$\downarrow 5745$$

$$c\int \sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2 d\sec^{-1}(cx)$$

$$\downarrow 3042$$

$$c\int \left(a+b\sec^{-1}(cx)\right)^2 \sin\left(\sec^{-1}(cx)\right) d\sec^{-1}(cx)$$

$$\downarrow 3777$$

$$c\left(2b\int \frac{a+b\sec^{-1}(cx)}{cx}d\sec^{-1}(cx) - \frac{\left(a+b\sec^{-1}(cx)\right)^2}{cx}\right)$$

$$\downarrow 3042$$

$$c\left(2b\int \left(a+b\sec^{-1}(cx)\right)\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right) d\sec^{-1}(cx) - \frac{\left(a+b\sec^{-1}(cx)\right)^2}{cx}\right)$$

$$\downarrow 3777$$

$$c\left(2b\left(b\int -\sqrt{1-\frac{1}{c^2x^2}}d\sec^{-1}(cx) + \sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)\right) - \frac{\left(a+b\sec^{-1}(cx)\right)^2}{cx}\right)$$

$$\downarrow 25$$

$$c\left(2b\left(\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right) - b\int \sqrt{1-\frac{1}{c^2x^2}}d\sec^{-1}(cx)\right) - \frac{\left(a+b\sec^{-1}(cx)\right)^2}{cx}\right)$$

$$\downarrow 3042$$

$$c\left(2b\left(\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right) - b\int \sin\left(\sec^{-1}(cx)\right) d\sec^{-1}(cx)\right) - \frac{\left(a+b\sec^{-1}(cx)\right)^2}{cx}\right)$$

$$\downarrow 3118$$

$$c\left(2b\left(\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)+\frac{b}{cx}\right)-\frac{\left(a+b\sec^{-1}(cx)\right)^2}{cx}\right)$$

input
```
Int[(a + b*ArcSec[c*x])^2/x^2,x]
```

output
```
c*(-((a + b*ArcSec[c*x])^2/(c*x)) + 2*b*(b/(c*x) + Sqrt[1 - 1/(c^2*x^2)]*(
a + b*ArcSec[c*x])))
```

3.20.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

3.20.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(48) = 96$.

Time $= 0.56$ (sec) , antiderivative size $= 114$, normalized size of antiderivative $= 2.28$

method	result
parts	$-\frac{a^2}{x} + b^2 c\left(-\frac{\operatorname{arcsec}(cx)^2}{cx} + \frac{2}{cx} + 2\operatorname{arcsec}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right) + 2abc\left(-\frac{\operatorname{arcsec}(cx)}{cx} + \frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x^2}\right.$
derivativedivides	$c\left(-\frac{a^2}{cx} + b^2\left(-\frac{\operatorname{arcsec}(cx)^2}{cx} + \frac{2}{cx} + 2\operatorname{arcsec}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right) + 2ab\left(-\frac{\operatorname{arcsec}(cx)}{cx} + \frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x}\right.\right.$
default	$c\left(-\frac{a^2}{cx} + b^2\left(-\frac{\operatorname{arcsec}(cx)^2}{cx} + \frac{2}{cx} + 2\operatorname{arcsec}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right) + 2ab\left(-\frac{\operatorname{arcsec}(cx)}{cx} + \frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x}\right.\right.$

input
```
int((a+b*arcsec(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/x+b^2*c*(-1/c/x*arcsec(c*x)^2+2/c/x+2*arcsec(c*x)*((c^2*x^2-1)/c^2/x^
2)^(1/2))+2*a*b*c*(-1/c/x*arcsec(c*x)+1/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^2/x^
2*(c^2*x^2-1))
```

3.20.5 Fricas [A] (verification not implemented)

Time $= 0.27$ (sec) , antiderivative size $= 57$, normalized size of antiderivative $= 1.14$

$$\int \frac{(a + b\sec^{-1}(cx))^2}{x^2}\,dx$$
$$= -\frac{b^2\operatorname{arcsec}(cx)^2 + 2\,ab\operatorname{arcsec}(cx) + a^2 - 2\,b^2 - 2\sqrt{c^2x^2-1}(b^2\operatorname{arcsec}(cx) + ab)}{x}$$

input
```
integrate((a+b*arcsec(c*x))^2/x^2,x, algorithm="fricas")
```

output
```
-(b^2*arcsec(c*x)^2 + 2*a*b*arcsec(c*x) + a^2 - 2*b^2 - 2*sqrt(c^2*x^2 - 1
)*(b^2*arcsec(c*x) + a*b))/x
```

3.20.6 Sympy [F]

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^2}\,dx = \int \frac{(a+b\operatorname{asec}(cx))^2}{x^2}\,dx$$

input
```
integrate((a+b*asec(c*x))**2/x**2,x)
```

output
```
Integral((a + b*asec(c*x))**2/x**2, x)
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.56

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^2}\,dx = 2\left(c\sqrt{-\frac{1}{c^2x^2}+1}-\frac{\operatorname{arcsec}(cx)}{x}\right)ab + 2\left(c\sqrt{-\frac{1}{c^2x^2}+1}\operatorname{arcsec}(cx)+\frac{1}{x}\right)b^2 - \frac{b^2\operatorname{arcsec}(cx)^2}{x} - \frac{a^2}{x}$$

input
```
integrate((a+b*arcsec(c*x))^2/x^2,x, algorithm="maxima")
```

output
```
2*(c*sqrt(-1/(c^2*x^2) + 1) - arcsec(c*x)/x)*a*b + 2*(c*sqrt(-1/(c^2*x^2)
+ 1)*arcsec(c*x) + 1/x)*b^2 - b^2*arcsec(c*x)^2/x - a^2/x
```

3.20.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(48) = 96.

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.10

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^2}\,dx = \left(2\,b^2\sqrt{-\frac{1}{c^2x^2}+1}\arccos\left(\frac{1}{cx}\right)+2\,ab\sqrt{-\frac{1}{c^2x^2}+1}-\frac{b^2\arccos\left(\frac{1}{cx}\right)^2}{cx}-\frac{2\,ab\arccos\left(\frac{1}{cx}\right)}{cx}-\frac{a^2}{cx}+\frac{2\,b^2}{cx}\right)c$$

input
```
integrate((a+b*arcsec(c*x))^2/x^2,x, algorithm="giac")
```

output
```
(2*b^2*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x)) + 2*a*b*sqrt(-1/(c^2*x^2) +
1) - b^2*arccos(1/(c*x))^2/(c*x) - 2*a*b*arccos(1/(c*x))/(c*x) - a^2/(c*x)
 + 2*b^2/(c*x))*c
```

3.20.9 Mupad [B] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.78

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^2}\,dx = 2\,b^2\,c\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\,\sqrt{1-\frac{1}{c^2\,x^2}} - \frac{b^2\left(\mathrm{acos}\left(\frac{1}{c\,x}\right)^2-2\right)}{x} - \frac{a^2}{x} + 2\,a\,b\,c\left(\sqrt{1-\frac{1}{c^2\,x^2}} - \frac{\mathrm{acos}\left(\frac{1}{c\,x}\right)}{c\,x}\right)$$

input
```
int((a + b*acos(1/(c*x)))^2/x^2,x)
```

output
```
2*b^2*c*acos(1/(c*x))*(1 - 1/(c^2*x^2))^(1/2) - (b^2*(acos(1/(c*x))^2 - 2)
)/x - a^2/x + 2*a*b*c*((1 - 1/(c^2*x^2))^(1/2) - acos(1/(c*x))/(c*x))
```

3.21 $\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^3}\,dx$

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3.21.1 Optimal result

Integrand size = 14, antiderivative size = 94

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^3}\,dx = \frac{b^2}{4x^2} - \frac{1}{2}abc^2\sec^{-1}(cx) - \frac{1}{4}b^2c^2\sec^{-1}(cx)^2 + \frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{2x} + \frac{1}{2}\left(c^2-\frac{1}{x^2}\right)\left(a+b\sec^{-1}(cx)\right)^2$$

output
```
1/4*b^2/x^2-1/2*a*b*c^2*arcsec(c*x)-1/4*b^2*c^2*arcsec(c*x)^2+1/2*(c^2-1/x
^2)*(a+b*arcsec(c*x))^2+1/2*b*c*(a+b*arcsec(c*x))*(1-1/c^2/x^2)^(1/2)/x
```

3.21.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^3}\,dx = \frac{-2a^2+b^2+2abc\sqrt{1-\frac{1}{c^2x^2}}x + 2b\left(-2a+bc\sqrt{1-\frac{1}{c^2x^2}}x\right)\sec^{-1}(cx) + b^2(-2+c^2x^2)\sec^{-1}(cx)^2 - 2abc^2x$$

input
```
Integrate[(a + b*ArcSec[c*x])^2/x^3,x]
```

output
```
(-2*a^2 + b^2 + 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x + 2*b*(-2*a + b*c*Sqrt[1 -
 1/(c^2*x^2)]*x)*ArcSec[c*x] + b^2*(-2 + c^2*x^2)*ArcSec[c*x]^2 - 2*a*b*c^
2*x^2*ArcSin[1/(c*x)])/(4*x^2)
```

3.21.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5745, 4904, 3042, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^3}\,dx$$

$$\downarrow 5745$$

$$c^2\int \frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{cx}\,d\sec^{-1}(cx)$$

$$\downarrow 4904$$

$$c^2\left(\frac{1}{2}\left(1-\frac{1}{c^2x^2}\right)\left(a+b\sec^{-1}(cx)\right)^2 - b\int\left(1-\frac{1}{c^2x^2}\right)\left(a+b\sec^{-1}(cx)\right)d\sec^{-1}(cx)\right)$$

$$\downarrow 3042$$

$$c^2\left(\frac{1}{2}\left(1-\frac{1}{c^2x^2}\right)\left(a+b\sec^{-1}(cx)\right)^2 - b\int\left(a+b\sec^{-1}(cx)\right)\sin\left(\sec^{-1}(cx)\right)^2 d\sec^{-1}(cx)\right)$$

$$\downarrow 3791$$

$$c^2\left(\frac{1}{2}\left(1-\frac{1}{c^2x^2}\right)\left(a+b\sec^{-1}(cx)\right)^2 - b\left(\frac{1}{2}\int\left(a+b\sec^{-1}(cx)\right)d\sec^{-1}(cx) - \frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{2cx} + \frac{1}{4}b\right.\right.$$

$$\downarrow 17$$

$$c^2\left(\frac{1}{2}\left(1-\frac{1}{c^2x^2}\right)\left(a+b\sec^{-1}(cx)\right)^2 - b\left(-\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{2cx} + \frac{\left(a+b\sec^{-1}(cx)\right)^2}{4b} + \frac{1}{4}b\left(1-\frac{1}{c^2x^2}\right)\right.\right.$$

input
```
Int[(a + b*ArcSec[c*x])^2/x^3,x]
```

output
```
c^2*(((1 - 1/(c^2*x^2))*(a + b*ArcSec[c*x])^2)/2 - b*((b*(1 - 1/(c^2*x^2))
)/4 - (Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcSec[c*x]))/(2*c*x) + (a + b*ArcSec[
c*x])^2/(4*b)))
```

3.21.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

3.21.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(82) = 164$.

Time $= 0.49$ (sec) , antiderivative size $= 177$, normalized size of antiderivative $= 1.88$

method	result
derivativedivides	$c^2\left(-\frac{a^2}{2c^2x^2}+b^2\left(-\frac{\operatorname{arcsec}(cx)^2}{2c^2x^2}+\frac{\operatorname{arcsec}(cx)\left(cx\operatorname{arcsec}(cx)+\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{2cx}-\frac{\operatorname{arcsec}(cx)^2}{4}-\frac{1}{4}+\frac{1}{4c^2x^2}\right)-$
default	$c^2\left(-\frac{a^2}{2c^2x^2}+b^2\left(-\frac{\operatorname{arcsec}(cx)^2}{2c^2x^2}+\frac{\operatorname{arcsec}(cx)\left(cx\operatorname{arcsec}(cx)+\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{2cx}-\frac{\operatorname{arcsec}(cx)^2}{4}-\frac{1}{4}+\frac{1}{4c^2x^2}\right)-$
parts	$-\frac{a^2}{2x^2}+b^2c^2\left(-\frac{\operatorname{arcsec}(cx)^2}{2c^2x^2}+\frac{\operatorname{arcsec}(cx)\left(cx\operatorname{arcsec}(cx)+\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{2cx}-\frac{\operatorname{arcsec}(cx)^2}{4}-\frac{1}{4}+\frac{1}{4c^2x^2}\right)+2c$

input
```
int((a+b*arcsec(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(-1/2*a^2/c^2/x^2+b^2*(-1/2/c^2/x^2*arcsec(c*x)^2+1/2*arcsec(c*x)*(c*x
*arcsec(c*x)+((c^2*x^2-1)/c^2/x^2)^(1/2))/c/x-1/4*arcsec(c*x)^2-1/4+1/4/c^
2/x^2)+2*a*b*(-1/2/c^2/x^2*arcsec(c*x)+1/4*(c^2*x^2-1)^(1/2)*(-arctan(1/(c
^2*x^2-1)^(1/2))*c^2*x^2+(c^2*x^2-1)^(1/2))/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^
3/x^3))
```

3.21.5 Fricas [A] (verification not implemented)

Time $= 0.26$ (sec) , antiderivative size $= 82$, normalized size of antiderivative $= 0.87$

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^3}\,dx$$
$$= \frac{\left(b^2c^2x^2-2\,b^2\right)\operatorname{arcsec}\left(cx\right)^2-2\,a^2+b^2+2\left(abc^2x^2-2\,ab\right)\operatorname{arcsec}\left(cx\right)+2\sqrt{c^2x^2-1}\left(b^2\operatorname{arcsec}\left(cx\right)+ab\right)}{4\,x^2}$$

input
```
integrate((a+b*arcsec(c*x))^2/x^3,x, algorithm="fricas")
```

output
```
1/4*((b^2*c^2*x^2 - 2*b^2)*arcsec(c*x)^2 - 2*a^2 + b^2 + 2*(a*b*c^2*x^2 -
2*a*b)*arcsec(c*x) + 2*sqrt(c^2*x^2 - 1)*(b^2*arcsec(c*x) + a*b))/x^2
```

3.21.6 Sympy [F]

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^3}\,dx = \int \frac{\left(a+b\,\mathrm{asec}\,(cx)\right)^2}{x^3}\,dx$$

input
```
integrate((a+b*asec(c*x))**2/x**3,x)
```

output
```
Integral((a + b*asec(c*x))**2/x**3, x)
```

3.21.7 Maxima [F]

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^3}\,dx = \int \frac{\left(b\,\mathrm{arcsec}\,(cx)+a\right)^2}{x^3}\,dx$$

input
```
integrate((a+b*arcsec(c*x))^2/x^3,x, algorithm="maxima")
```

output
```
-1/2*a*b*((c^4*x*sqrt(-1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) -
c^3*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)))/c + 2*arcsec(c*x)/x^2) - 1/8*(4*(c
^2*(log(c*x + 1) + log(c*x - 1) - 2*log(x))*log(c)^2 - 4*c^2*integrate(1/2
*x^2*log(c^2*x^2)/(c^2*x^5 - x^3), x)*log(c) + 8*c^2*integrate(1/2*x^2*log
(x)/(c^2*x^5 - x^3), x)*log(c) - 4*c^2*integrate(1/2*x^2*log(c^2*x^2)*log(
x)/(c^2*x^5 - x^3), x) + 4*c^2*integrate(1/2*x^2*log(x)^2/(c^2*x^5 - x^3),
 x) + 2*c^2*integrate(1/2*x^2*log(c^2*x^2)/(c^2*x^5 - x^3), x) - (c^2*log(
c*x + 1) + c^2*log(c*x - 1) - 2*c^2*log(x) + 1/x^2)*log(c)^2 + 4*integrate
(1/2*log(c^2*x^2)/(c^2*x^5 - x^3), x)*log(c) - 8*integrate(1/2*log(x)/(c^2
*x^5 - x^3), x)*log(c) - 4*integrate(1/2*sqrt(c*x + 1)*sqrt(c*x - 1)*arcta
n(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^5 - x^3), x) + 4*integrate(1/2*log(c
^2*x^2)*log(x)/(c^2*x^5 - x^3), x) - 4*integrate(1/2*log(x)^2/(c^2*x^5 - x
^3), x) - 2*integrate(1/2*log(c^2*x^2)/(c^2*x^5 - x^3), x))*x^2 + 4*arctan
(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - log(c^2*x^2)^2)*b^2/x^2 - 1/2*a^2/x^2
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.56

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^3}\,dx$$

$$=\frac{1}{8}\left(2\,b^2c\arccos\left(\frac{1}{cx}\right)^2+4\,abc\arccos\left(\frac{1}{cx}\right)-b^2c+\frac{4\,b^2\sqrt{-\frac{1}{c^2x^2}+1}\arccos\left(\frac{1}{cx}\right)}{x}+\frac{4\,ab\sqrt{-\frac{1}{c^2x^2}+1}}{x}-\frac{4}{-}\right.$$

input
```
integrate((a+b*arcsec(c*x))^2/x^3,x, algorithm="giac")
```

output
```
1/8*(2*b^2*c*arccos(1/(c*x))^2 + 4*a*b*c*arccos(1/(c*x)) - b^2*c + 4*b^2*s
qrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x))/x + 4*a*b*sqrt(-1/(c^2*x^2) + 1)/x -
 4*b^2*arccos(1/(c*x))^2/(c*x^2) - 8*a*b*arccos(1/(c*x))/(c*x^2) - 4*a^2/(
c*x^2) + 2*b^2/(c*x^2))*c
```

3.21.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^3}\,dx=\int \frac{\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)^2}{x^3}\,dx$$

input
```
int((a + b*acos(1/(c*x)))^2/x^3,x)
```

output
```
int((a + b*acos(1/(c*x)))^2/x^3, x)
```

3.22 $\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^4}\,dx$

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3.22.1 Optimal result

Integrand size = 14, antiderivative size = 102

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^4}\,dx = \frac{2b^2}{27x^3}+\frac{4b^2c^2}{9x}+\frac{4}{9}bc^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right) + \frac{2bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{9x^2}-\frac{\left(a+b\sec^{-1}(cx)\right)^2}{3x^3}$$

output
```
2/27*b^2/x^3+4/9*b^2*c^2/x-1/3*(a+b*arcsec(c*x))^2/x^3+4/9*b*c^3*(a+b*arcs
ec(c*x))*(1-1/c^2/x^2)^(1/2)+2/9*b*c*(a+b*arcsec(c*x))*(1-1/c^2/x^2)^(1/2)
/x^2
```

3.22.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.06

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^4}\,dx = \frac{-9a^2+6abc\sqrt{1-\frac{1}{c^2x^2}}x(1+2c^2x^2)+2b^2(1+6c^2x^2)+6b\left(-3a+bc\sqrt{1-\frac{1}{c^2x^2}}x(1+2c^2x^2)\right)\sec^{-1}(cx)-}{27x^3}$$

input
```
Integrate[(a + b*ArcSec[c*x])^2/x^4,x]
```

output
```
(-9*a^2 + 6*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2) + 2*b^2*(1 + 6*c
^2*x^2) + 6*b*(-3*a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2))*ArcSec[
c*x] - 9*b^2*ArcSec[c*x]^2)/(27*x^3)
```

3.22.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {5745, 4905, 3042, 3791, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^4}\,dx$$

$$\downarrow 5745$$

$$c^3\int \frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{c^2x^2}\,d\sec^{-1}(cx)$$

$$\downarrow 4905$$

$$c^3\left(\frac{2}{3}b\int \frac{a+b\sec^{-1}(cx)}{c^3x^3}\,d\sec^{-1}(cx)-\frac{\left(a+b\sec^{-1}(cx)\right)^2}{3c^3x^3}\right)$$

$$\downarrow 3042$$

$$c^3\left(\frac{2}{3}b\int \left(a+b\sec^{-1}(cx)\right)\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^3 d\sec^{-1}(cx)-\frac{\left(a+b\sec^{-1}(cx)\right)^2}{3c^3x^3}\right)$$

$$\downarrow 3791$$

$$c^3\left(\frac{2}{3}b\left(\frac{2}{3}\int \frac{a+b\sec^{-1}(cx)}{cx}\,d\sec^{-1}(cx)+\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{3c^2x^2}+\frac{b}{9c^3x^3}\right)-\frac{\left(a+b\sec^{-1}(cx)\right)^2}{3c^3x^3}\right)$$

$$\downarrow 3042$$

$$c^3\left(\frac{2}{3}b\left(\frac{2}{3}\int \left(a+b\sec^{-1}(cx)\right)\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)d\sec^{-1}(cx)+\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{3c^2x^2}+\frac{b}{9c^3x^3}\right)-\frac{(a+}{}\right.$$

$$\downarrow 3777$$

$$c^3\left(\frac{2}{3}b\left(\frac{2}{3}\left(b\int -\sqrt{1-\frac{1}{c^2x^2}}\,d\sec^{-1}(cx)+\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)\right)+\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{3c^2x^2}+\frac{b}{9c^3x^3}\right.\right.$$

$$\downarrow 25$$

$$c^3\left(\frac{2}{3}b\left(\frac{2}{3}\left(\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)-b\int \sqrt{1-\frac{1}{c^2x^2}}\,d\sec^{-1}(cx)\right)+\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{3c^2x^2}+\frac{b}{9c^3x^3}\right.\right.$$

$$\downarrow 3042$$

$$c^3\left(\frac{2}{3}b\left(\frac{2}{3}\left(\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)-b\int \sin\left(\sec^{-1}(cx)\right)d\sec^{-1}(cx)\right)+\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{3c^2x^2}+\frac{b}{9c^3x^3}\right.\right.$$

$$\downarrow 3118$$

$$c^3\left(\frac{2}{3}b\left(\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{3c^2x^2}+\frac{2}{3}\left(\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)+\frac{b}{cx}\right)+\frac{b}{9c^3x^3}\right)-\frac{\left(a+b\sec^{-1}(cx)\right)^2}{3c^3x^3}\right.$$

input
```
Int[(a + b*ArcSec[c*x])^2/x^4,x]
```

output
```
c^3*(-1/3*(a + b*ArcSec[c*x])^2/(c^3*x^3) + (2*b*(b/(9*c^3*x^3) + (Sqrt[1
- 1/(c^2*x^2)]*(a + b*ArcSec[c*x]))/(3*c^2*x^2) + (2*(b/(c*x) + Sqrt[1 - 1
/(c^2*x^2)]*(a + b*ArcSec[c*x])))/3))/3)
```

3.22.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4905
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

3.22.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.50

method	result
parts	$-\frac{a^2}{3x^3}+b^2c^3\left(-\frac{\operatorname{arcsec}(cx)^2}{3c^3x^3}+\frac{2\operatorname{arcsec}(cx)(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{9c^2x^2}+\frac{2}{27c^3x^3}+\frac{4}{9cx}\right)+2ab\,c^3\left(-\frac{\operatorname{arcsec}(c$
derivativedivides	$c^3\left(-\frac{a^2}{3c^3x^3}+b^2\left(-\frac{\operatorname{arcsec}(cx)^2}{3c^3x^3}+\frac{2\operatorname{arcsec}(cx)(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{9c^2x^2}+\frac{2}{27c^3x^3}+\frac{4}{9cx}\right)+2ab\left(-\frac{\operatorname{arcsec}($
default	$c^3\left(-\frac{a^2}{3c^3x^3}+b^2\left(-\frac{\operatorname{arcsec}(cx)^2}{3c^3x^3}+\frac{2\operatorname{arcsec}(cx)(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{9c^2x^2}+\frac{2}{27c^3x^3}+\frac{4}{9cx}\right)+2ab\left(-\frac{\operatorname{arcsec}($

input
```
int((a+b*arcsec(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2/x^3+b^2*c^3*(-1/3/c^3/x^3*arcsec(c*x)^2+2/9*arcsec(c*x)*(2*c^2*x^
2+1)/c^2/x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)+2/27/c^3/x^3+4/9/c/x)+2*a*b*c^3*(
-1/3/c^3/x^3*arcsec(c*x)+1/9*(c^2*x^2-1)*(2*c^2*x^2+1)/((c^2*x^2-1)/c^2/x^
2)^(1/2)/c^4/x^4)
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^4}\,dx$$
$$= \frac{12\,b^2c^2x^2 - 9\,b^2\operatorname{arcsec}(cx)^2 - 18\,ab\operatorname{arcsec}(cx) - 9\,a^2 + 2\,b^2 + 6\,(2\,abc^2x^2 + ab + (2\,b^2c^2x^2 + b^2)\operatorname{arcsec}(c$$
$$27\,x^3$$

input
```
integrate((a+b*arcsec(c*x))^2/x^4,x, algorithm="fricas")
```

output
```
1/27*(12*b^2*c^2*x^2 - 9*b^2*arcsec(c*x)^2 - 18*a*b*arcsec(c*x) - 9*a^2 +
2*b^2 + 6*(2*a*b*c^2*x^2 + a*b + (2*b^2*c^2*x^2 + b^2)*arcsec(c*x))*sqrt(c
^2*x^2 - 1))/x^3
```

3.22.6 Sympy [F]

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^4}\,dx = \int \frac{(a+b\operatorname{asec}(cx))^2}{x^4}\,dx$$

input
```
integrate((a+b*asec(c*x))**2/x**4,x)
```

output
```
Integral((a + b*asec(c*x))**2/x**4, x)
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.61

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^4}\,dx$$
$$= -\frac{2}{9}ab\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{3\,\mathrm{arcsec}\,(cx)}{x^3}\right)-\frac{b^2\,\mathrm{arcsec}\,(cx)^2}{3\,x^3}-\frac{a^2}{3\,x^3}$$
$$+\frac{2\left(\left(6\,c^3x^2+c\right)\sqrt{cx+1}\sqrt{cx-1}+3\left(2\,c^5x^4-c^3x^2-c\right)\arctan\left(\sqrt{cx+1}\sqrt{cx-1}\right)\right)b^2}{27\sqrt{cx+1}\sqrt{cx-1}cx^3}$$

input
```
integrate((a+b*arcsec(c*x))^2/x^4,x, algorithm="maxima")
```

output
```
-2/9*a*b*((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c
+ 3*arcsec(c*x)/x^3) - 1/3*b^2*arcsec(c*x)^2/x^3 - 1/3*a^2/x^3 + 2/27*((6*
c^3*x^2 + c)*sqrt(c*x + 1)*sqrt(c*x - 1) + 3*(2*c^5*x^4 - c^3*x^2 - c)*arc
tan(sqrt(c*x + 1)*sqrt(c*x - 1)))*b^2/(sqrt(c*x + 1)*sqrt(c*x - 1)*c*x^3)
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.65

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^4}\,dx$$
$$= \frac{1}{27}\left(12\,b^2c^2\sqrt{-\frac{1}{c^2x^2}+1}\arccos\left(\frac{1}{cx}\right)+12\,abc^2\sqrt{-\frac{1}{c^2x^2}+1}+\frac{12\,b^2c}{x}+\frac{6\,b^2\sqrt{-\frac{1}{c^2x^2}+1}\arccos\left(\frac{1}{cx}\right)}{x^2}+\frac{6}{}\right.$$

input
```
integrate((a+b*arcsec(c*x))^2/x^4,x, algorithm="giac")
```

output
```
1/27*(12*b^2*c^2*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x)) + 12*a*b*c^2*sqrt(
-1/(c^2*x^2) + 1) + 12*b^2*c/x + 6*b^2*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*
x))/x^2 + 6*a*b*sqrt(-1/(c^2*x^2) + 1)/x^2 - 9*b^2*arccos(1/(c*x))^2/(c*x^
3) - 18*a*b*arccos(1/(c*x))/(c*x^3) - 9*a^2/(c*x^3) + 2*b^2/(c*x^3))*c
```

3.22.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^4}\,dx = \int \frac{\left(a+b\,\mathrm{acos}\left(\frac{1}{cx}\right)\right)^2}{x^4}\,dx$$

input
```
int((a + b*acos(1/(c*x)))^2/x^4,x)
```

output
```
int((a + b*acos(1/(c*x)))^2/x^4, x)
```

3.23 $\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^5}\,dx$

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3.23.1 Optimal result

Integrand size = 14, antiderivative size = 134

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^5}\,dx = \frac{b^2}{32x^4}+\frac{3b^2c^2}{32x^2}+\frac{3}{16}abc^4\sec^{-1}(cx) + \frac{3}{32}b^2c^4\sec^{-1}(cx)^2+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{8x^3} + \frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{16x}-\frac{\left(a+b\sec^{-1}(cx)\right)^2}{4x^4}$$

output

```
1/32*b^2/x^4+3/32*b^2*c^2/x^2+3/16*a*b*c^4*arcsec(c*x)+3/32*b^2*c^4*arcsec
(c*x)^2-1/4*(a+b*arcsec(c*x))^2/x^4+1/8*b*c*(a+b*arcsec(c*x))*(1-1/c^2/x^2
)^(1/2)/x^3+3/16*b*c^3*(a+b*arcsec(c*x))*(1-1/c^2/x^2)^(1/2)/x
```

3.23.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^5}\,dx = \frac{-8a^2+b^2+4abc\sqrt{1-\frac{1}{c^2x^2}}x+3b^2c^2x^2+6abc^3\sqrt{1-\frac{1}{c^2x^2}}x^3+2b\left(-8a+bc\sqrt{1-\frac{1}{c^2x^2}}x(2+3c^2x^2)\right)\sec^{-}}{32x^4}$$

input
```
Integrate[(a + b*ArcSec[c*x])^2/x^5,x]
```

output
```
(-8*a^2 + b^2 + 4*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x + 3*b^2*c^2*x^2 + 6*a*b*c^
3*Sqrt[1 - 1/(c^2*x^2)]*x^3 + 2*b*(-8*a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(2 +
 3*c^2*x^2))*ArcSec[c*x] + b^2*(-8 + 3*c^4*x^4)*ArcSec[c*x]^2 - 6*a*b*c^4*
x^4*ArcSin[1/(c*x)])/(32*x^4)
```

3.23.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5745, 4905, 3042, 3791, 3042, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^5}\,dx$$

$$\downarrow 5745$$

$$c^4\int \frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{c^3x^3}\,d\sec^{-1}(cx)$$

$$\downarrow 4905$$

$$c^4\left(\frac{1}{2}b\int \frac{a+b\sec^{-1}(cx)}{c^4x^4}\,d\sec^{-1}(cx) - \frac{\left(a+b\sec^{-1}(cx)\right)^2}{4c^4x^4}\right)$$

$$\downarrow 3042$$

$$c^4\left(\frac{1}{2}b\int \left(a+b\sec^{-1}(cx)\right)\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^4 d\sec^{-1}(cx) - \frac{\left(a+b\sec^{-1}(cx)\right)^2}{4c^4x^4}\right)$$

$$\downarrow 3791$$

$$c^4\left(\frac{1}{2}b\left(\frac{3}{4}\int \frac{a+b\sec^{-1}(cx)}{c^2x^2}\,d\sec^{-1}(cx) + \frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{4c^3x^3} + \frac{b}{16c^4x^4}\right) - \frac{\left(a+b\sec^{-1}(cx)\right)^2}{4c^4x^4}\right)$$

$$\downarrow 3042$$

$$c^4\left(\frac{1}{2}b\left(\frac{3}{4}\int\left(a+b\sec^{-1}(cx)\right)\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^2 d\sec^{-1}(cx)+\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{4c^3x^3}+\frac{b}{16c^4x^4}\right)-\frac{(a}{}\right.$$

$$\downarrow 3791$$

$$c^4\left(\frac{1}{2}b\left(\frac{3}{4}\left(\frac{1}{2}\int\left(a+b\sec^{-1}(cx)\right)d\sec^{-1}(cx)+\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{2cx}+\frac{b}{4c^2x^2}\right)+\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-}\right.}{4c^3x^3}\right.$$

$$\downarrow 17$$

$$c^4\left(\frac{1}{2}b\left(\frac{3}{4}\left(\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{2cx}+\frac{\left(a+b\sec^{-1}(cx)\right)^2}{4b}+\frac{b}{4c^2x^2}\right)+\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{4c^3x^3}+\frac{b}{16c^4x}\right.\right.$$

input
```
Int[(a + b*ArcSec[c*x])^2/x^5,x]
```

output
```
c^4*(-1/4*(a + b*ArcSec[c*x])^2/(c^4*x^4) + (b*(b/(16*c^4*x^4) + (Sqrt[1 -
 1/(c^2*x^2)]*(a + b*ArcSec[c*x]))/(4*c^3*x^3) + (3*(b/(4*c^2*x^2) + (Sqrt
[1 - 1/(c^2*x^2)]*(a + b*ArcSec[c*x]))/(2*c*x) + (a + b*ArcSec[c*x])^2/(4*
b)))/4))/2)
```

3.23.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)  Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4905
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

3.23.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(116) = 232$.

Time = 0.82 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.98

method	result
parts	$-\frac{a^2}{4x^4}+b^2c^4\left(-\frac{\operatorname{arcsec}(cx)^2}{4c^4x^4}+\frac{\operatorname{arcsec}(cx)\left(3c^3x^3\operatorname{arcsec}(cx)+3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}+2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{16c^3x^3}-\frac{3\operatorname{arcsec}(cx)^2}{32}+$
derivativedivides	$c^4\left(-\frac{a^2}{4c^4x^4}+b^2\left(-\frac{\operatorname{arcsec}(cx)^2}{4c^4x^4}+\frac{\operatorname{arcsec}(cx)\left(3c^3x^3\operatorname{arcsec}(cx)+3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}+2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{16c^3x^3}-\frac{3\operatorname{arcsec}(cx$
default	$c^4\left(-\frac{a^2}{4c^4x^4}+b^2\left(-\frac{\operatorname{arcsec}(cx)^2}{4c^4x^4}+\frac{\operatorname{arcsec}(cx)\left(3c^3x^3\operatorname{arcsec}(cx)+3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}+2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{16c^3x^3}-\frac{3\operatorname{arcsec}(cx$

input
```
int((a+b*arcsec(c*x))^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^2/x^4+b^2*c^4*(-1/4/c^4/x^4*arcsec(c*x)^2+1/16*arcsec(c*x)*(3*c^3*x
^3*arcsec(c*x)+3*c^2*x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)+2*((c^2*x^2-1)/c^2/x^
2)^(1/2))/c^3/x^3-3/32*arcsec(c*x)^2+1/128*(3*c^2*x^2+2)^2/c^4/x^4)-1/2*a*
b/x^4*arcsec(c*x)-3/16*a*b*c^3*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/
2)/x*arctan(1/(c^2*x^2-1)^(1/2))+3/16*a*b*c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x
^2)^(1/2)/x^3+1/8*a*b/c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^5
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^5}\,dx$$
$$= \frac{3\,b^2c^2x^2+\left(3\,b^2c^4x^4-8\,b^2\right)\operatorname{arcsec}(cx)^2-8\,a^2+b^2+2\left(3\,abc^4x^4-8\,ab\right)\operatorname{arcsec}(cx)+2\left(3\,abc^2x^2+2\,ab+\right.}{32\,x^4}$$

input
```
integrate((a+b*arcsec(c*x))^2/x^5,x, algorithm="fricas")
```

output
```
1/32*(3*b^2*c^2*x^2 + (3*b^2*c^4*x^4 - 8*b^2)*arcsec(c*x)^2 - 8*a^2 + b^2
+ 2*(3*a*b*c^4*x^4 - 8*a*b)*arcsec(c*x) + 2*(3*a*b*c^2*x^2 + 2*a*b + (3*b^
2*c^2*x^2 + 2*b^2)*arcsec(c*x))*sqrt(c^2*x^2 - 1))/x^4
```

3.23.6 Sympy [F]

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^5}\,dx = \int \frac{\left(a+b\operatorname{asec}(cx)\right)^2}{x^5}\,dx$$

input
```
integrate((a+b*asec(c*x))**2/x**5,x)
```

output
```
Integral((a + b*asec(c*x))**2/x**5, x)
```

3.23.7 Maxima [F]

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^5}\,dx = \int \frac{\left(b\operatorname{arcsec}(cx)+a\right)^2}{x^5}\,dx$$

input
```
integrate((a+b*arcsec(c*x))^2/x^5,x, algorithm="maxima")
```

output
```
1/16*a*b*((3*c^5*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)) + (3*c^8*x^3*(-1/(c^2*
x^2) + 1)^(3/2) + 5*c^6*x*sqrt(-1/(c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2) -
1)^2 - 2*c^2*x^2*(1/(c^2*x^2) - 1) + 1))/c - 8*arcsec(c*x)/x^4) - 1/16*(4*
(2*(c^2*log(c*x + 1) + c^2*log(c*x - 1) - 2*c^2*log(x) + 1/x^2)*c^2*log(c)
^2 - 16*c^2*integrate(1/4*x^2*log(c^2*x^2)/(c^2*x^7 - x^5), x)*log(c) + 32
*c^2*integrate(1/4*x^2*log(x)/(c^2*x^7 - x^5), x)*log(c) - 16*c^2*integrat
e(1/4*x^2*log(c^2*x^2)*log(x)/(c^2*x^7 - x^5), x) + 16*c^2*integrate(1/4*x
^2*log(x)^2/(c^2*x^7 - x^5), x) + 4*c^2*integrate(1/4*x^2*log(c^2*x^2)/(c^
2*x^7 - x^5), x) - (2*c^4*log(c*x + 1) + 2*c^4*log(c*x - 1) - 4*c^4*log(x)
 + (2*c^2*x^2 + 1)/x^4)*log(c)^2 + 16*integrate(1/4*log(c^2*x^2)/(c^2*x^7
- x^5), x)*log(c) - 32*integrate(1/4*log(x)/(c^2*x^7 - x^5), x)*log(c) - 8
*integrate(1/4*sqrt(c*x + 1)*sqrt(c*x - 1)*arctan(sqrt(c*x + 1)*sqrt(c*x -
 1))/(c^2*x^7 - x^5), x) + 16*integrate(1/4*log(c^2*x^2)*log(x)/(c^2*x^7 -
 x^5), x) - 16*integrate(1/4*log(x)^2/(c^2*x^7 - x^5), x) - 4*integrate(1/
4*log(c^2*x^2)/(c^2*x^7 - x^5), x))*x^4 + 4*arctan(sqrt(c*x + 1)*sqrt(c*x
- 1))^2 - log(c^2*x^2)^2)*b^2/x^4 - 1/4*a^2/x^4
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.60

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^5}\,dx$$

$$=\frac{1}{256}\left(24\,b^2c^3\arccos\left(\frac{1}{cx}\right)^2+48\,abc^3\arccos\left(\frac{1}{cx}\right)-15\,b^2c^3+\frac{48\,b^2c^2\sqrt{-\frac{1}{c^2x^2}+1}\arccos\left(\frac{1}{cx}\right)}{x}+\frac{48\,abc^2}{}\right.$$

input
```
integrate((a+b*arcsec(c*x))^2/x^5,x, algorithm="giac")
```

output
```
1/256*(24*b^2*c^3*arccos(1/(c*x))^2 + 48*a*b*c^3*arccos(1/(c*x)) - 15*b^2*
c^3 + 48*b^2*c^2*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x))/x + 48*a*b*c^2*sqr
t(-1/(c^2*x^2) + 1)/x + 24*b^2*c/x^2 + 32*b^2*sqrt(-1/(c^2*x^2) + 1)*arcco
s(1/(c*x))/x^3 + 32*a*b*sqrt(-1/(c^2*x^2) + 1)/x^3 - 64*b^2*arccos(1/(c*x)
)^2/(c*x^4) - 128*a*b*arccos(1/(c*x))/(c*x^4) - 64*a^2/(c*x^4) + 8*b^2/(c*
x^4))*c
```

3.23.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^5}\,dx = \int \frac{\left(a+b\,\mathrm{acos}\left(\frac{1}{cx}\right)\right)^2}{x^5}\,dx$$

input
```
int((a + b*acos(1/(c*x)))^2/x^5,x)
```

output
```
int((a + b*acos(1/(c*x)))^2/x^5, x)
```

3.24 $\int x^3(a+b\sec^{-1}(cx))^3\,dx$

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3.24.1 Optimal result

Integrand size = 14, antiderivative size = 207

$$\begin{aligned}\int x^3(a+b\sec^{-1}(cx))^3\,dx = &-\frac{b^3\sqrt{1-\frac{1}{c^2x^2}}x}{4c^3}+\frac{b^2x^2(a+b\sec^{-1}(cx))}{4c^2}\\&+\frac{ib(a+b\sec^{-1}(cx))^2}{2c^4}-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x(a+b\sec^{-1}(cx))^2}{2c^3}\\&-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^3(a+b\sec^{-1}(cx))^2}{4c}+\frac{1}{4}x^4(a+b\sec^{-1}(cx))^3\\&-\frac{b^2(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{c^4}\\&+\frac{ib^3\,\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2c^4}\end{aligned}$$

output

```
1/4*b^2*x^2*(a+b*arcsec(c*x))/c^2+1/2*I*b*(a+b*arcsec(c*x))^2/c^4+1/4*x^4*
(a+b*arcsec(c*x))^3-b^2*(a+b*arcsec(c*x))*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2
))^2)/c^4+1/2*I*b^3*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)/c^4-1/4*b^
3*x*(1-1/c^2/x^2)^(1/2)/c^3-1/2*b*x*(a+b*arcsec(c*x))^2*(1-1/c^2/x^2)^(1/2
)/c^3-1/4*b*x^3*(a+b*arcsec(c*x))^2*(1-1/c^2/x^2)^(1/2)/c
```

3.24.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.39

$$\int x^3\left(a+b\sec^{-1}(cx)\right)^3\,dx$$
$$=\frac{-2a^2bc\sqrt{1-\frac{1}{c^2x^2}}x-b^3c\sqrt{1-\frac{1}{c^2x^2}}x+ab^2c^2x^2-a^2bc^3\sqrt{1-\frac{1}{c^2x^2}}x^3+a^3c^4x^4-b^2\left(-3ac^4x^4+b\left(-2i+2\right.\right.}{}$$

input
```
Integrate[x^3*(a + b*ArcSec[c*x])^3,x]
```

output
```
(-2*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x - b^3*c*Sqrt[1 - 1/(c^2*x^2)]*x + a*b^
2*c^2*x^2 - a^2*b*c^3*Sqrt[1 - 1/(c^2*x^2)]*x^3 + a^3*c^4*x^4 - b^2*(-3*a*
c^4*x^4 + b*(-2*I + 2*c*Sqrt[1 - 1/(c^2*x^2)]*x + c^3*Sqrt[1 - 1/(c^2*x^2)
]*x^3))*ArcSec[c*x]^2 + b^3*c^4*x^4*ArcSec[c*x]^3 + b*ArcSec[c*x]*(c*x*(b^
2*c*x + 3*a^2*c^3*x^3 - 2*a*b*Sqrt[1 - 1/(c^2*x^2)]*(2 + c^2*x^2)) - 4*b^2
*Log[1 + E^((2*I)*ArcSec[c*x])]) - 4*a*b^2*Log[1/(c*x)] + (2*I)*b^3*PolyLo
g[2, -E^((2*I)*ArcSec[c*x])])/(4*c^4)
```

3.24.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5745, 4909, 3042, 4674, 3042, 4254, 24, 4672, 25, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\sec^{-1}(cx)\right)^3\,dx$$
$$\downarrow 5745$$
$$\frac{\int c^5\sqrt{1-\frac{1}{c^2x^2}}x^5\left(a+b\sec^{-1}(cx)\right)^3\,d\sec^{-1}(cx)}{c^4}$$
$$\downarrow 4909$$
$$\frac{\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{4}b\int c^4x^4\left(a+b\sec^{-1}(cx)\right)^2\,d\sec^{-1}(cx)}{c^4}$$
$$\downarrow 3042$$

$$\frac{\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{4}b\int\left(a+b\sec^{-1}(cx)\right)^2\csc\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^4 d\sec^{-1}(cx)}{c^4}$$

$$\downarrow 4674$$

$$\frac{\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{4}b\left(\frac{2}{3}\int c^2x^2\left(a+b\sec^{-1}(cx)\right)^2 d\sec^{-1}(cx)+\frac{1}{3}b^2\int c^2x^2d\sec^{-1}(cx)-\frac{1}{3}bc^2x^2\left(a+b\sec^{-1}\right.\right.}{c^4}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{4}b\left(\frac{2}{3}\int\left(a+b\sec^{-1}(cx)\right)^2\csc\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^2 d\sec^{-1}(cx)+\frac{1}{3}b^2\int\csc\left(\sec^{-1}(cx)+\frac{\pi}{2}\right.\right.}{c^4}$$

$$\downarrow 4254$$

$$\frac{\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{4}b\left(\frac{2}{3}\int\left(a+b\sec^{-1}(cx)\right)^2\csc\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^2 d\sec^{-1}(cx)-\frac{1}{3}b^2\int 1d\left(-c\sqrt{1-\frac{1}{c^2x^2}}x\right.\right.}{c^4}$$

$$\downarrow 24$$

$$\frac{\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{4}b\left(\frac{2}{3}\int\left(a+b\sec^{-1}(cx)\right)^2\csc\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^2 d\sec^{-1}(cx)-\frac{1}{3}bc^2x^2\left(a+b\sec^{-1}(cx)\right)\right.}{c^4}$$

$$\downarrow 4672$$

$$\frac{\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{4}b\left(\frac{2}{3}\left(2b\int-c\sqrt{1-\frac{1}{c^2x^2}}x\left(a+b\sec^{-1}(cx)\right)d\sec^{-1}(cx)+cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right.\right.\right.}{c^4}$$

$$\downarrow 25$$

$$\frac{\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{4}b\left(\frac{2}{3}\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\int c\sqrt{1-\frac{1}{c^2x^2}}x\left(a+b\sec^{-1}(cx)\right)d\sec^{-1}(cx\right.\right.\right.}{c^4}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{4}b\left(\frac{2}{3}\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\int\left(a+b\sec^{-1}(cx)\right)\tan\left(\sec^{-1}(cx)\right)d\sec^{-1}\right.\right.\right.}{c^4}$$

$$\downarrow 4202$$

$$\frac{\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{4}b\left(\frac{2}{3}\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\left(\frac{i\left(a+b\sec^{-1}(cx)\right)^2}{2b}-2i\int\frac{e^{2i\sec^{-1}(cx)}\left(a+b\sec^{-1}\right.\right.\right.\right.}{c^4}$$

$$\downarrow 2620$$

$$\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{4}b\left(\frac{2}{3}\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\left(\frac{i\left(a+b\sec^{-1}(cx)\right)^2}{2b}-2i\left(\frac{1}{2}ib\int\log\left(1+e^{2i\sec}\right.\right.\right.\right.$$

$$\downarrow 2715$$

$$\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{4}b\left(\frac{2}{3}\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\left(\frac{i\left(a+b\sec^{-1}(cx)\right)^2}{2b}-2i\left(\frac{1}{4}b\int e^{-2i\sec^{-1}(cx)}\log\right.\right.\right.\right.$$

$$\downarrow 2838$$

$$\frac{1}{4}c^4x^4\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{4}b\left(\frac{2}{3}\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\left(\frac{i\left(a+b\sec^{-1}(cx)\right)^2}{2b}-2i\left(-\frac{1}{2}i\log\left(1+e^{2i\sec^{-}}\right.\right.\right.\right.\right.$$

input
```
Int[x^3*(a + b*ArcSec[c*x])^3,x]
```

output
```
((c^4*x^4*(a + b*ArcSec[c*x])^3)/4 - (3*b*((b^2*c*Sqrt[1 - 1/(c^2*x^2)]*x)
/3 - (b*c^2*x^2*(a + b*ArcSec[c*x]))/3 + (c^3*Sqrt[1 - 1/(c^2*x^2)]*x^3*(a
 + b*ArcSec[c*x])^2)/3 + (2*(c*Sqrt[1 - 1/(c^2*x^2)]*x*(a + b*ArcSec[c*x])
^2 - 2*b*(((I/2)*(a + b*ArcSec[c*x])^2)/b - (2*I)*((-1/2*I)*(a + b*ArcSec[
c*x])*Log[1 + E^((2*I)*ArcSec[c*x])] - (b*PolyLog[2, -E^((2*I)*ArcSec[c*x]
)])/4))))/3))/4)/c^4
```

3.24.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5745

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

3.24.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.76

method	result
derivativedivides	$\frac{a^3c^4x^4}{4}+b^3\left(\frac{\operatorname{arcsec}(cx)^3c^4x^4}{4}-\frac{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,\operatorname{arcsec}(cx)^2c^3x^3}{4}-\frac{\operatorname{arcsec}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}{2}+\frac{i\operatorname{arcsec}(cx)^2}{2}+\frac{c^2x^2\operatorname{arcsec}(cx)}{4}-\frac{xc\sqrt{\frac{c^2}{}}}{}\right.$
default	$\frac{a^3c^4x^4}{4}+b^3\left(\frac{\operatorname{arcsec}(cx)^3c^4x^4}{4}-\frac{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,\operatorname{arcsec}(cx)^2c^3x^3}{4}-\frac{\operatorname{arcsec}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}{2}+\frac{i\operatorname{arcsec}(cx)^2}{2}+\frac{c^2x^2\operatorname{arcsec}(cx)}{4}-\frac{xc\sqrt{\frac{c^2}{}}}{}\right.$
parts	$\frac{a^3x^4}{4}+\frac{b^3\left(\frac{\operatorname{arcsec}(cx)^3c^4x^4}{4}-\frac{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,\operatorname{arcsec}(cx)^2c^3x^3}{4}-\frac{\operatorname{arcsec}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}{2}+\frac{i\operatorname{arcsec}(cx)^2}{2}+\frac{c^2x^2\operatorname{arcsec}(cx)}{4}-\frac{xc\sqrt{\frac{c^2}{}}}{}\right.}{c^4}$

input

```
int(x^3*(a+b*arcsec(c*x))^3,x,method=_RETURNVERBOSE)
```

output

```
1/c^4*(1/4*a^3*c^4*x^4+b^3*(1/4*arcsec(c*x)^3*c^4*x^4-1/4*((c^2*x^2-1)/c^2
/x^2)^(1/2)*arcsec(c*x)^2*c^3*x^3-1/2*arcsec(c*x)^2*((c^2*x^2-1)/c^2/x^2)^
(1/2)*c*x+1/2*I*arcsec(c*x)^2+1/4*c^2*x^2*arcsec(c*x)-1/4*x*c*((c^2*x^2-1)
/c^2/x^2)^(1/2)-1/4*I-arcsec(c*x)*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)+1/
2*I*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2))+3*a*b^2*(1/4*arcsec(c*x)^
2*c^4*x^4-1/6*arcsec(c*x)*((c^2*x^2-1)/c^2/x^2)^(1/2)*c^3*x^3+1/12*c^2*x^2
-1/3*arcsec(c*x)*c*x*((c^2*x^2-1)/c^2/x^2)^(1/2)-1/3*ln(1/c/x))+3*a^2*b*(1
/4*c^4*x^4*arcsec(c*x)-1/12*(c^2*x^2-1)*(c^2*x^2+2)/((c^2*x^2-1)/c^2/x^2)^
(1/2)/c/x))
```

3.24.5 Fricas [F]

$$\int x^3\left(a+b\sec^{-1}(cx)\right)^3\,dx = \int (b\operatorname{arcsec}(cx)+a)^3x^3\,dx$$

input
```
integrate(x^3*(a+b*arcsec(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^3*arcsec(c*x)^3 + 3*a*b^2*x^3*arcsec(c*x)^2 + 3*a^2*b*x^3*a
rcsec(c*x) + a^3*x^3, x)
```

3.24.6 Sympy [F]

$$\int x^3\left(a+b\sec^{-1}(cx)\right)^3\,dx = \int x^3(a+b\operatorname{asec}(cx))^3\,dx$$

input
```
integrate(x**3*(a+b*asec(c*x))**3,x)
```

output
```
Integral(x**3*(a + b*asec(c*x))**3, x)
```

3.24.7 Maxima [F]

$$\int x^3\left(a+b\sec^{-1}(cx)\right)^3\,dx = \int (b\operatorname{arcsec}(cx)+a)^3x^3\,dx$$

input
```
integrate(x^3*(a+b*arcsec(c*x))^3,x, algorithm="maxima")
```

output
```
3/4*a*b^2*x^4*arcsec(c*x)^2 + 1/4*a^3*x^4 + 1/4*(3*x^4*arcsec(c*x) - (c^2*
x^3*(-1/(c^2*x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*a^2*b + 1/
16*(4*x^4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3*x^4*arctan(sqrt(c*x +
1)*sqrt(c*x - 1))*log(c^2*x^2)^2 - 16*integrate(3/16*((4*x^3*arctan(sqrt(c
*x + 1)*sqrt(c*x - 1))^2 - x^3*log(c^2*x^2)^2)*sqrt(c*x + 1)*sqrt(c*x - 1)
 + 4*(4*c^2*x^5*log(c)^2 - 4*x^3*log(c)^2 + 4*(c^2*x^5 - x^3)*log(x)^2 - (
(4*c^2*log(c) + c^2)*x^5 - x^3*(4*log(c) + 1) + 4*(c^2*x^5 - x^3)*log(x))*
log(c^2*x^2) + 8*(c^2*x^5*log(c) - x^3*log(c))*log(x))*arctan(sqrt(c*x + 1
)*sqrt(c*x - 1)))/(c^2*x^2 - 1), x))*b^3 + 1/4*((c^2*x^2 + 2*log(x^2))*sqr
t(c*x + 1)*sqrt(c*x - 1) - 2*(c^4*x^4 + c^2*x^2 - 2)*arctan(sqrt(c*x + 1)*
sqrt(c*x - 1)))*a*b^2/(sqrt(c*x + 1)*sqrt(c*x - 1)*c^4)
```

3.24.8 Giac [F]

$$\int x^3 \left(a + b \sec^{-1}(cx)\right)^3 \, dx = \int (b \operatorname{arcsec}(cx) + a)^3 x^3 \, dx$$

input
```
integrate(x^3*(a+b*arcsec(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)^3*x^3, x)
```

3.24.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a + b \sec^{-1}(cx)\right)^3 \, dx = \int x^3 \left(a + b \operatorname{acos}\left(\frac{1}{cx}\right)\right)^3 dx$$

input
```
int(x^3*(a + b*acos(1/(c*x)))^3,x)
```

output
```
int(x^3*(a + b*acos(1/(c*x)))^3, x)
```

3.25 $\int x^2(a+b\sec^{-1}(cx))^3\,dx$

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3.25.1 Optimal result

Integrand size = 14, antiderivative size = 236

$$\begin{aligned}\int x^2(a+b\sec^{-1}(cx))^3\,dx = &\frac{b^2x(a+b\sec^{-1}(cx))}{c^2}-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^2(a+b\sec^{-1}(cx))^2}{2c}\\&+\frac{1}{3}x^3(a+b\sec^{-1}(cx))^3\\&+\frac{ib(a+b\sec^{-1}(cx))^2\arctan\left(e^{i\sec^{-1}(cx)}\right)}{c^3}\\&-\frac{b^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^3}\\&-\frac{ib^2(a+b\sec^{-1}(cx))\operatorname{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)}{c^3}\\&+\frac{ib^2(a+b\sec^{-1}(cx))\operatorname{PolyLog}\left(2,ie^{i\sec^{-1}(cx)}\right)}{c^3}\\&+\frac{b^3\operatorname{PolyLog}\left(3,-ie^{i\sec^{-1}(cx)}\right)}{c^3}-\frac{b^3\operatorname{PolyLog}\left(3,ie^{i\sec^{-1}(cx)}\right)}{c^3}\end{aligned}$$

output
```
b^2*x*(a+b*arcsec(c*x))/c^2+1/3*x^3*(a+b*arcsec(c*x))^3+I*b*(a+b*arcsec(c*
x))^2*arctan(1/c/x+I*(1-1/c^2/x^2)^(1/2))/c^3-b^3*arctanh((1-1/c^2/x^2)^(1
/2))/c^3-I*b^2*(a+b*arcsec(c*x))*polylog(2,-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)
))/c^3+I*b^2*(a+b*arcsec(c*x))*polylog(2,I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))/
c^3+b^3*polylog(3,-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))/c^3-b^3*polylog(3,I*(1
/c/x+I*(1-1/c^2/x^2)^(1/2)))/c^3-1/2*b*x^2*(a+b*arcsec(c*x))^2*(1-1/c^2/x^
2)^(1/2)/c
```

3.25.2 Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.71

$$\int x^2 \left(a + b \sec^{-1}(cx)\right)^3 \, dx$$

$$= \frac{6ab^2cx - 3a^2bc^2\sqrt{1 - \frac{1}{c^2x^2}}x^2 + 2a^3c^3x^3 + 6b^3cx\sec^{-1}(cx) - 6ab^2c^2\sqrt{1 - \frac{1}{c^2x^2}}x^2\sec^{-1}(cx) + 6a^2bc^3x^3\sec^{-1}(cx) + 6b^3c^3x^3\sec^{-1}(cx)^2 - 6ab^2\log\left(1 - ie^{i\sec^{-1}(cx)}\right) + 6ab^2\log\left(1 + ie^{i\sec^{-1}(cx)}\right) - 3a^2b\log\left(\left(1 + \sqrt{1 - \frac{1}{c^2x^2}}\right)cx\right) - 6ib^2\left(a + b\sec^{-1}(cx)\right)\operatorname{Li}_2\left(-ie^{i\sec^{-1}(cx)}\right) + 6ib^2\left(a + b\sec^{-1}(cx)\right)\operatorname{Li}_2\left(ie^{i\sec^{-1}(cx)}\right) + 6b^3\operatorname{Li}_3\left(-ie^{i\sec^{-1}(cx)}\right) - 6b^3\operatorname{Li}_3\left(ie^{i\sec^{-1}(cx)}\right)}{6c^3}$$

input
```
Integrate[x^2*(a + b*ArcSec[c*x])^3,x]
```

output
```
(6*a*b^2*c*x - 3*a^2*b*c^2*Sqrt[1 - 1/(c^2*x^2)]*x^2 + 2*a^3*c^3*x^3 + 6*b
^3*c*x*ArcSec[c*x] - 6*a*b^2*c^2*Sqrt[1 - 1/(c^2*x^2)]*x^2*ArcSec[c*x] + 6
*a^2*b*c^3*x^3*ArcSec[c*x] - 3*b^3*c^2*Sqrt[1 - 1/(c^2*x^2)]*x^2*ArcSec[c*
x]^2 + 6*a*b^2*c^3*x^3*ArcSec[c*x]^2 + 2*b^3*c^3*x^3*ArcSec[c*x]^3 + (6*I)
*b^3*ArcSec[c*x]^2*ArcTan[E^(I*ArcSec[c*x])] - 6*b^3*ArcTanh[Sqrt[1 - 1/(c
^2*x^2)]] - 6*a*b^2*ArcSec[c*x]*Log[1 - I*E^(I*ArcSec[c*x])] + 6*a*b^2*Arc
Sec[c*x]*Log[1 + I*E^(I*ArcSec[c*x])] - 3*a^2*b*Log[(1 + Sqrt[1 - 1/(c^2*x
^2)])*x] - (6*I)*b^2*(a + b*ArcSec[c*x])*PolyLog[2, (-I)*E^(I*ArcSec[c*x])
] + (6*I)*b^2*(a + b*ArcSec[c*x])*PolyLog[2, I*E^(I*ArcSec[c*x])] + 6*b^3*
PolyLog[3, (-I)*E^(I*ArcSec[c*x])] - 6*b^3*PolyLog[3, I*E^(I*ArcSec[c*x])]
)/(6*c^3)
```

3.25.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5745, 4909, 3042, 4674, 3042, 4257, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b \sec^{-1}(cx)\right)^3 dx$$

$$\downarrow 5745$$

$$\frac{\int c^4 \sqrt{1-\frac{1}{c^2x^2}} x^4 \left(a+b \sec^{-1}(cx)\right)^3 d\sec^{-1}(cx)}{c^3}$$

$$\downarrow 4909$$

$$\frac{\frac{1}{3}c^3x^3\left(a+b \sec^{-1}(cx)\right)^3 - b\int c^3x^3\left(a+b \sec^{-1}(cx)\right)^2 d\sec^{-1}(cx)}{c^3}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{3}c^3x^3\left(a+b \sec^{-1}(cx)\right)^3 - b\int \left(a+b \sec^{-1}(cx)\right)^2 \csc\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^3 d\sec^{-1}(cx)}{c^3}$$

$$\downarrow 4674$$

$$\frac{\frac{1}{3}c^3x^3\left(a+b \sec^{-1}(cx)\right)^3 - b\left(\frac{1}{2}\int cx\left(a+b \sec^{-1}(cx)\right)^2 d\sec^{-1}(cx) + b^2\int cx d\sec^{-1}(cx) + \frac{1}{2}c^2x^2\sqrt{1-\frac{1}{c^2x^2}}\left(a+b \mathrm{s}\right.\right.}{c^3}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{3}c^3x^3\left(a+b \sec^{-1}(cx)\right)^3 - b\left(\frac{1}{2}\int \left(a+b \sec^{-1}(cx)\right)^2 \csc\left(\sec^{-1}(cx)+\frac{\pi}{2}\right) d\sec^{-1}(cx) + b^2\int \csc\left(\sec^{-1}(cx)+\frac{\pi}{2}\right) d\mathrm{s}\right.}{c^3}$$

$$\downarrow 4257$$

$$\frac{\frac{1}{3}c^3x^3\left(a+b \sec^{-1}(cx)\right)^3 - b\left(\frac{1}{2}\int \left(a+b \sec^{-1}(cx)\right)^2 \csc\left(\sec^{-1}(cx)+\frac{\pi}{2}\right) d\sec^{-1}(cx) + \frac{1}{2}c^2x^2\sqrt{1-\frac{1}{c^2x^2}}\left(a+b \sec^{-}\right.\right.}{c^3}$$

$$\downarrow 4669$$

$$\frac{\frac{1}{3}c^3x^3\left(a+b \sec^{-1}(cx)\right)^3 - b\left(\frac{1}{2}\left(-2b\int \left(a+b \sec^{-1}(cx)\right)\log\left(1-ie^{i \sec^{-1}(cx)}\right) d\sec^{-1}(cx) + 2b\int \left(a+b \sec^{-1}(cx)\right)\right.\right.}$$

$$\downarrow 3011$$

$$\frac{1}{3}c^3x^3\left(a+b\sec^{-1}(cx)\right)^3-b\left(\frac{1}{2}\left(2b\left(i\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)\right)-ib\int \text{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right.\right.\right.$$

$$\downarrow 2720$$

$$\frac{1}{3}c^3x^3\left(a+b\sec^{-1}(cx)\right)^3-b\left(\frac{1}{2}\left(2b\left(i\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)\right)-b\int e^{-i\sec^{-1}(cx)}\,\text{PolyLog}\left(2,-\right.\right.\right.$$

$$\downarrow 7143$$

$$\frac{1}{3}c^3x^3\left(a+b\sec^{-1}(cx)\right)^3-b\left(\frac{1}{2}\left(-2i\arctan\left(e^{i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)^2+2b\left(i\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)\left(a+\right.\right.\right.\right.$$

input
```
Int[x^2*(a + b*ArcSec[c*x])^3,x]
```

output
```
((c^3*x^3*(a + b*ArcSec[c*x])^3)/3 - b*(-(b*c*x*(a + b*ArcSec[c*x])) + (c^
2*Sqrt[1 - 1/(c^2*x^2)]*x^2*(a + b*ArcSec[c*x])^2)/2 + b^2*ArcTanh[Sqrt[1
- 1/(c^2*x^2)]] + ((-2*I)*(a + b*ArcSec[c*x])^2*ArcTan[E^(I*ArcSec[c*x])]
+ 2*b*(I*(a + b*ArcSec[c*x])*PolyLog[2, (-I)*E^(I*ArcSec[c*x])] - b*PolyLo
g[3, (-I)*E^(I*ArcSec[c*x])]) - 2*b*(I*(a + b*ArcSec[c*x])*PolyLog[2, I*E^
(I*ArcSec[c*x])] - b*PolyLog[3, I*E^(I*ArcSec[c*x])]))/2))/c^3
```

3.25.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))  Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
 - 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.25.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(286) = 572$.

Time = 1.44 (sec) , antiderivative size = 574, normalized size of antiderivative = 2.43

method	result
derivativedivides	$\frac{a^3c^3x^3}{3}+b^3\left(\frac{\operatorname{arcsec}(cx)\left(2c^2x^2\operatorname{arcsec}(cx)^2-3\operatorname{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}+6\right)cx}{6}+\frac{\operatorname{arcsec}(cx)^2\ln\left(1+i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{2}-i\operatorname{arcsec}($
default	$\frac{a^3c^3x^3}{3}+b^3\left(\frac{\operatorname{arcsec}(cx)\left(2c^2x^2\operatorname{arcsec}(cx)^2-3\operatorname{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}+6\right)cx}{6}+\frac{\operatorname{arcsec}(cx)^2\ln\left(1+i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{2}-i\operatorname{arcsec}($
parts	$\frac{a^3x^3}{3}+b^3\left(\frac{\operatorname{arcsec}(cx)\left(2c^2x^2\operatorname{arcsec}(cx)^2-3\operatorname{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}+6\right)cx}{6}+\frac{\operatorname{arcsec}(cx)^2\ln\left(1+i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{2}-i\operatorname{arcsec}($

input
```
int(x^2*(a+b*arcsec(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(1/3*a^3*c^3*x^3+b^3*(1/6*arcsec(c*x)*(2*c^2*x^2*arcsec(c*x)^2-3*arc
sec(c*x)*c*x*((c^2*x^2-1)/c^2/x^2)^(1/2)+6)*c*x+1/2*arcsec(c*x)^2*ln(1+I*(
1/c/x+I*(1-1/c^2/x^2)^(1/2)))-I*arcsec(c*x)*polylog(2,-I*(1/c/x+I*(1-1/c^2
/x^2)^(1/2)))+polylog(3,-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-1/2*arcsec(c*x)^
2*ln(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))+I*arcsec(c*x)*polylog(2,I*(1/c/x+I
*(1-1/c^2/x^2)^(1/2)))-polylog(3,I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))+2*I*arct
an(1/c/x+I*(1-1/c^2/x^2)^(1/2)))+3*a*b^2*(1/3*(c^2*x^2*arcsec(c*x)^2-arcse
c(c*x)*c*x*((c^2*x^2-1)/c^2/x^2)^(1/2)+1)*c*x+1/3*arcsec(c*x)*ln(1+I*(1/c/
x+I*(1-1/c^2/x^2)^(1/2)))-1/3*arcsec(c*x)*ln(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1
/2)))-1/3*I*dilog(1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))+1/3*I*dilog(1-I*(1/c/
x+I*(1-1/c^2/x^2)^(1/2))))+3*a^2*b*(1/3*c^3*x^3*arcsec(c*x)-1/6*(c^2*x^2-1
)^(1/2)*(c*x*(c^2*x^2-1)^(1/2)+ln(c*x+(c^2*x^2-1)^(1/2)))/((c^2*x^2-1)/c^2
/x^2)^(1/2)/c/x))
```

3.25.5 Fricas [F]

$$\int x^2 \left(a + b \sec^{-1}(cx)\right)^3 \, dx = \int (b \operatorname{arcsec}(cx) + a)^3 x^2 \, dx$$

input
```
integrate(x^2*(a+b*arcsec(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^2*arcsec(c*x)^3 + 3*a*b^2*x^2*arcsec(c*x)^2 + 3*a^2*b*x^2*a
rcsec(c*x) + a^3*x^2, x)
```

3.25.6 Sympy [F]

$$\int x^2 \left(a + b \sec^{-1}(cx)\right)^3 \, dx = \int x^2 (a + b \operatorname{asec}(cx))^3 \, dx$$

input
```
integrate(x**2*(a+b*asec(c*x))**3,x)
```

output
```
Integral(x**2*(a + b*asec(c*x))**3, x)
```

3.25.7 Maxima [F]

$$\int x^2 \left(a + b \sec^{-1}(cx)\right)^3 \, dx = \int (b \operatorname{arcsec}(cx) + a)^3 x^2 \, dx$$

input
```
integrate(x^2*(a+b*arcsec(c*x))^3,x, algorithm="maxima")
```

output
```
1/3*b^3*x^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 1/4*b^3*x^3*arctan(sqr
t(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 - 1/2*a*b^2*c^2*(2*(c^2*x^3 + 3*x
)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5)*log(c)^2 - 12*b^3*c^2*int
egrate(1/4*x^4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^2 - 1), x)*log(c
)^2 + 12*b^3*c^2*integrate(1/4*x^4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log
(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) - 24*b^3*c^2*integrate(1/4*x^4*arctan(s
qrt(c*x + 1)*sqrt(c*x - 1))*log(x)/(c^2*x^2 - 1), x)*log(c) + 12*a*b^2*c^2
*integrate(1/4*x^4*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) - 24*a*b^2*c^2*in
tegrate(1/4*x^4*log(x)/(c^2*x^2 - 1), x)*log(c) + 1/3*a^3*x^3 + 12*b^3*c^2
*integrate(1/4*x^4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)*log(x)
/(c^2*x^2 - 1), x) - 12*b^3*c^2*integrate(1/4*x^4*arctan(sqrt(c*x + 1)*sqr
t(c*x - 1))*log(x)^2/(c^2*x^2 - 1), x) + 12*a*b^2*c^2*integrate(1/4*x^4*ar
ctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2/(c^2*x^2 - 1), x) + 4*b^3*c^2*integrat
e(1/4*x^4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)/(c^2*x^2 - 1),
x) - 3*a*b^2*c^2*integrate(1/4*x^4*log(c^2*x^2)^2/(c^2*x^2 - 1), x) + 12*a
*b^2*c^2*integrate(1/4*x^4*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) - 12*a*b^
2*c^2*integrate(1/4*x^4*log(x)^2/(c^2*x^2 - 1), x) + 3/2*a*b^2*(2*x/c^2 -
log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*log(c)^2 + 12*b^3*integrate(1/4*x^2*a
rctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^2 - 1), x)*log(c)^2 - 12*b^3*int
egrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)/(c^2*x^...
```

3.25.8 Giac [F]

$$\int x^2 \left(a + b \sec^{-1}(cx)\right)^3 \, dx = \int \left(b \operatorname{arcsec}(cx) + a\right)^3 x^2 \, dx$$

input
```
integrate(x^2*(a+b*arcsec(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)^3*x^2, x)
```

3.25.9 Mupad [F(-1)]

Timed out.

$$\int x^2 (a + b \sec^{-1}(cx))^3 \, dx = \int x^2 \left(a + b \operatorname{acos}\left(\frac{1}{c\,x} \right) \right)^3 dx$$

input
```
int(x^2*(a + b*acos(1/(c*x)))^3,x)
```

output
```
int(x^2*(a + b*acos(1/(c*x)))^3, x)
```

3.26 $\int x(a+b\sec^{-1}(cx))^3\,dx$

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3.26.1 Optimal result

Integrand size = 12, antiderivative size = 126

$$\begin{aligned}\int x\left(a+b\sec^{-1}(cx)\right)^3\,dx = {}& \frac{3ib\left(a+b\sec^{-1}(cx)\right)^2}{2c^2} - \frac{3b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(a+b\sec^{-1}(cx)\right)^2}{2c} \\ & + \frac{1}{2}x^2\left(a+b\sec^{-1}(cx)\right)^3 \\ & - \frac{3b^2\left(a+b\sec^{-1}(cx)\right)\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{c^2} \\ & + \frac{3ib^3\,\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2c^2}\end{aligned}$$

output

```
3/2*I*b*(a+b*arcsec(c*x))^2/c^2+1/2*x^2*(a+b*arcsec(c*x))^3-3*b^2*(a+b*arc
sec(c*x))*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)/c^2+3/2*I*b^3*polylog(2,-(
1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)/c^2-3/2*b*x*(a+b*arcsec(c*x))^2*(1-1/c^2/x
^2)^(1/2)/c
```

3.26.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.46

$$\int x\left(a+b\sec^{-1}(cx)\right)^3\,dx$$
$$=\frac{-3b^2\left(-ac^2x^2+b\left(-i+c\sqrt{1-\frac{1}{c^2x^2}}x\right)\right)\sec^{-1}(cx)^2+b^3c^2x^2\sec^{-1}(cx)^3-3b\sec^{-1}(cx)\left(acx\left(2b\sqrt{1-\frac{1}{c^2x^2}}\right.\right.}{}$$

input
```
Integrate[x*(a + b*ArcSec[c*x])^3,x]
```

output
```
(-3*b^2*(-(a*c^2*x^2) + b*(-I + c*Sqrt[1 - 1/(c^2*x^2)]*x))*ArcSec[c*x]^2
+ b^3*c^2*x^2*ArcSec[c*x]^3 - 3*b*ArcSec[c*x]*(a*c*x*(2*b*Sqrt[1 - 1/(c^2*
x^2)] - a*c*x) + 2*b^2*Log[1 + E^((2*I)*ArcSec[c*x])]) + a*(a*c*x*(-3*b*Sq
rt[1 - 1/(c^2*x^2)] + a*c*x) - 6*b^2*Log[1/(c*x)]) + (3*I)*b^3*PolyLog[2,
-E^((2*I)*ArcSec[c*x])])/(2*c^2)
```

3.26.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5745, 4909, 3042, 4672, 25, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\sec^{-1}(cx)\right)^3\,dx$$
$$\downarrow 5745$$
$$\frac{\int c^3\sqrt{1-\frac{1}{c^2x^2}}x^3\left(a+b\sec^{-1}(cx)\right)^3\,d\sec^{-1}(cx)}{c^2}$$
$$\downarrow 4909$$
$$\frac{\frac{1}{2}c^2x^2\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{2}b\int c^2x^2\left(a+b\sec^{-1}(cx)\right)^2\,d\sec^{-1}(cx)}{c^2}$$
$$\downarrow 3042$$
$$\frac{\frac{1}{2}c^2x^2\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{2}b\int\left(a+b\sec^{-1}(cx)\right)^2\csc\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^2\,d\sec^{-1}(cx)}{c^2}$$

$$\downarrow 4672$$

$$\frac{\frac{1}{2}c^2x^2\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{2}b\left(2b\int -c\sqrt{1-\frac{1}{c^2x^2}}x\left(a+b\sec^{-1}(cx)\right)d\sec^{-1}(cx)+cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2\right.}{c^2}$$

$$\downarrow 25$$

$$\frac{\frac{1}{2}c^2x^2\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{2}b\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\int c\sqrt{1-\frac{1}{c^2x^2}}x\left(a+b\sec^{-1}(cx)\right)d\sec^{-1}(cx)\right)}{c^2}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{2}c^2x^2\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{2}b\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\int \left(a+b\sec^{-1}(cx)\right)\tan\left(\sec^{-1}(cx)\right)d\sec^{-1}(cx\right.}{c^2}$$

$$\downarrow 4202$$

$$\frac{\frac{1}{2}c^2x^2\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{2}b\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\left(\frac{i\left(a+b\sec^{-1}(cx)\right)^2}{2b}-2i\int\frac{e^{2i\sec^{-1}(cx)}\left(a+b\sec^{-1}(cx)\right.}{1+e^{2i\sec^{-1}(cx)}}\right.\right.}{c^2}$$

$$\downarrow 2620$$

$$\frac{\frac{1}{2}c^2x^2\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{2}b\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\left(\frac{i\left(a+b\sec^{-1}(cx)\right)^2}{2b}-2i\left(\frac{1}{2}ib\int\log\left(1+e^{2i\sec^{-1}(\right.\right.\right.\right.}{c^2}$$

$$\downarrow 2715$$

$$\frac{\frac{1}{2}c^2x^2\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{2}b\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\left(\frac{i\left(a+b\sec^{-1}(cx)\right)^2}{2b}-2i\left(\frac{1}{4}b\int e^{-2i\sec^{-1}(cx)}\log\left(1\right.\right.\right.\right.}{c^2}$$

$$\downarrow 2838$$

$$\frac{\frac{1}{2}c^2x^2\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{2}b\left(cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\left(\frac{i\left(a+b\sec^{-1}(cx)\right)^2}{2b}-2i\left(-\frac{1}{2}i\log\left(1+e^{2i\sec^{-1}(cx\right.\right.\right.\right.}{c^2}$$

input
```
Int[x*(a + b*ArcSec[c*x])^3,x]
```

output
```
((c^2*x^2*(a + b*ArcSec[c*x])^3)/2 - (3*b*(c*Sqrt[1 - 1/(c^2*x^2)]*x*(a +
b*ArcSec[c*x])^2 - 2*b*(((I/2)*(a + b*ArcSec[c*x])^2)/b - (2*I)*((-1/2*I)*
(a + b*ArcSec[c*x])*Log[1 + E^((2*I)*ArcSec[c*x])] - (b*PolyLog[2, -E^((2*
I)*ArcSec[c*x])])/4))))/2)/c^2
```

3.26.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

3.26.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.00

method	result
derivativedivides	$\frac{a^3c^2x^2}{2}+b^3\left(\frac{\operatorname{arcsec}(cx)^2\left(c^2x^2\operatorname{arcsec}(cx)-3xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}-3i\right)}{2}+3i\operatorname{arcsec}(cx)^2-3\operatorname{arcsec}(cx)\ln\left(1+\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2\right)+$
default	$\frac{a^3c^2x^2}{2}+b^3\left(\frac{\operatorname{arcsec}(cx)^2\left(c^2x^2\operatorname{arcsec}(cx)-3xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}-3i\right)}{2}+3i\operatorname{arcsec}(cx)^2-3\operatorname{arcsec}(cx)\ln\left(1+\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2\right)+$
parts	$\frac{a^3x^2}{2}+\frac{b^3\left(\frac{\operatorname{arcsec}(cx)^2\left(c^2x^2\operatorname{arcsec}(cx)-3xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}-3i\right)}{2}+3i\operatorname{arcsec}(cx)^2-3\operatorname{arcsec}(cx)\ln\left(1+\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2\right)+\right.}{c^2}$

input
```
int(x*(a+b*arcsec(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(1/2*a^3*c^2*x^2+b^3*(1/2*arcsec(c*x)^2*(c^2*x^2*arcsec(c*x)-3*x*c*(
(c^2*x^2-1)/c^2/x^2)^(1/2)-3*I)+3*I*arcsec(c*x)^2-3*arcsec(c*x)*ln(1+(1/c/
x+I*(1-1/c^2/x^2)^(1/2))^2)+3/2*I*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))
^2))+3*a*b^2*(1/2*c^2*x^2*arcsec(c*x)^2-arcsec(c*x)*c*x*((c^2*x^2-1)/c^2/x
^2)^(1/2)-ln(1/c/x))+3*a^2*b*(1/2*c^2*x^2*arcsec(c*x)-1/2/((c^2*x^2-1)/c^2
/x^2)^(1/2)/c/x*(c^2*x^2-1)))
```

3.26.5 Fricas [F]

$$\int x\left(a+b\sec^{-1}(cx)\right)^3\,dx = \int (b\operatorname{arcsec}(cx)+a)^3 x\,dx$$

input
```
integrate(x*(a+b*arcsec(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x*arcsec(c*x)^3 + 3*a*b^2*x*arcsec(c*x)^2 + 3*a^2*b*x*arcsec(
c*x) + a^3*x, x)
```

3.26.6 Sympy [F]

$$\int x\left(a+b\sec^{-1}(cx)\right)^3\,dx = \int x(a+b\operatorname{asec}(cx))^3\,dx$$

input
```
integrate(x*(a+b*asec(c*x))**3,x)
```

output
```
Integral(x*(a + b*asec(c*x))**3, x)
```

3.26.7 Maxima [F]

$$\int x\left(a+b\sec^{-1}(cx)\right)^3\,dx = \int (b\operatorname{arcsec}(cx)+a)^3 x\,dx$$

input
```
integrate(x*(a+b*arcsec(c*x))^3,x, algorithm="maxima")
```

output
```
3/2*a*b^2*x^2*arcsec(c*x)^2 + 1/2*a^3*x^2 + 3/2*(x^2*arcsec(c*x) - x*sqrt(
-1/(c^2*x^2) + 1)/c)*a^2*b - 3*(x*sqrt(-1/(c^2*x^2) + 1)*arcsec(c*x)/c - l
og(x)/c^2)*a*b^2 + 1/8*(4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3*x^
2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 - 8*integrate(3/8*((4
*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - x*log(c^2*x^2)^2)*sqrt(c*x + 1)
*sqrt(c*x - 1) + 4*(2*c^2*x^3*log(c)^2 - 2*x*log(c)^2 + 2*(c^2*x^3 - x)*lo
g(x)^2 - ((2*c^2*log(c) + c^2)*x^3 - x*(2*log(c) + 1) + 2*(c^2*x^3 - x)*lo
g(x))*log(c^2*x^2) + 4*(c^2*x^3*log(c) - x*log(c))*log(x))*arctan(sqrt(c*x
 + 1)*sqrt(c*x - 1)))/(c^2*x^2 - 1), x))*b^3
```

3.26.8 Giac [F]

$$\int x(a+b\sec^{-1}(cx))^3\,dx = \int (b\operatorname{arcsec}(cx)+a)^3 x\,dx$$

input
```
integrate(x*(a+b*arcsec(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)^3*x, x)
```

3.26.9 Mupad [F(-1)]

Timed out.

$$\int x(a+b\sec^{-1}(cx))^3\,dx = \int x\left(a+b\operatorname{acos}\left(\frac{1}{c\,x}\right)\right)^3 dx$$

input
```
int(x*(a + b*acos(1/(c*x)))^3,x)
```

output
```
int(x*(a + b*acos(1/(c*x)))^3, x)
```

3.27 $\int (a + b \sec^{-1}(cx))^3 \, dx$

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3.27.1 Optimal result

Integrand size = 10, antiderivative size = 158

$$\int (a + b \sec^{-1}(cx))^3 \, dx = x(a + b \sec^{-1}(cx))^3 + \frac{6ib(a + b \sec^{-1}(cx))^2 \arctan\left(e^{i \sec^{-1}(cx)}\right)}{c} - \frac{6ib^2(a + b \sec^{-1}(cx)) \operatorname{PolyLog}\left(2, -ie^{i \sec^{-1}(cx)}\right)}{c} + \frac{6ib^2(a + b \sec^{-1}(cx)) \operatorname{PolyLog}\left(2, ie^{i \sec^{-1}(cx)}\right)}{c} + \frac{6b^3 \operatorname{PolyLog}\left(3, -ie^{i \sec^{-1}(cx)}\right)}{c} - \frac{6b^3 \operatorname{PolyLog}\left(3, ie^{i \sec^{-1}(cx)}\right)}{c}$$

output

```
x*(a+b*arcsec(c*x))^3+6*I*b*(a+b*arcsec(c*x))^2*arctan(1/c/x+I*(1-1/c^2/x^
2)^(1/2))/c-6*I*b^2*(a+b*arcsec(c*x))*polylog(2,-I*(1/c/x+I*(1-1/c^2/x^2)^
(1/2)))/c+6*I*b^2*(a+b*arcsec(c*x))*polylog(2,I*(1/c/x+I*(1-1/c^2/x^2)^(1/
2)))/c+6*b^3*polylog(3,-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))/c-6*b^3*polylog(3
,I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))/c
```

3.27.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.83

$$\int \left(a+b\sec^{-1}(cx)\right)^3\,dx$$
$$= \frac{a^3cx+3a^2bcx\sec^{-1}(cx)+3ab^2cx\sec^{-1}(cx)^2+b^3cx\sec^{-1}(cx)^3-6ab^2\sec^{-1}(cx)\log\left(1-ie^{i\sec^{-1}(cx)}\right)-3}{}$$

input
```
Integrate[(a + b*ArcSec[c*x])^3,x]
```

output
```
(a^3*c*x + 3*a^2*b*c*x*ArcSec[c*x] + 3*a*b^2*c*x*ArcSec[c*x]^2 + b^3*c*x*A
rcSec[c*x]^3 - 6*a*b^2*ArcSec[c*x]*Log[1 - I*E^(I*ArcSec[c*x])] - 3*b^3*Ar
cSec[c*x]^2*Log[1 - I*E^(I*ArcSec[c*x])] + 6*a*b^2*ArcSec[c*x]*Log[1 + I*E
^(I*ArcSec[c*x])] + 3*b^3*ArcSec[c*x]^2*Log[1 + I*E^(I*ArcSec[c*x])] - 3*a
^2*b*Log[c*(1 + Sqrt[1 - 1/(c^2*x^2)])*x] - (6*I)*b^2*(a + b*ArcSec[c*x])*
PolyLog[2, (-I)*E^(I*ArcSec[c*x])] + (6*I)*b^2*(a + b*ArcSec[c*x])*PolyLog
[2, I*E^(I*ArcSec[c*x])] + 6*b^3*PolyLog[3, (-I)*E^(I*ArcSec[c*x])] - 6*b^
3*PolyLog[3, I*E^(I*ArcSec[c*x])])/c
```

3.27.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5739, 4909, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sec^{-1}(cx)\right)^3\,dx$$
$$\downarrow 5739$$
$$\frac{\int c^2\sqrt{1-\frac{1}{c^2x^2}}x^2\left(a+b\sec^{-1}(cx)\right)^3\,d\sec^{-1}(cx)}{c}$$
$$\downarrow 4909$$
$$\frac{cx\left(a+b\sec^{-1}(cx)\right)^3-3b\int cx\left(a+b\sec^{-1}(cx)\right)^2\,d\sec^{-1}(cx)}{c}$$
$$\downarrow 3042$$

$$\frac{cx\left(a+b\sec^{-1}(cx)\right)^3-3b\int\left(a+b\sec^{-1}(cx)\right)^2\csc\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)d\sec^{-1}(cx)}{c}$$

$$\downarrow 4669$$

$$\frac{cx\left(a+b\sec^{-1}(cx)\right)^3-3b\left(-2b\int\left(a+b\sec^{-1}(cx)\right)\log\left(1-ie^{i\sec^{-1}(cx)}\right)d\sec^{-1}(cx)+2b\int\left(a+b\sec^{-1}(cx)\right)\log\left(\right.\right.}{c}$$

$$\downarrow 3011$$

$$cx\left(a+b\sec^{-1}(cx)\right)^3-3b\left(2b\left(i\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)-ib\int\text{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)d\text{se}\right.\right.$$

$$\downarrow 2720$$

$$cx\left(a+b\sec^{-1}(cx)\right)^3-3b\left(2b\left(i\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)-b\int e^{-i\sec^{-1}(cx)}\text{PolyLog}\left(2,-ie^{i\sec}\right.\right.\right.$$

$$\downarrow 7143$$

$$cx\left(a+b\sec^{-1}(cx)\right)^3-3b\left(-2i\arctan\left(e^{i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)^2+2b\left(i\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)\left(a+b\sec\right.\right.\right.$$

input
```
Int[(a + b*ArcSec[c*x])^3,x]
```

output
```
(c*x*(a + b*ArcSec[c*x])^3 - 3*b*((-2*I)*(a + b*ArcSec[c*x])^2*ArcTan[E^(I
*ArcSec[c*x])] + 2*b*(I*(a + b*ArcSec[c*x])*PolyLog[2, (-I)*E^(I*ArcSec[c*
x])] - b*PolyLog[3, (-I)*E^(I*ArcSec[c*x])]) - 2*b*(I*(a + b*ArcSec[c*x])*
PolyLog[2, I*E^(I*ArcSec[c*x])] - b*PolyLog[3, I*E^(I*ArcSec[c*x])])))/c
```

3.27.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))  Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)  Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)  Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))  Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5739
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[1/c  Subst[
Int[(a + b*x)^n*Sec[x]*Tan[x], x], x, ArcSec[c*x]], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.27.4 Maple [F]

$$\int (a + b \operatorname{arcsec}(cx))^3 \, dx$$

input
```
int((a+b*arcsec(c*x))^3,x)
```

output
```
int((a+b*arcsec(c*x))^3,x)
```

3.27.5 Fricas [F]

$$\int \left(a + b \sec^{-1}(cx)\right)^3 \, dx = \int \left(b \operatorname{arcsec}(cx) + a\right)^3 \, dx$$

input
```
integrate((a+b*arcsec(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*arcsec(c*x)^3 + 3*a*b^2*arcsec(c*x)^2 + 3*a^2*b*arcsec(c*x) +
 a^3, x)
```

3.27.6 Sympy [F]

$$\int \left(a + b \sec^{-1}(cx)\right)^3 \, dx = \int \left(a + b \operatorname{asec}(cx)\right)^3 \, dx$$

input
```
integrate((a+b*asec(c*x))**3,x)
```

output
```
Integral((a + b*asec(c*x))**3, x)
```

3.27.7 Maxima [F]

$$\int \left(a + b \sec^{-1}(cx)\right)^3 \, dx = \int \left(b \operatorname{arcsec}(cx) + a\right)^3 \, dx$$

input
```
integrate((a+b*arcsec(c*x))^3,x, algorithm="maxima")
```

output
```
-3/2*a*b^2*c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*log(c)^2 -
12*b^3*c^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^2
- 1), x)*log(c)^2 + b^3*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3/4*b^3*
x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 + 12*b^3*c^2*integrat
e(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)/(c^2*x^2 - 1),
x)*log(c) - 24*b^3*c^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1
))*log(x)/(c^2*x^2 - 1), x)*log(c) + 12*a*b^2*c^2*integrate(1/4*x^2*log(c^
2*x^2)/(c^2*x^2 - 1), x)*log(c) - 24*a*b^2*c^2*integrate(1/4*x^2*log(x)/(c
^2*x^2 - 1), x)*log(c) + 12*b^3*c^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)
*sqrt(c*x - 1))*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) - 12*b^3*c^2*integra
te(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(x)^2/(c^2*x^2 - 1), x)
+ 12*a*b^2*c^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2/(c^
2*x^2 - 1), x) + 12*b^3*c^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*
x - 1))*log(c^2*x^2)/(c^2*x^2 - 1), x) - 3*a*b^2*c^2*integrate(1/4*x^2*log
(c^2*x^2)^2/(c^2*x^2 - 1), x) + 12*a*b^2*c^2*integrate(1/4*x^2*log(c^2*x^2
)*log(x)/(c^2*x^2 - 1), x) - 12*a*b^2*c^2*integrate(1/4*x^2*log(x)^2/(c^2*
x^2 - 1), x) - 3/2*a*b^2*(log(c*x + 1)/c - log(c*x - 1)/c)*log(c)^2 + 12*b
^3*integrate(1/4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^2 - 1), x)*log
(c)^2 - 12*b^3*integrate(1/4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x
^2)/(c^2*x^2 - 1), x)*log(c) + 24*b^3*integrate(1/4*arctan(sqrt(c*x + 1...
```

3.27.8 Giac [F]

$$\int \left(a + b \sec^{-1}(cx)\right)^3 \, dx = \int \left(b \operatorname{arcsec}(cx) + a\right)^3 \, dx$$

input
```
integrate((a+b*arcsec(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)^3, x)
```

3.27.9 Mupad [F(-1)]

Timed out.

$$\int \left(a + b \sec^{-1}(cx)\right)^3 \, dx = \int \left(a + b \operatorname{acos}\left(\frac{1}{c\,x}\right)\right)^3 \, dx$$

input
```
int((a + b*acos(1/(c*x)))^3,x)
```

output
```
int((a + b*acos(1/(c*x)))^3, x)
```

3.28 $\int \frac{(a+b\sec^{-1}(cx))^3}{x}\,dx$

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3.28.1 Optimal result

Integrand size = 14, antiderivative size = 128

$$\begin{aligned}\int \frac{(a+b\sec^{-1}(cx))^3}{x}\,dx = {}& \frac{i(a+b\sec^{-1}(cx))^4}{4b} - (a+b\sec^{-1}(cx))^3\log\left(1+e^{2i\sec^{-1}(cx)}\right)\\ &+\frac{3}{2}ib(a+b\sec^{-1}(cx))^2\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)\\ &-\frac{3}{2}b^2(a+b\sec^{-1}(cx))\operatorname{PolyLog}\left(3,-e^{2i\sec^{-1}(cx)}\right)\\ &-\frac{3}{4}ib^3\operatorname{PolyLog}\left(4,-e^{2i\sec^{-1}(cx)}\right)\end{aligned}$$

output

```
1/4*I*(a+b*arcsec(c*x))^4/b-(a+b*arcsec(c*x))^3*ln(1+(1/c/x+I*(1-1/c^2/x^2
)^(1/2))^2)+3/2*I*b*(a+b*arcsec(c*x))^2*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^
(1/2))^2)-3/2*b^2*(a+b*arcsec(c*x))*polylog(3,-(1/c/x+I*(1-1/c^2/x^2)^(1/2
))^2)-3/4*I*b^3*polylog(4,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)
```

3.28.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.59

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^3}{x}\,dx = \frac{1}{4}\Big(6ia^2b\sec^{-1}(cx)^2 + 4iab^2\sec^{-1}(cx)^3 + ib^3\sec^{-1}(cx)^4 - 12a^2b\sec^{-1}(cx)\log\left(1+e^{2i\sec^{-1}(cx)}\right) - 12ab^2\sec^{-1}(cx)^2\log\left(1+e^{2i\sec^{-1}(cx)}\right) - 4b^3\sec^{-1}(cx)^3\log\left(1+e^{2i\sec^{-1}(cx)}\right) + 4a^3\log(cx) + 6ib\left(a+b\sec^{-1}(cx)\right)^2\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right) - 6b^2\left(a+b\sec^{-1}(cx)\right)\operatorname{PolyLog}\left(3,-e^{2i\sec^{-1}(cx)}\right) - 3ib^3\operatorname{PolyLog}\left(4,-e^{2i\sec^{-1}(cx)}\right)\Big)$$

input
```
Integrate[(a + b*ArcSec[c*x])^3/x,x]
```

output
```
((6*I)*a^2*b*ArcSec[c*x]^2 + (4*I)*a*b^2*ArcSec[c*x]^3 + I*b^3*ArcSec[c*x]
^4 - 12*a^2*b*ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] - 12*a*b^2*ArcSec
[c*x]^2*Log[1 + E^((2*I)*ArcSec[c*x])] - 4*b^3*ArcSec[c*x]^3*Log[1 + E^((2
*I)*ArcSec[c*x])] + 4*a^3*Log[c*x] + (6*I)*b*(a + b*ArcSec[c*x])^2*PolyLog
[2, -E^((2*I)*ArcSec[c*x])] - 6*b^2*(a + b*ArcSec[c*x])*PolyLog[3, -E^((2*
I)*ArcSec[c*x])] - (3*I)*b^3*PolyLog[4, -E^((2*I)*ArcSec[c*x])])/4
```

3.28.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5745, 3042, 4202, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^3}{x}\,dx$$

$$\downarrow 5745$$

$$\int cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^3 d\sec^{-1}(cx)$$

$$\downarrow 3042$$

$$\int \tan\left(\sec^{-1}(cx)\right)\left(a+b\sec^{-1}(cx)\right)^3 d\sec^{-1}(cx)$$

$$\downarrow 4202$$

$$\frac{i\left(a+b\sec^{-1}(cx)\right)^4}{4b} - 2i\int \frac{e^{2i\sec^{-1}(cx)}\left(a+b\sec^{-1}(cx)\right)^3}{1+e^{2i\sec^{-1}(cx)}} d\sec^{-1}(cx)$$

$$\downarrow 2620$$

$$\frac{i\left(a+b\sec^{-1}(cx)\right)^4}{4b} -$$
$$2i\left(\frac{3}{2}ib\int \left(a+b\sec^{-1}(cx)\right)^2 \log\left(1+e^{2i\sec^{-1}(cx)}\right) d\sec^{-1}(cx) - \frac{1}{2}i\log\left(1+e^{2i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)^3\right)$$

$$\downarrow 3011$$

$$\frac{i\left(a+b\sec^{-1}(cx)\right)^4}{4b} -$$
$$2i\left(\frac{3}{2}ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)^2 - ib\int \left(a+b\sec^{-1}(cx)\right)\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right) d\sec\right.$$

$$\downarrow 7163$$

$$\frac{i\left(a+b\sec^{-1}(cx)\right)^4}{4b} -$$
$$2i\left(\frac{3}{2}ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)^2 - ib\left(\frac{1}{2}ib\int \text{PolyLog}\left(3,-e^{2i\sec^{-1}(cx)}\right) d\sec^{-1}(cx) - \frac{1}{2}i\right.\right.\right.$$

$$\downarrow 2720$$

$$\frac{i\left(a+b\sec^{-1}(cx)\right)^4}{4b} -$$
$$2i\left(\frac{3}{2}ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)^2 - ib\left(\frac{1}{4}b\int e^{-2i\sec^{-1}(cx)}\,\text{PolyLog}\left(3,-e^{2i\sec^{-1}(cx)}\right) de^{2i\,s}\right.\right.\right.$$

$$\downarrow 7143$$

$$\frac{i\left(a+b\sec^{-1}(cx)\right)^4}{4b} -$$
$$2i\left(\frac{3}{2}ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)^2 - ib\left(\frac{1}{4}b\,\text{PolyLog}\left(4,-e^{2i\sec^{-1}(cx)}\right) - \frac{1}{2}i\,\text{PolyLog}\left(3,-e\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcSec[c*x])^3/x,x]
```

output
```
((I/4)*(a + b*ArcSec[c*x])^4)/b - (2*I)*((-1/2*I)*(a + b*ArcSec[c*x])^3*Lo
g[1 + E^((2*I)*ArcSec[c*x])] + ((3*I)/2)*b*((I/2)*(a + b*ArcSec[c*x])^2*Po
lyLog[2, -E^((2*I)*ArcSec[c*x])] - I*b*((-1/2*I)*(a + b*ArcSec[c*x])*PolyL
og[3, -E^((2*I)*ArcSec[c*x])] + (b*PolyLog[4, -E^((2*I)*ArcSec[c*x])])/4))
)
```

3.28.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.28.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(177) = 354$.

Time $= 0.86$ (sec) , antiderivative size $= 364$, normalized size of antiderivative $= 2.84$

method	result
parts	$a^3 \ln(x) + b^3\left(\frac{i \operatorname{arcsec}(cx)^4}{4} - \operatorname{arcsec}(cx)^3 \ln\left(1 + \left(\frac{1}{cx} + i\sqrt{1 - \frac{1}{c^2x^2}}\right)^2\right) + \frac{3i \operatorname{arcsec}(cx)^2 \text{poly}$
derivativedivides	$a^3 \ln(cx) + b^3\left(\frac{i \operatorname{arcsec}(cx)^4}{4} - \operatorname{arcsec}(cx)^3 \ln\left(1 + \left(\frac{1}{cx} + i\sqrt{1 - \frac{1}{c^2x^2}}\right)^2\right) + \frac{3i \operatorname{arcsec}(cx)^2 \text{pol}$
default	$a^3 \ln(cx) + b^3\left(\frac{i \operatorname{arcsec}(cx)^4}{4} - \operatorname{arcsec}(cx)^3 \ln\left(1 + \left(\frac{1}{cx} + i\sqrt{1 - \frac{1}{c^2x^2}}\right)^2\right) + \frac{3i \operatorname{arcsec}(cx)^2 \text{pol}$

input
```
int((a+b*arcsec(c*x))^3/x,x,method=_RETURNVERBOSE)
```

output
```
a^3*ln(x)+b^3*(1/4*I*arcsec(c*x)^4-arcsec(c*x)^3*ln(1+(1/c/x+I*(1-1/c^2/x^
2)^(1/2))^2)+3/2*I*arcsec(c*x)^2*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^
2)-3/2*arcsec(c*x)*polylog(3,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)-3/4*I*polyl
og(4,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2))+3*a*b^2*(1/3*I*arcsec(c*x)^3-arcse
c(c*x)^2*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)+I*arcsec(c*x)*polylog(2,-(1
/c/x+I*(1-1/c^2/x^2)^(1/2))^2)-1/2*polylog(3,-(1/c/x+I*(1-1/c^2/x^2)^(1/2)
)^2))+3*a^2*b*(1/2*I*arcsec(c*x)^2-arcsec(c*x)*ln(1+(1/c/x+I*(1-1/c^2/x^2)
^(1/2))^2)+1/2*I*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2))
```

3.28.5 Fricas [F]

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)^3}{x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))^3/x,x, algorithm="fricas")
```

output
```
integral((b^3*arcsec(c*x)^3 + 3*a*b^2*arcsec(c*x)^2 + 3*a^2*b*arcsec(c*x)
+ a^3)/x, x)
```

3.28.6 Sympy [F]

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x}\,dx = \int \frac{(a+b\operatorname{asec}(cx))^3}{x}\,dx$$

input
```
integrate((a+b*asec(c*x))**3/x,x)
```

output
```
Integral((a + b*asec(c*x))**3/x, x)
```

3.28.7 Maxima [F]

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)^3}{x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))^3/x,x, algorithm="maxima")
```

output
```
-3/2*a*b^2*c^2*(log(c*x + 1)/c^2 + log(c*x - 1)/c^2)*log(c)^2 - 12*b^3*c^2
*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^3 - x), x)*l
og(c)^2 + 12*b^3*c^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))
*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) - 24*b^3*c^2*integrate(1/4*x^2*arct
an(sqrt(c*x + 1)*sqrt(c*x - 1))*log(x)/(c^2*x^3 - x), x)*log(c) + 12*a*b^2
*c^2*integrate(1/4*x^2*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) - 24*a*b^2*c^
2*integrate(1/4*x^2*log(x)/(c^2*x^3 - x), x)*log(c) + b^3*arctan(sqrt(c*x
+ 1)*sqrt(c*x - 1))^3*log(x) - 3/4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))
*log(c^2*x^2)^2*log(x) + 24*b^3*c^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)
*sqrt(c*x - 1))*log(c^2*x^2)*log(x)/(c^2*x^3 - x), x) - 12*b^3*c^2*integra
te(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(x)^2/(c^2*x^3 - x), x)
+ 12*a*b^2*c^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2/(c^
2*x^3 - x), x) - 3*a*b^2*c^2*integrate(1/4*x^2*log(c^2*x^2)^2/(c^2*x^3 - x
), x) + 12*a*b^2*c^2*integrate(1/4*x^2*log(c^2*x^2)*log(x)/(c^2*x^3 - x),
x) - 12*a*b^2*c^2*integrate(1/4*x^2*log(x)^2/(c^2*x^3 - x), x) + 12*a^2*b*
c^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^3 - x), x
) + 3/2*a*b^2*(log(c*x + 1) + log(c*x - 1) - 2*log(x))*log(c)^2 + 12*b^3*i
ntegrate(1/4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^3 - x), x)*log(c)^
2 - 12*b^3*integrate(1/4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)/
(c^2*x^3 - x), x)*log(c) + 24*b^3*integrate(1/4*arctan(sqrt(c*x + 1)*sq...
```

3.28.8 Giac [F]

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)^3}{x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))^3/x,x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)^3/x, x)
```

3.28.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x}\,dx = \int \frac{\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)^3}{x}\,dx$$

input
```
int((a + b*acos(1/(c*x)))^3/x,x)
```

output
```
int((a + b*acos(1/(c*x)))^3/x, x)
```

3.29 $\int \frac{(a+b\sec^{-1}(cx))^3}{x^2}\,dx$

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3.29.1 Optimal result

Integrand size = 14, antiderivative size = 80

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^2}\,dx = -6b^3c\sqrt{1-\frac{1}{c^2x^2}} + \frac{6b^2(a+b\sec^{-1}(cx))}{x} + 3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2 - \frac{(a+b\sec^{-1}(cx))^3}{x}$$

output
```
6*b^2*(a+b*arcsec(c*x))/x-(a+b*arcsec(c*x))^3/x-6*b^3*c*(1-1/c^2/x^2)^(1/2
)+3*b*c*(a+b*arcsec(c*x))^2*(1-1/c^2/x^2)^(1/2)
```

3.29.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.76

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^2}\,dx = \frac{-a^3+6ab^2+3a^2bc\sqrt{1-\frac{1}{c^2x^2}}x-6b^3c\sqrt{1-\frac{1}{c^2x^2}}x+3b\left(-a^2+2b^2+2abc\sqrt{1-\frac{1}{c^2x^2}}x\right)\sec^{-1}(cx)+3b^2\left(}{x}$$

input
```
Integrate[(a + b*ArcSec[c*x])^3/x^2,x]
```

output
```
(-a^3 + 6*a*b^2 + 3*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x - 6*b^3*c*Sqrt[1 - 1/(
c^2*x^2)]*x + 3*b*(-a^2 + 2*b^2 + 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x)*ArcSec[
c*x] + 3*b^2*(-a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x)*ArcSec[c*x]^2 - b^3*ArcSec
[c*x]^3)/x
```

3.29.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5745, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^3}{x^2}\,dx$$

$$\downarrow 5745$$

$$c\int \sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^3 d\sec^{-1}(cx)$$

$$\downarrow 3042$$

$$c\int \left(a+b\sec^{-1}(cx)\right)^3 \sin\left(\sec^{-1}(cx)\right) d\sec^{-1}(cx)$$

$$\downarrow 3777$$

$$c\left(3b\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{cx} d\sec^{-1}(cx) - \frac{\left(a+b\sec^{-1}(cx)\right)^3}{cx}\right)$$

$$\downarrow 3042$$

$$c\left(3b\int \left(a+b\sec^{-1}(cx)\right)^2 \sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right) d\sec^{-1}(cx) - \frac{\left(a+b\sec^{-1}(cx)\right)^3}{cx}\right)$$

$$\downarrow 3777$$

$$c\left(3b\left(2b\int -\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right) d\sec^{-1}(cx) + \sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2\right) - \frac{\left(a+b\sec^{-1}(cx)\right)^3}{cx}\right)$$

$$\downarrow 25$$

$$c\left(3b\left(\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\int\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)d\sec^{-1}(cx)\right)-\frac{\left(a+b\sec^{-1}(cx)\right)^3}{cx}\right)$$

$$\downarrow 3042$$

$$c\left(3b\left(\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\int\left(a+b\sec^{-1}(cx)\right)\sin\left(\sec^{-1}(cx)\right)d\sec^{-1}(cx)\right)-\frac{\left(a+b\sec^{-1}(cx)\right)^3}{cx}\right)$$

$$\downarrow 3777$$

$$c\left(3b\left(\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\left(b\int\frac{1}{cx}d\sec^{-1}(cx)-\frac{a+b\sec^{-1}(cx)}{cx}\right)\right)-\frac{\left(a+b\sec^{-1}(cx)\right)^3}{cx}\right)$$

$$\downarrow 3042$$

$$c\left(3b\left(\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\left(b\int\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)d\sec^{-1}(cx)-\frac{a+b\sec^{-1}(cx)}{cx}\right)\right)-\frac{\left(a+b\sec^{-1}(cx)\right)^3}{cx}\right)$$

$$\downarrow 3117$$

$$c\left(3b\left(\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\left(b\sqrt{1-\frac{1}{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{cx}\right)\right)-\frac{\left(a+b\sec^{-1}(cx)\right)^3}{cx}\right)$$

input
```
Int[(a + b*ArcSec[c*x])^3/x^2,x]
```

output
```
c*(-((a + b*ArcSec[c*x])^3/(c*x)) + 3*b*(Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcS
ec[c*x])^2 - 2*b*(b*Sqrt[1 - 1/(c^2*x^2)] - (a + b*ArcSec[c*x])/(c*x))))
```

3.29.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

3.29.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(76) = 152$.

Time $= 0.68$ (sec) , antiderivative size $= 196$, normalized size of antiderivative $= 2.45$

method	result
parts	$-\frac{a^3}{x}+b^3c\left(-\frac{\operatorname{arcsec}(cx)^3}{cx}+3\sqrt{\frac{c^2x^2-1}{c^2x^2}}\operatorname{arcsec}(cx)^2-6\sqrt{\frac{c^2x^2-1}{c^2x^2}}+\frac{6\operatorname{arcsec}(cx)}{cx}\right)+3ab^2c\left(-$
derivativedivides	$c\left(-\frac{a^3}{cx}+b^3\left(-\frac{\operatorname{arcsec}(cx)^3}{cx}+3\sqrt{\frac{c^2x^2-1}{c^2x^2}}\operatorname{arcsec}(cx)^2-6\sqrt{\frac{c^2x^2-1}{c^2x^2}}+\frac{6\operatorname{arcsec}(cx)}{cx}\right)+3ab^2\left(-\right.\right.$
default	$c\left(-\frac{a^3}{cx}+b^3\left(-\frac{\operatorname{arcsec}(cx)^3}{cx}+3\sqrt{\frac{c^2x^2-1}{c^2x^2}}\operatorname{arcsec}(cx)^2-6\sqrt{\frac{c^2x^2-1}{c^2x^2}}+\frac{6\operatorname{arcsec}(cx)}{cx}\right)+3ab^2\left(-\right.\right.$

input
```
int((a+b*arcsec(c*x))^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^3/x+b^3*c*(-1/c/x*arcsec(c*x)^3+3*((c^2*x^2-1)/c^2/x^2)^(1/2)*arcsec(c*
x)^2-6*((c^2*x^2-1)/c^2/x^2)^(1/2)+6/c/x*arcsec(c*x))+3*a*b^2*c*(-1/c/x*ar
csec(c*x)^2+2/c/x+2*arcsec(c*x)*((c^2*x^2-1)/c^2/x^2)^(1/2))+3*a^2*b*c*(-1
/c/x*arcsec(c*x)+1/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^2/x^2*(c^2*x^2-1))
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.22

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^3}{x^2}\,dx =$$
$$-\frac{b^3\operatorname{arcsec}(cx)^3+3\,ab^2\operatorname{arcsec}(cx)^2+a^3-6\,ab^2+3\left(a^2b-2\,b^3\right)\operatorname{arcsec}(cx)-3\left(b^3\operatorname{arcsec}(cx)^2+2\,ab^2\right.}{x}$$

input
```
integrate((a+b*arcsec(c*x))^3/x^2,x, algorithm="fricas")
```

output
```
-(b^3*arcsec(c*x)^3 + 3*a*b^2*arcsec(c*x)^2 + a^3 - 6*a*b^2 + 3*(a^2*b - 2
*b^3)*arcsec(c*x) - 3*(b^3*arcsec(c*x)^2 + 2*a*b^2*arcsec(c*x) + a^2*b - 2
*b^3)*sqrt(c^2*x^2 - 1))/x
```

3.29.6 Sympy [F]

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^3}{x^2}\,dx = \int \frac{\left(a+b\operatorname{asec}(cx)\right)^3}{x^2}\,dx$$

input
```
integrate((a+b*asec(c*x))**3/x**2,x)
```

output
```
Integral((a + b*asec(c*x))**3/x**2, x)
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.82

$$\begin{aligned}
&\int \frac{\left(a+b\sec^{-1}(cx)\right)^3}{x^2}\,dx \\
&= -\frac{b^3\operatorname{arcsec}(cx)^3}{x}+3\left(c\sqrt{-\frac{1}{c^2x^2}+1}-\frac{\operatorname{arcsec}(cx)}{x}\right)a^2b \\
&\quad+6\left(c\sqrt{-\frac{1}{c^2x^2}+1}\operatorname{arcsec}(cx)+\frac{1}{x}\right)ab^2 \\
&\quad+3\left(c\sqrt{-\frac{1}{c^2x^2}+1}\operatorname{arcsec}(cx)^2-2\,c\sqrt{-\frac{1}{c^2x^2}+1}+\frac{2\operatorname{arcsec}(cx)}{x}\right)b^3 \\
&\quad-\frac{3\,ab^2\operatorname{arcsec}(cx)^2}{x}-\frac{a^3}{x}
\end{aligned}$$

input
```
integrate((a+b*arcsec(c*x))^3/x^2,x, algorithm="maxima")
```

output
```
-b^3*arcsec(c*x)^3/x + 3*(c*sqrt(-1/(c^2*x^2) + 1) - arcsec(c*x)/x)*a^2*b
+ 6*(c*sqrt(-1/(c^2*x^2) + 1)*arcsec(c*x) + 1/x)*a*b^2 + 3*(c*sqrt(-1/(c^2
*x^2) + 1)*arcsec(c*x)^2 - 2*c*sqrt(-1/(c^2*x^2) + 1) + 2*arcsec(c*x)/x)*b
^3 - 3*a*b^2*arcsec(c*x)^2/x - a^3/x
```

3.29.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(76) = 152$.

Time $= 0.31$ (sec) , antiderivative size $= 196$, normalized size of antiderivative $= 2.45$

$$\int \frac{\left(a + b\sec^{-1}(cx)\right)^3}{x^2}\,dx$$
$$= \left(3\,b^3\sqrt{-\frac{1}{c^2x^2}+1}\arccos\left(\frac{1}{cx}\right)^2 + 6\,ab^2\sqrt{-\frac{1}{c^2x^2}+1}\arccos\left(\frac{1}{cx}\right) - \frac{b^3\arccos\left(\frac{1}{cx}\right)^3}{cx} + 3\,a^2b\sqrt{-\frac{1}{c^2x^2}+1}\right.$$

input
```
integrate((a+b*arcsec(c*x))^3/x^2,x, algorithm="giac")
```

output
```
(3*b^3*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x))^2 + 6*a*b^2*sqrt(-1/(c^2*x^2
) + 1)*arccos(1/(c*x)) - b^3*arccos(1/(c*x))^3/(c*x) + 3*a^2*b*sqrt(-1/(c^
2*x^2) + 1) - 6*b^3*sqrt(-1/(c^2*x^2) + 1) - 3*a*b^2*arccos(1/(c*x))^2/(c*
x) - 3*a^2*b*arccos(1/(c*x))/(c*x) + 6*b^3*arccos(1/(c*x))/(c*x) - a^3/(c*
x) + 6*a*b^2/(c*x))*c
```

3.29.9 Mupad [B] (verification not implemented)

Time $= 0.96$ (sec) , antiderivative size $= 156$, normalized size of antiderivative $= 1.95$

$$\begin{aligned}\int \frac{\left(a + b\sec^{-1}(cx)\right)^3}{x^2}\,dx = {} & \frac{b^3\left(6\operatorname{acos}\left(\frac{1}{c\,x}\right) - \operatorname{acos}\left(\frac{1}{c\,x}\right)^3\right)}{x} - \frac{a^3}{x} \\ & + 3\,a^2\,b\,c\left(\sqrt{1-\frac{1}{c^2\,x^2}} - \frac{\operatorname{acos}\left(\frac{1}{c\,x}\right)}{c\,x}\right) \\ & + b^3\,c\sqrt{1-\frac{1}{c^2\,x^2}}\left(3\operatorname{acos}\left(\frac{1}{c\,x}\right)^2 - 6\right) \\ & + 3\,a\,b^2\,c\left(2\operatorname{acos}\left(\frac{1}{c\,x}\right)\sqrt{1-\frac{1}{c^2\,x^2}} - \frac{\operatorname{acos}\left(\frac{1}{c\,x}\right)^2 - 2}{c\,x}\right)\end{aligned}$$

input
```
int((a + b*acos(1/(c*x)))^3/x^2,x)
```

output
```
(b^3*(6*acos(1/(c*x)) - acos(1/(c*x))^3))/x - a^3/x + 3*a^2*b*c*((1 - 1/(c
^2*x^2))^(1/2) - acos(1/(c*x))/(c*x)) + b^3*c*(1 - 1/(c^2*x^2))^(1/2)*(3*a
cos(1/(c*x))^2 - 6) + 3*a*b^2*c*(2*acos(1/(c*x))*(1 - 1/(c^2*x^2))^(1/2) -
 (acos(1/(c*x))^2 - 2)/(c*x))
```

3.30 $\int \frac{\left(a+b\sec^{-1}(cx)\right)^3}{x^3}\,dx$

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3.30.1 Optimal result

Integrand size = 14, antiderivative size = 137

$$\begin{aligned}\int \frac{\left(a+b\sec^{-1}(cx)\right)^3}{x^3}\,dx = &-\frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{8x}+\frac{3}{8}b^3c^2\sec^{-1}(cx)\\ &-\frac{3}{4}b^2\left(c^2-\frac{1}{x^2}\right)\left(a+b\sec^{-1}(cx)\right)\\ &+\frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{4x}\\ &-\frac{1}{4}c^2\left(a+b\sec^{-1}(cx)\right)^3+\frac{1}{2}\left(c^2-\frac{1}{x^2}\right)\left(a+b\sec^{-1}(cx)\right)^3\end{aligned}$$

output

```
3/8*b^3*c^2*arcsec(c*x)-3/4*b^2*(c^2-1/x^2)*(a+b*arcsec(c*x))-1/4*c^2*(a+b
*arcsec(c*x))^3+1/2*(c^2-1/x^2)*(a+b*arcsec(c*x))^3-3/8*b^3*c*(1-1/c^2/x^2
)^(1/2)/x+3/4*b*c*(a+b*arcsec(c*x))^2*(1-1/c^2/x^2)^(1/2)/x
```

3.30.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.35

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^3}\,dx$$
$$= \frac{-4a^3+6ab^2+6a^2bc\sqrt{1-\frac{1}{c^2x^2}}x-3b^3c\sqrt{1-\frac{1}{c^2x^2}}x+6b\left(-2a^2+b^2+2abc\sqrt{1-\frac{1}{c^2x^2}}x\right)\sec^{-1}(cx)+6b^2}{8}$$

input
```
Integrate[(a + b*ArcSec[c*x])^3/x^3,x]
```

output
```
(-4*a^3 + 6*a*b^2 + 6*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x - 3*b^3*c*Sqrt[1 - 1
/(c^2*x^2)]*x + 6*b*(-2*a^2 + b^2 + 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x)*ArcSe
c[c*x] + 6*b^2*(b*c*Sqrt[1 - 1/(c^2*x^2)]*x + a*(-2 + c^2*x^2))*ArcSec[c*x
]^2 + 2*b^3*(-2 + c^2*x^2)*ArcSec[c*x]^3 + 3*b*(-2*a^2 + b^2)*c^2*x^2*ArcS
in[1/(c*x)])/(8*x^2)
```

3.30.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5745, 4904, 3042, 3792, 17, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^3}\,dx$$
$$\downarrow 5745$$
$$c^2\int \frac{\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^3}{cx}\,d\sec^{-1}(cx)$$
$$\downarrow 4904$$
$$c^2\left(\frac{1}{2}\left(1-\frac{1}{c^2x^2}\right)(a+b\sec^{-1}(cx))^3-\frac{3}{2}b\int\left(1-\frac{1}{c^2x^2}\right)(a+b\sec^{-1}(cx))^2\,d\sec^{-1}(cx)\right)$$
$$\downarrow 3042$$
$$c^2\left(\frac{1}{2}\left(1-\frac{1}{c^2x^2}\right)(a+b\sec^{-1}(cx))^3-\frac{3}{2}b\int(a+b\sec^{-1}(cx))^2\sin\left(\sec^{-1}(cx)\right)^2\,d\sec^{-1}(cx)\right)$$

$$\downarrow 3792$$

$$c^2\left(\frac{1}{2}\left(1-\frac{1}{c^2x^2}\right)\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{2}b\left(\frac{1}{2}\int\left(a+b\sec^{-1}(cx)\right)^2 d\sec^{-1}(cx)-\frac{1}{2}b^2\int\left(1-\frac{1}{c^2x^2}\right)d\sec^{-1}(cx)\right.\right.$$

$$\downarrow 17$$

$$c^2\left(\frac{1}{2}\left(1-\frac{1}{c^2x^2}\right)\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{2}b\left(-\frac{1}{2}b^2\int\left(1-\frac{1}{c^2x^2}\right)d\sec^{-1}(cx)-\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{2cx}+\frac{1}{2}\right.\right.$$

$$\downarrow 3042$$

$$c^2\left(\frac{1}{2}\left(1-\frac{1}{c^2x^2}\right)\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{2}b\left(-\frac{1}{2}b^2\int\sin\left(\sec^{-1}(cx)\right)^2 d\sec^{-1}(cx)-\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{2cx}\right.\right.$$

$$\downarrow 3115$$

$$c^2\left(\frac{1}{2}\left(1-\frac{1}{c^2x^2}\right)\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{2}b\left(-\frac{1}{2}b^2\left(\frac{1}{2}\int 1d\sec^{-1}(cx)-\frac{\sqrt{1-\frac{1}{c^2x^2}}}{2cx}\right)-\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right.\right.\right.\right.$$

$$\downarrow 24$$

$$c^2\left(\frac{1}{2}\left(1-\frac{1}{c^2x^2}\right)\left(a+b\sec^{-1}(cx)\right)^3-\frac{3}{2}b\left(-\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{2cx}+\frac{1}{2}b\left(1-\frac{1}{c^2x^2}\right)\left(a+b\sec^{-1}(cx)\right)+\right.\right.$$

input
```
Int[(a + b*ArcSec[c*x])^3/x^3,x]
```

output
```
c^2*(((1 - 1/(c^2*x^2))*(a + b*ArcSec[c*x])^3)/2 - (3*b*(-1/2*(b^2*(-1/2*S
qrt[1 - 1/(c^2*x^2)]/(c*x) + ArcSec[c*x]/2)) + (b*(1 - 1/(c^2*x^2))*(a + b
*ArcSec[c*x]))/2 - (Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcSec[c*x])^2)/(2*c*x) +
 (a + b*ArcSec[c*x])^3/(6*b)))/2)
```

3.30.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

3.30.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. 2(121) = 242.

Time = 0.81 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.14

method	result
derivativedivides	$c^2\left(-\frac{a^3}{2c^2x^2}+b^3\left(-\frac{\operatorname{arcsec}(cx)^3}{2c^2x^2}+\frac{3\operatorname{arcsec}(cx)^2\left(cx\operatorname{arcsec}(cx)+\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{4cx}+\frac{3\operatorname{arcsec}(cx)}{4c^2x^2}-\frac{3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{8cx}\right.\right.$
default	$c^2\left(-\frac{a^3}{2c^2x^2}+b^3\left(-\frac{\operatorname{arcsec}(cx)^3}{2c^2x^2}+\frac{3\operatorname{arcsec}(cx)^2\left(cx\operatorname{arcsec}(cx)+\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{4cx}+\frac{3\operatorname{arcsec}(cx)}{4c^2x^2}-\frac{3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{8cx}\right.\right.$
parts	$-\frac{a^3}{2x^2}+b^3c^2\left(-\frac{\operatorname{arcsec}(cx)^3}{2c^2x^2}+\frac{3\operatorname{arcsec}(cx)^2\left(cx\operatorname{arcsec}(cx)+\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{4cx}+\frac{3\operatorname{arcsec}(cx)}{4c^2x^2}-\frac{3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{8cx}-\frac{3}{}\right.$

input
```
int((a+b*arcsec(c*x))^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(-1/2*a^3/c^2/x^2+b^3*(-1/2/c^2/x^2*arcsec(c*x)^3+3/4*arcsec(c*x)^2*(c
*x*arcsec(c*x)+((c^2*x^2-1)/c^2/x^2)^(1/2))/c/x+3/4/c^2/x^2*arcsec(c*x)-3/
8*((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x-3/8*arcsec(c*x)-1/2*arcsec(c*x)^3)+3*a*b
^2*(-1/2/c^2/x^2*arcsec(c*x)^2+1/2*arcsec(c*x)*(c*x*arcsec(c*x)+((c^2*x^2-
1)/c^2/x^2)^(1/2))/c/x-1/4*arcsec(c*x)^2-1/4+1/4/c^2/x^2)+3*a^2*b*(-1/2/c^
2/x^2*arcsec(c*x)-1/4*(c^2*x^2-1)^(1/2)*(arctan(1/(c^2*x^2-1)^(1/2))*c^2*x
^2-(c^2*x^2-1)^(1/2))/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^3/c^3))
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^3}{x^3}\,dx$$
$$=\frac{2\left(b^3c^2x^2-2\,b^3\right)\operatorname{arcsec}\left(cx\right)^3-4\,a^3+6\,ab^2+6\left(ab^2c^2x^2-2\,ab^2\right)\operatorname{arcsec}\left(cx\right)^2+3\left(\left(2\,a^2b-b^3\right)c^2x^2-4\,a^2b\right.}{8\,x^2}$$

input
```
integrate((a+b*arcsec(c*x))^3/x^3,x, algorithm="fricas")
```

output
```
1/8*(2*(b^3*c^2*x^2 - 2*b^3)*arcsec(c*x)^3 - 4*a^3 + 6*a*b^2 + 6*(a*b^2*c^
2*x^2 - 2*a*b^2)*arcsec(c*x)^2 + 3*((2*a^2*b - b^3)*c^2*x^2 - 4*a^2*b + 2*
b^3)*arcsec(c*x) + 3*(2*b^3*arcsec(c*x)^2 + 4*a*b^2*arcsec(c*x) + 2*a^2*b
- b^3)*sqrt(c^2*x^2 - 1))/x^2
```

3.30.6 Sympy [F]

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^3}\,dx = \int \frac{(a+b\operatorname{asec}(cx))^3}{x^3}\,dx$$

input
```
integrate((a+b*asec(c*x))**3/x**3,x)
```

output
```
Integral((a + b*asec(c*x))**3/x**3, x)
```

3.30.7 Maxima [F]

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^3}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)^3}{x^3}\,dx$$

input
```
integrate((a+b*arcsec(c*x))^3/x^3,x, algorithm="maxima")
```

output

```
-3/4*a^2*b*((c^4*x*sqrt(-1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1)
- c^3*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)))/c + 2*arcsec(c*x)/x^2) - 1/2*a^3
/x^2 - 1/8*(4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3*b^3*arctan(sqr
t(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 + 12*(a*b^2*c^2*(log(c*x + 1) + l
og(c*x - 1) - 2*log(x))*log(c)^2 + 16*b^3*c^2*integrate(1/8*x^2*arctan(sqr
t(c*x + 1)*sqrt(c*x - 1))/(c^2*x^5 - x^3), x)*log(c)^2 - 16*b^3*c^2*integr
ate(1/8*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)/(c^2*x^5 - x^
3), x)*log(c) + 32*b^3*c^2*integrate(1/8*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x
 - 1))*log(x)/(c^2*x^5 - x^3), x)*log(c) - 16*a*b^2*c^2*integrate(1/8*x^2*
log(c^2*x^2)/(c^2*x^5 - x^3), x)*log(c) + 32*a*b^2*c^2*integrate(1/8*x^2*l
og(x)/(c^2*x^5 - x^3), x)*log(c) - 16*b^3*c^2*integrate(1/8*x^2*arctan(sqr
t(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)*log(x)/(c^2*x^5 - x^3), x) + 16*b^3
*c^2*integrate(1/8*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(x)^2/(c^2*x
^5 - x^3), x) - 16*a*b^2*c^2*integrate(1/8*x^2*arctan(sqrt(c*x + 1)*sqrt(c
*x - 1))^2/(c^2*x^5 - x^3), x) + 8*b^3*c^2*integrate(1/8*x^2*arctan(sqrt(c
*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)/(c^2*x^5 - x^3), x) + 4*a*b^2*c^2*inte
grate(1/8*x^2*log(c^2*x^2)^2/(c^2*x^5 - x^3), x) - 16*a*b^2*c^2*integrate(
1/8*x^2*log(c^2*x^2)*log(x)/(c^2*x^5 - x^3), x) + 16*a*b^2*c^2*integrate(1
/8*x^2*log(x)^2/(c^2*x^5 - x^3), x) - (c^2*log(c*x + 1) + c^2*log(c*x - 1)
 - 2*c^2*log(x) + 1/x^2)*a*b^2*log(c)^2 - 16*b^3*integrate(1/8*arctan(s...
```

3.30.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(121) = 242$.

Time $= 0.30$ (sec) , antiderivative size $= 278$, normalized size of antiderivative $= 2.03$

$$\int \frac{\left(a + b\sec^{-1}(cx)\right)^3}{x^3}\,dx$$

$$= \frac{1}{8}\left(2\,b^3 c \arccos\left(\frac{1}{cx}\right)^3 + 6\,ab^2 c \arccos\left(\frac{1}{cx}\right)^2 + 6\,a^2 bc \arccos\left(\frac{1}{cx}\right) - 3\,b^3 c \arccos\left(\frac{1}{cx}\right) + \frac{6\,b^3\sqrt{-\frac{1}{c^2x^2}}}{}\right.$$

input

```
integrate((a+b*arcsec(c*x))^3/x^3,x, algorithm="giac")
```

output
```
1/8*(2*b^3*c*arccos(1/(c*x))^3 + 6*a*b^2*c*arccos(1/(c*x))^2 + 6*a^2*b*c*a
rccos(1/(c*x)) - 3*b^3*c*arccos(1/(c*x)) + 6*b^3*sqrt(-1/(c^2*x^2) + 1)*ar
ccos(1/(c*x))^2/x - 3*a*b^2*c + 12*a*b^2*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(
c*x))/x - 4*b^3*arccos(1/(c*x))^3/(c*x^2) + 6*a^2*b*sqrt(-1/(c^2*x^2) + 1)
/x - 3*b^3*sqrt(-1/(c^2*x^2) + 1)/x - 12*a*b^2*arccos(1/(c*x))^2/(c*x^2) -
 12*a^2*b*arccos(1/(c*x))/(c*x^2) + 6*b^3*arccos(1/(c*x))/(c*x^2) - 4*a^3/
(c*x^2) + 6*a*b^2/(c*x^2))*c
```

3.30.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^3}\,dx = \int \frac{\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^3}{x^3}\,dx$$

input
```
int((a + b*acos(1/(c*x)))^3/x^3,x)
```

output
```
int((a + b*acos(1/(c*x)))^3/x^3, x)
```

3.31 $\int \frac{(a+b\sec^{-1}(cx))^3}{x^4}\,dx$

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3.31.1 Optimal result

Integrand size = 14, antiderivative size = 170

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^4}\,dx = -\frac{14}{9}b^3c^3\sqrt{1-\frac{1}{c^2x^2}} + \frac{2}{27}b^3c^3\left(1-\frac{1}{c^2x^2}\right)^{3/2} + \frac{2b^2(a+b\sec^{-1}(cx))}{9x^3} + \frac{4b^2c^2(a+b\sec^{-1}(cx))}{3x} + \frac{2}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2 + \frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{3x^2} - \frac{(a+b\sec^{-1}(cx))^3}{3x^3}$$

output

```
2/27*b^3*c^3*(1-1/c^2/x^2)^(3/2)+2/9*b^2*(a+b*arcsec(c*x))/x^3+4/3*b^2*c^2
*(a+b*arcsec(c*x))/x-1/3*(a+b*arcsec(c*x))^3/x^3-14/9*b^3*c^3*(1-1/c^2/x^2
)^(1/2)+2/3*b*c^3*(a+b*arcsec(c*x))^2*(1-1/c^2/x^2)^(1/2)+1/3*b*c*(a+b*arc
sec(c*x))^2*(1-1/c^2/x^2)^(1/2)/x^2
```

3.31.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.20

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^4}\,dx = \frac{-9a^3+9a^2bc\sqrt{1-\frac{1}{c^2x^2}}x(1+2c^2x^2)+6ab^2(1+6c^2x^2)-2b^3c\sqrt{1-\frac{1}{c^2x^2}}x(1+20c^2x^2)+3b\left(-9a^2+6ab\right.}{}$$

input
```
Integrate[(a + b*ArcSec[c*x])^3/x^4,x]
```

output
```
(-9*a^3 + 9*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2) + 6*a*b^2*(1 +
 6*c^2*x^2) - 2*b^3*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 20*c^2*x^2) + 3*b*(-9*a
^2 + 6*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2) + 2*b^2*(1 + 6*c^2*x^
2))*ArcSec[c*x] + 9*b^2*(-3*a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2
))*ArcSec[c*x]^2 - 9*b^3*ArcSec[c*x]^3)/(27*x^3)
```

3.31.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {5745, 4905, 3042, 3792, 3042, 3113, 2009, 3777, 25, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^3}{x^4}\,dx$$

$$\downarrow 5745$$

$$c^3\int \frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^3}{c^2x^2}\,d\sec^{-1}(cx)$$

$$\downarrow 4905$$

$$c^3\left(b\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{c^3x^3}\,d\sec^{-1}(cx)-\frac{\left(a+b\sec^{-1}(cx)\right)^3}{3c^3x^3}\right)$$

$$\downarrow 3042$$

$$c^3\left(b\int \left(a+b\sec^{-1}(cx)\right)^2\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^3 d\sec^{-1}(cx)-\frac{\left(a+b\sec^{-1}(cx)\right)^3}{3c^3x^3}\right)$$

$$\downarrow 3792$$

$$c^3\left(b\left(\frac{2}{3}\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{cx}\,d\sec^{-1}(cx)-\frac{2}{9}b^2\int \frac{1}{c^3x^3}\,d\sec^{-1}(cx)+\frac{2b\left(a+b\sec^{-1}(cx)\right)}{9c^3x^3}+\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec\right.}{3c^2x^2}\right.$$

$$\downarrow 3042$$

$$c^3\left(b\left(\frac{2}{3}\int \left(a+b\sec^{-1}(cx)\right)^2\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)d\sec^{-1}(cx)-\frac{2}{9}b^2\int \sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^3 d\sec^{-1}(cx)+\frac{2b(a\right.\right.$$

$$\downarrow 3113$$

$$c^3\left(b\left(\frac{2}{3}\int \left(a+b\sec^{-1}(cx)\right)^2\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)d\sec^{-1}(cx)+\frac{2}{9}b^2\int \frac{1}{c^2x^2}d\left(-\sqrt{1-\frac{1}{c^2x^2}}\right)+\frac{2b(a+b\sec^{-1}(\right.\right.$$

$$\downarrow 2009$$

$$c^3\left(b\left(\frac{2}{3}\int \left(a+b\sec^{-1}(cx)\right)^2\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)d\sec^{-1}(cx)+\frac{2b\left(a+b\sec^{-1}(cx)\right)}{9c^3x^3}+\frac{\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(c\right.\right.$$

$$\downarrow 3777$$

$$c^3\left(b\left(\frac{2}{3}\left(2b\int -\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)d\sec^{-1}(cx)+\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2\right)+\frac{2b\left(a+b\sec^{-1}(cx)\right.}{9c^3x^3}\right.\right.$$

$$\downarrow 25$$

$$c^3\left(b\left(\frac{2}{3}\left(\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\int \sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)d\sec^{-1}(cx)\right)+\frac{2b\left(a+b\sec^{-1}(cx)\right)}{9c^3x^3}\right.\right.$$

$$\downarrow 3042$$

$$c^3\left(b\left(\frac{2}{3}\left(\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\int \left(a+b\sec^{-1}(cx)\right)\sin\left(\sec^{-1}(cx)\right)d\sec^{-1}(cx)\right)+\frac{2b(a+b\sec^{-1}(c\right.\right.$$

$$\downarrow 3777$$

$$c^3\left(b\left(\frac{2}{3}\left(\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\left(b\int \frac{1}{cx}d\sec^{-1}(cx)-\frac{a+b\sec^{-1}(cx)}{cx}\right)\right)+\frac{2b\left(a+b\sec^{-1}(cx)\right)}{9c^3x^3}+\right.\right.$$

$$\downarrow 3042$$

$$c^3\left(b\left(\frac{2}{3}\left(\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\left(b\int\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)d\sec^{-1}(cx)-\frac{a+b\sec^{-1}(cx)}{cx}\right)\right)+\frac{2b(a\right.\right.\right.$$

$$\downarrow 3117$$

$$c^3\left(b\left(\frac{2b\left(a+b\sec^{-1}(cx)\right)}{9c^3x^3}+\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{3c^2x^2}+\frac{2}{3}\left(\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2-2b\left(b\sqrt{1-\frac{1}{c^2x}}\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcSec[c*x])^3/x^4,x]
```

output
```
c^3*(-1/3*(a + b*ArcSec[c*x])^3/(c^3*x^3) + b*((2*b^2*(-Sqrt[1 - 1/(c^2*x^
2)] + (1 - 1/(c^2*x^2))^(3/2)/3))/9 + (2*b*(a + b*ArcSec[c*x]))/(9*c^3*x^3
) + (Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcSec[c*x])^2)/(3*c^2*x^2) + (2*(Sqrt[1
 - 1/(c^2*x^2)]*(a + b*ArcSec[c*x])^2 - 2*b*(b*Sqrt[1 - 1/(c^2*x^2)] - (a
+ b*ArcSec[c*x])/(c*x))))/3))
```

3.31.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4905
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

3.31.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(148) = 296$.

Time = 1.16 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.76

method	result
derivativedivides	$c^3\left(-\frac{a^3}{3c^3x^3}+b^3\left(-\frac{\operatorname{arcsec}(cx)^3}{3c^3x^3}+\frac{\operatorname{arcsec}(cx)^2(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3c^2x^2}-\frac{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3}+\frac{4\operatorname{arcsec}(cx)}{3cx}+\frac{2\operatorname{arcse}}{9c^3}\right.\right.$
default	$c^3\left(-\frac{a^3}{3c^3x^3}+b^3\left(-\frac{\operatorname{arcsec}(cx)^3}{3c^3x^3}+\frac{\operatorname{arcsec}(cx)^2(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3c^2x^2}-\frac{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3}+\frac{4\operatorname{arcsec}(cx)}{3cx}+\frac{2\operatorname{arcse}}{9c^3}\right.\right.$
parts	$-\frac{a^3}{3x^3}+b^3c^3\left(-\frac{\operatorname{arcsec}(cx)^3}{3c^3x^3}+\frac{\operatorname{arcsec}(cx)^2(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3c^2x^2}-\frac{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3}+\frac{4\operatorname{arcsec}(cx)}{3cx}+\frac{2\operatorname{arcsec}(cx}{9c^3x^3}\right.$

input
```
int((a+b*arcsec(c*x))^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
c^3*(-1/3*a^3/c^3/x^3+b^3*(-1/3/c^3/x^3*arcsec(c*x)^3+1/3*arcsec(c*x)^2*(2
*c^2*x^2+1)/c^2/x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)-4/3*((c^2*x^2-1)/c^2/x^2)^
(1/2)+4/3/c/x*arcsec(c*x)+2/9/c^3/x^3*arcsec(c*x)-2/27*(2*c^2*x^2+1)/c^2/x
^2*((c^2*x^2-1)/c^2/x^2)^(1/2))+3*a*b^2*(-1/3/c^3/x^3*arcsec(c*x)^2+2/9*ar
csec(c*x)*(2*c^2*x^2+1)/c^2/x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)+2/27/c^3/x^3+4
/9/c/x)+3*a^2*b*(-1/3/c^3/x^3*arcsec(c*x)+1/9*(c^2*x^2-1)*(2*c^2*x^2+1)/((
c^2*x^2-1)/c^2/x^2)^(1/2)/c^4/x^4))
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.01

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^4}\,dx$$
$$= \frac{36\,ab^2c^2x^2 - 9\,b^3\operatorname{arcsec}(cx)^3 - 27\,ab^2\operatorname{arcsec}(cx)^2 - 9\,a^3 + 6\,ab^2 + 3\,(12\,b^3c^2x^2 - 9\,a^2b + 2\,b^3)\operatorname{arcsec}(cx)}$$

input
```
integrate((a+b*arcsec(c*x))^3/x^4,x, algorithm="fricas")
```

output
```
1/27*(36*a*b^2*c^2*x^2 - 9*b^3*arcsec(c*x)^3 - 27*a*b^2*arcsec(c*x)^2 - 9*
a^3 + 6*a*b^2 + 3*(12*b^3*c^2*x^2 - 9*a^2*b + 2*b^3)*arcsec(c*x) + (2*(9*a
^2*b - 20*b^3)*c^2*x^2 + 9*a^2*b - 2*b^3 + 9*(2*b^3*c^2*x^2 + b^3)*arcsec(
c*x)^2 + 18*(2*a*b^2*c^2*x^2 + a*b^2)*arcsec(c*x))*sqrt(c^2*x^2 - 1))/x^3
```

3.31.6 Sympy [F]

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^4}\,dx = \int \frac{(a+b\operatorname{asec}(cx))^3}{x^4}\,dx$$

input
```
integrate((a+b*asec(c*x))**3/x**4,x)
```

output
```
Integral((a + b*asec(c*x))**3/x**4, x)
```

3.31.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 575 vs. $2(148) = 296$.

Time = 0.70 (sec) , antiderivative size = 575, normalized size of antiderivative = 3.38

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^3}{x^4}\,dx =$$

$$-\frac{1}{216}\left(\frac{72\left(c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}\right)\operatorname{arcsec}(cx)^2}{c}+\frac{72\,c^4\left(\frac{c^2\arcsin\left(\frac{1}{c|x|}\right)+\frac{2\sqrt{c^2x^2-1}c}{x}-\frac{\sqrt{c^2x^2-1}}{x^2}}{c}-\frac{c^2}{c}\right.\right.$$

$$-\frac{1}{3}\,a^2b\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{3\operatorname{arcsec}(cx)}{x^3}\right)$$

$$-\frac{b^3\operatorname{arcsec}(cx)^3}{3\,x^3}-\frac{ab^2\operatorname{arcsec}(cx)^2}{x^3}-\frac{a^3}{3\,x^3}$$

$$+\frac{2\left((6\,c^3x^2+c)\sqrt{cx+1}\sqrt{cx-1}+3\left(2\,c^5x^4-c^3x^2-c\right)\arctan\left(\sqrt{cx+1}\sqrt{cx-1}\right)\right)ab^2}{9\sqrt{cx+1}\sqrt{cx-1}cx^3}$$

input
```
integrate((a+b*arcsec(c*x))^3/x^4,x, algorithm="maxima")
```

output
```
-1/216*(72*(c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))*a
rcsec(c*x)^2/c + (72*c^4*((c^2*arcsin(1/(c*abs(x))) + 2*sqrt(c^2*x^2 - 1)*
c/x - sqrt(c^2*x^2 - 1)/x^2)/c - (c^2*arcsin(1/(c*abs(x))) - 2*sqrt(c^2*x^
2 - 1)*c/x - sqrt(c^2*x^2 - 1)/x^2)/c - 4*arctan(sqrt(c*x + 1)*sqrt(c*x -
1))/x) + c^2*((9*c^4*arcsin(1/(c*abs(x))) + 16*sqrt(c^2*x^2 - 1)*c^3/x - 9
*sqrt(c^2*x^2 - 1)*c^2/x^2 + 8*sqrt(c^2*x^2 - 1)*c/x^3 - 6*sqrt(c^2*x^2 -
1)/x^4)/c - (9*c^4*arcsin(1/(c*abs(x))) - 16*sqrt(c^2*x^2 - 1)*c^3/x - 9*s
qrt(c^2*x^2 - 1)*c^2/x^2 - 8*sqrt(c^2*x^2 - 1)*c/x^3 - 6*sqrt(c^2*x^2 - 1)
/x^4)/c - 48*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/x^3))/c^2)*b^3 - 1/3*a^2*
b*((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c + 3*arc
sec(c*x)/x^3) - 1/3*b^3*arcsec(c*x)^3/x^3 - a*b^2*arcsec(c*x)^2/x^3 - 1/3*
a^3/x^3 + 2/9*((6*c^3*x^2 + c)*sqrt(c*x + 1)*sqrt(c*x - 1) + 3*(2*c^5*x^4
- c^3*x^2 - c)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)))*a*b^2/(sqrt(c*x + 1)*s
qrt(c*x - 1)*c*x^3)
```

3.31.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. 2(148) = 296.

Time = 0.32 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.98

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^4}\,dx$$

$$= \frac{1}{27}\left(18\,b^3c^2\sqrt{-\frac{1}{c^2x^2}+1}\arccos\left(\frac{1}{cx}\right)^2 + 36\,ab^2c^2\sqrt{-\frac{1}{c^2x^2}+1}\arccos\left(\frac{1}{cx}\right) + 18\,a^2bc^2\sqrt{-\frac{1}{c^2x^2}+1} - 4\right.$$

input
```
integrate((a+b*arcsec(c*x))^3/x^4,x, algorithm="giac")
```

output
```
1/27*(18*b^3*c^2*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x))^2 + 36*a*b^2*c^2*s
qrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x)) + 18*a^2*b*c^2*sqrt(-1/(c^2*x^2) + 1
) - 40*b^3*c^2*sqrt(-1/(c^2*x^2) + 1) + 36*b^3*c*arccos(1/(c*x))/x + 9*b^3
*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x))^2/x^2 + 36*a*b^2*c/x + 18*a*b^2*sq
rt(-1/(c^2*x^2) + 1)*arccos(1/(c*x))/x^2 - 9*b^3*arccos(1/(c*x))^3/(c*x^3)
 + 9*a^2*b*sqrt(-1/(c^2*x^2) + 1)/x^2 - 2*b^3*sqrt(-1/(c^2*x^2) + 1)/x^2 -
 27*a*b^2*arccos(1/(c*x))^2/(c*x^3) - 27*a^2*b*arccos(1/(c*x))/(c*x^3) + 6
*b^3*arccos(1/(c*x))/(c*x^3) - 9*a^3/(c*x^3) + 6*a*b^2/(c*x^3))*c
```

3.31.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^4}\,dx = \int \frac{\left(a+b\operatorname{acos}\left(\frac{1}{c\,x}\right)\right)^3}{x^4}\,dx$$

input
```
int((a + b*acos(1/(c*x)))^3/x^4,x)
```

output
```
int((a + b*acos(1/(c*x)))^3/x^4, x)
```

3.32 $\int \frac{(a+b\sec^{-1}(cx))^3}{x^5}\,dx$

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3.32.1 Optimal result

Integrand size = 14, antiderivative size = 208

$$\begin{aligned}\int \frac{(a+b\sec^{-1}(cx))^3}{x^5}\,dx = &-\frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{128x^3}-\frac{45b^3c^3\sqrt{1-\frac{1}{c^2x^2}}}{256x}\\ &-\frac{45}{256}b^3c^4\sec^{-1}(cx)+\frac{3b^2(a+b\sec^{-1}(cx))}{32x^4}\\ &+\frac{9b^2c^2(a+b\sec^{-1}(cx))}{32x^2}+\frac{3bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2}{16x^3}\\ &+\frac{9bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2}{32x}\\ &+\frac{3}{32}c^4(a+b\sec^{-1}(cx))^3-\frac{(a+b\sec^{-1}(cx))^3}{4x^4}\end{aligned}$$

output

```
-45/256*b^3*c^4*arcsec(c*x)+3/32*b^2*(a+b*arcsec(c*x))/x^4+9/32*b^2*c^2*(a
+b*arcsec(c*x))/x^2+3/32*c^4*(a+b*arcsec(c*x))^3-1/4*(a+b*arcsec(c*x))^3/x
^4-3/128*b^3*c*(1-1/c^2/x^2)^(1/2)/x^3-45/256*b^3*c^3*(1-1/c^2/x^2)^(1/2)/
x+3/16*b*c*(a+b*arcsec(c*x))^2*(1-1/c^2/x^2)^(1/2)/x^3+9/32*b*c^3*(a+b*arc
sec(c*x))^2*(1-1/c^2/x^2)^(1/2)/x
```

3.32.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.36

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^5}\,dx$$
$$= \frac{-64a^3+24ab^2+48a^2bc\sqrt{1-\frac{1}{c^2x^2}}x-6b^3c\sqrt{1-\frac{1}{c^2x^2}}x+72ab^2c^2x^2+72a^2bc^3\sqrt{1-\frac{1}{c^2x^2}}x^3-45b^3c^3\sqrt{1-}$$

input
```
Integrate[(a + b*ArcSec[c*x])^3/x^5,x]
```

output
```
(-64*a^3 + 24*a*b^2 + 48*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x - 6*b^3*c*Sqrt[1
- 1/(c^2*x^2)]*x + 72*a*b^2*c^2*x^2 + 72*a^2*b*c^3*Sqrt[1 - 1/(c^2*x^2)]*x
^3 - 45*b^3*c^3*Sqrt[1 - 1/(c^2*x^2)]*x^3 + 24*b*(-8*a^2 + b^2*(1 + 3*c^2*
x^2) + 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(2 + 3*c^2*x^2))*ArcSec[c*x] + 24*b
^2*(b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(2 + 3*c^2*x^2) + a*(-8 + 3*c^4*x^4))*ArcS
ec[c*x]^2 + 8*b^3*(-8 + 3*c^4*x^4)*ArcSec[c*x]^3 + 9*b*(-8*a^2 + 5*b^2)*c^
4*x^4*ArcSin[1/(c*x)])/(256*x^4)
```

3.32.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.29, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5745, 4905, 3042, 3792, 3042, 3115, 3042, 3115, 24, 3792, 17, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^5}\,dx$$
$$\downarrow 5745$$
$$c^4\int \frac{\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^3}{c^3x^3}\,d\sec^{-1}(cx)$$
$$\downarrow 4905$$
$$c^4\left(\frac{3}{4}b\int \frac{(a+b\sec^{-1}(cx))^2}{c^4x^4}\,d\sec^{-1}(cx)-\frac{(a+b\sec^{-1}(cx))^3}{4c^4x^4}\right)$$
$$\downarrow 3042$$

$$c^4\left(\frac{3}{4}b\int\left(a+b\sec^{-1}(cx)\right)^2\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^4 d\sec^{-1}(cx)-\frac{\left(a+b\sec^{-1}(cx)\right)^3}{4c^4x^4}\right)$$

$$\downarrow 3792$$

$$c^4\left(\frac{3}{4}b\left(\frac{3}{4}\int\frac{\left(a+b\sec^{-1}(cx)\right)^2}{c^2x^2}d\sec^{-1}(cx)-\frac{1}{8}b^2\int\frac{1}{c^4x^4}d\sec^{-1}(cx)+\frac{b\left(a+b\sec^{-1}(cx)\right)}{8c^4x^4}+\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec\right.}{4c^3x^3}\right.$$

$$\downarrow 3042$$

$$c^4\left(\frac{3}{4}b\left(\frac{3}{4}\int\left(a+b\sec^{-1}(cx)\right)^2\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^2 d\sec^{-1}(cx)-\frac{1}{8}b^2\int\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^4 d\sec^{-1}(cx)+\frac{b\left(\right.}{}\right.\right.$$

$$\downarrow 3115$$

$$c^4\left(\frac{3}{4}b\left(\frac{3}{4}\int\left(a+b\sec^{-1}(cx)\right)^2\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^2 d\sec^{-1}(cx)-\frac{1}{8}b^2\left(\frac{3}{4}\int\frac{1}{c^2x^2}d\sec^{-1}(cx)+\frac{\sqrt{1-\frac{1}{c^2x^2}}}{4c^3x^3}\right)+\right.\right.$$

$$\downarrow 3042$$

$$c^4\left(\frac{3}{4}b\left(\frac{3}{4}\int\left(a+b\sec^{-1}(cx)\right)^2\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^2 d\sec^{-1}(cx)-\frac{1}{8}b^2\left(\frac{3}{4}\int\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^2 d\sec^{-1}(cx)-\right.\right.\right.$$

$$\downarrow 3115$$

$$c^4\left(\frac{3}{4}b\left(\frac{3}{4}\int\left(a+b\sec^{-1}(cx)\right)^2\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^2 d\sec^{-1}(cx)-\frac{1}{8}b^2\left(\frac{3}{4}\left(\frac{1}{2}\int 1 d\sec^{-1}(cx)+\frac{\sqrt{1-\frac{1}{c^2x^2}}}{2cx}\right)+\right.\right.\right.$$

$$\downarrow 24$$

$$c^4\left(\frac{3}{4}b\left(\frac{3}{4}\int\left(a+b\sec^{-1}(cx)\right)^2\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^2 d\sec^{-1}(cx)+\frac{b\left(a+b\sec^{-1}(cx)\right)}{8c^4x^4}+\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(\right.}{4c^3x^3}\right.\right.$$

$$\downarrow 3792$$

$$c^4\left(\frac{3}{4}b\left(\frac{3}{4}\left(\frac{1}{2}\int\left(a+b\sec^{-1}(cx)\right)^2 d\sec^{-1}(cx)-\frac{1}{2}b^2\int\frac{1}{c^2x^2}d\sec^{-1}(cx)+\frac{b\left(a+b\sec^{-1}(cx)\right)}{2c^2x^2}+\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+\right.}{2c}\right.\right.\right.$$

$$\downarrow 17$$

$$c^4\left(\frac{3}{4}b\left(\frac{3}{4}\left(-\frac{1}{2}b^2\int\frac{1}{c^2x^2}d\sec^{-1}(cx)+\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{2cx}+\frac{b\left(a+b\sec^{-1}(cx)\right)}{2c^2x^2}+\frac{\left(a+b\sec^{-1}(cx)\right)^3}{6b}\right.\right.\right.$$

$$\downarrow 3042$$

$$c^4\left(\frac{3}{4}b\left(\frac{3}{4}\left(-\frac{1}{2}b^2\int\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)^2 d\sec^{-1}(cx)+\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{2cx}+\frac{b\left(a+b\sec^{-1}(cx)\right)}{2c^2x^2}+\frac{(a\right.\right.\right.$$

$$\downarrow 3115$$

$$c^4\left(\frac{3}{4}b\left(\frac{3}{4}\left(-\frac{1}{2}b^2\left(\frac{1}{2}\int 1d\sec^{-1}(cx)+\frac{\sqrt{1-\frac{1}{c^2x^2}}}{2cx}\right)+\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{2cx}+\frac{b\left(a+b\sec^{-1}(cx)\right)}{2c^2x^2}+\frac{(a\right.\right.\right.$$

$$\downarrow 24$$

$$c^4\left(\frac{3}{4}b\left(\frac{3}{4}\left(\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{2cx}+\frac{b\left(a+b\sec^{-1}(cx)\right)}{2c^2x^2}+\frac{\left(a+b\sec^{-1}(cx)\right)^3}{6b}-\frac{1}{2}b^2\left(\frac{\sqrt{1-\frac{1}{c^2x^2}}}{2cx}+\frac{1}{2}\sec\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcSec[c*x])^3/x^5,x]
```

output
```
c^4*(-1/4*(a + b*ArcSec[c*x])^3/(c^4*x^4) + (3*b*(-1/8*(b^2*(Sqrt[1 - 1/(c
^2*x^2)]/(4*c^3*x^3) + (3*(Sqrt[1 - 1/(c^2*x^2)]/(2*c*x) + ArcSec[c*x]/2))
/4)) + (b*(a + b*ArcSec[c*x]))/(8*c^4*x^4) + (Sqrt[1 - 1/(c^2*x^2)]*(a + b
*ArcSec[c*x])^2)/(4*c^3*x^3) + (3*(-1/2*(b^2*(Sqrt[1 - 1/(c^2*x^2)]/(2*c*x
) + ArcSec[c*x]/2)) + (b*(a + b*ArcSec[c*x]))/(2*c^2*x^2) + (Sqrt[1 - 1/(c
^2*x^2)]*(a + b*ArcSec[c*x])^2)/(2*c*x) + (a + b*ArcSec[c*x])^3/(6*b)))/4)
)/4)
```

3.32.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4905
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

3.32.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(182) = 364$.

Time = 1.10 (sec) , antiderivative size = 470, normalized size of antiderivative = 2.26

method	result
parts	$-\frac{a^3}{4x^4}+b^3c^4\left(-\frac{\operatorname{arcsec}(cx)^3}{4c^4x^4}+\frac{3\operatorname{arcsec}(cx)^2\left(3c^3x^3\operatorname{arcsec}(cx)+3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}+2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{32c^3x^3}+\frac{3\operatorname{arcsec}(cx)}{32c^4x^4}$
derivativedivides	$c^4\left(-\frac{a^3}{4c^4x^4}+b^3\left(-\frac{\operatorname{arcsec}(cx)^3}{4c^4x^4}+\frac{3\operatorname{arcsec}(cx)^2\left(3c^3x^3\operatorname{arcsec}(cx)+3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}+2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{32c^3x^3}+\frac{3\operatorname{arcsec}$
default	$c^4\left(-\frac{a^3}{4c^4x^4}+b^3\left(-\frac{\operatorname{arcsec}(cx)^3}{4c^4x^4}+\frac{3\operatorname{arcsec}(cx)^2\left(3c^3x^3\operatorname{arcsec}(cx)+3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}+2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{32c^3x^3}+\frac{3\operatorname{arcsec}$

input
```
int((a+b*arcsec(c*x))^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^3/x^4+b^3*c^4*(-1/4/c^4/x^4*arcsec(c*x)^3+3/32*arcsec(c*x)^2*(3*c^3
*x^3*arcsec(c*x)+3*c^2*x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)+2*((c^2*x^2-1)/c^2/
x^2)^(1/2))/c^3/x^3+3/32*arcsec(c*x)/c^4/x^4-3/256*(3*c^2*x^2+2)/c^3/x^3*(
(c^2*x^2-1)/c^2/x^2)^(1/2)-45/256*arcsec(c*x)+9/32/c^2/x^2*arcsec(c*x)-9/6
4*((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x-3/16*arcsec(c*x)^3)+3*a*b^2*c^4*(-1/4/c^
4/x^4*arcsec(c*x)^2+1/16*arcsec(c*x)*(3*c^3*x^3*arcsec(c*x)+3*c^2*x^2*((c^
2*x^2-1)/c^2/x^2)^(1/2)+2*((c^2*x^2-1)/c^2/x^2)^(1/2))/c^3/x^3-3/32*arcsec
(c*x)^2+1/128*(3*c^2*x^2+2)^2/c^4/x^4)-3/4*a^2*b/x^4*arcsec(c*x)-9/32*a^2*
b*c^3*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*arctan(1/(c^2*x^2-1)
^(1/2))+9/32*a^2*b*c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^3+3/16*a^2*
b/c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^5
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b\operatorname{sec}^{-1}(cx)\right)^3}{x^5}\,dx$$
$$=\frac{72\,ab^2c^2x^2+8\left(3\,b^3c^4x^4-8\,b^3\right)\operatorname{arcsec}\left(cx\right)^3-64\,a^3+24\,ab^2+24\left(3\,ab^2c^4x^4-8\,ab^2\right)\operatorname{arcsec}\left(cx\right)^2+3\left(3\left(8\right.\right.}{}$$

input
```
integrate((a+b*arcsec(c*x))^3/x^5,x, algorithm="fricas")
```

output
```
1/256*(72*a*b^2*c^2*x^2 + 8*(3*b^3*c^4*x^4 - 8*b^3)*arcsec(c*x)^3 - 64*a^3
 + 24*a*b^2 + 24*(3*a*b^2*c^4*x^4 - 8*a*b^2)*arcsec(c*x)^2 + 3*(3*(8*a^2*b
 - 5*b^3)*c^4*x^4 + 24*b^3*c^2*x^2 - 64*a^2*b + 8*b^3)*arcsec(c*x) + 3*(3*
(8*a^2*b - 5*b^3)*c^2*x^2 + 16*a^2*b - 2*b^3 + 8*(3*b^3*c^2*x^2 + 2*b^3)*a
rcsec(c*x)^2 + 16*(3*a*b^2*c^2*x^2 + 2*a*b^2)*arcsec(c*x))*sqrt(c^2*x^2 -
1))/x^4
```

3.32.6 Sympy [F]

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^5}\,dx = \int \frac{(a+b\,\text{asec}\,(cx))^3}{x^5}\,dx$$

input
```
integrate((a+b*asec(c*x))**3/x**5,x)
```

output
```
Integral((a + b*asec(c*x))**3/x**5, x)
```

3.32.7 Maxima [F]

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^5}\,dx = \int \frac{(b\,\text{arcsec}\,(cx)+a)^3}{x^5}\,dx$$

input
```
integrate((a+b*arcsec(c*x))^3/x^5,x, algorithm="maxima")
```

output

```
3/32*a^2*b*((3*c^5*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)) + (3*c^8*x^3*(-1/(c^
2*x^2) + 1)^(3/2) + 5*c^6*x*sqrt(-1/(c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2)
- 1)^2 - 2*c^2*x^2*(1/(c^2*x^2) - 1) + 1))/c - 8*arcsec(c*x)/x^4) - 1/4*a^
3/x^4 - 1/16*(4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3*b^3*arctan(s
qrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 + 12*(2*(c^2*log(c*x + 1) + c^2
*log(c*x - 1) - 2*c^2*log(x) + 1/x^2)*a*b^2*c^2*log(c)^2 + 64*b^3*c^2*inte
grate(1/16*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^7 - x^5), x)*log
(c)^2 - 64*b^3*c^2*integrate(1/16*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*
log(c^2*x^2)/(c^2*x^7 - x^5), x)*log(c) + 128*b^3*c^2*integrate(1/16*x^2*a
rctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(x)/(c^2*x^7 - x^5), x)*log(c) - 64*
a*b^2*c^2*integrate(1/16*x^2*log(c^2*x^2)/(c^2*x^7 - x^5), x)*log(c) + 128
*a*b^2*c^2*integrate(1/16*x^2*log(x)/(c^2*x^7 - x^5), x)*log(c) - 64*b^3*c
^2*integrate(1/16*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)*log
(x)/(c^2*x^7 - x^5), x) + 64*b^3*c^2*integrate(1/16*x^2*arctan(sqrt(c*x +
1)*sqrt(c*x - 1))*log(x)^2/(c^2*x^7 - x^5), x) - 64*a*b^2*c^2*integrate(1/
16*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2/(c^2*x^7 - x^5), x) + 16*b^3*
c^2*integrate(1/16*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)/(c
^2*x^7 - x^5), x) + 16*a*b^2*c^2*integrate(1/16*x^2*log(c^2*x^2)^2/(c^2*x^
7 - x^5), x) - 64*a*b^2*c^2*integrate(1/16*x^2*log(c^2*x^2)*log(x)/(c^2*x^
7 - x^5), x) + 64*a*b^2*c^2*integrate(1/16*x^2*log(x)^2/(c^2*x^7 - x^5)...
```

3.32.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(182) = 364$.

Time $= 0.31$ (sec) , antiderivative size $= 427$, normalized size of antiderivative $= 2.05$

$$\int \frac{\left(a + b\sec^{-1}(cx)\right)^3}{x^5}\,dx$$

$$= \frac{1}{256}\left(24\,b^3c^3\arccos\left(\frac{1}{cx}\right)^3 + 72\,ab^2c^3\arccos\left(\frac{1}{cx}\right)^2 + 72\,a^2bc^3\arccos\left(\frac{1}{cx}\right) - 45\,b^3c^3\arccos\left(\frac{1}{cx}\right) + \frac{72}{-}\right.$$

input

```
integrate((a+b*arcsec(c*x))^3/x^5,x, algorithm="giac")
```

output
```
1/256*(24*b^3*c^3*arccos(1/(c*x))^3 + 72*a*b^2*c^3*arccos(1/(c*x))^2 + 72*
a^2*b*c^3*arccos(1/(c*x)) - 45*b^3*c^3*arccos(1/(c*x)) + 72*b^3*c^2*sqrt(-
1/(c^2*x^2) + 1)*arccos(1/(c*x))^2/x - 45*a*b^2*c^3 + 144*a*b^2*c^2*sqrt(-
1/(c^2*x^2) + 1)*arccos(1/(c*x))/x + 72*a^2*b*c^2*sqrt(-1/(c^2*x^2) + 1)/x
 - 45*b^3*c^2*sqrt(-1/(c^2*x^2) + 1)/x + 72*b^3*c*arccos(1/(c*x))/x^2 + 48
*b^3*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x))^2/x^3 + 72*a*b^2*c/x^2 + 96*a*
b^2*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x))/x^3 - 64*b^3*arccos(1/(c*x))^3/
(c*x^4) + 48*a^2*b*sqrt(-1/(c^2*x^2) + 1)/x^3 - 6*b^3*sqrt(-1/(c^2*x^2) +
1)/x^3 - 192*a*b^2*arccos(1/(c*x))^2/(c*x^4) - 192*a^2*b*arccos(1/(c*x))/(
c*x^4) + 24*b^3*arccos(1/(c*x))/(c*x^4) - 64*a^3/(c*x^4) + 24*a*b^2/(c*x^4
))*c
```

3.32.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^5}\,dx = \int \frac{\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)^3}{x^5}\,dx$$

input
```
int((a + b*acos(1/(c*x)))^3/x^5,x)
```

output
```
int((a + b*acos(1/(c*x)))^3/x^5, x)
```

3.33 $\int \frac{x}{a+b\sec^{-1}(cx)}\,dx$

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3.33.8 Giac [N/A] . . . 294
3.33.9 Mupad [N/A] . . . 294

3.33.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{x}{a+b\sec^{-1}(cx)}\,dx = \text{Int}\left(\frac{x}{a+b\sec^{-1}(cx)},x\right)$$

output
```
Unintegrable(x/(a+b*arcsec(c*x)),x)
```

3.33.2 Mathematica [N/A]

Not integrable

Time = 2.47 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a+b\sec^{-1}(cx)}\,dx = \int \frac{x}{a+b\sec^{-1}(cx)}\,dx$$

input
```
Integrate[x/(a + b*ArcSec[c*x]),x]
```

output
```
Integrate[x/(a + b*ArcSec[c*x]), x]
```

3.33.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a + b \sec^{-1}(cx)}\,dx$$
$$\downarrow 5771$$
$$\int \frac{x}{a + b \sec^{-1}(cx)}dx$$

input
```
Int[x/(a + b*ArcSec[c*x]),x]
```

output
```
$Aborted
```

3.33.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.33.4 Maple [N/A] (verified)

Not integrable

Time = 0.71 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{a + b \operatorname{arcsec}(cx)}dx$$

input
```
int(x/(a+b*arcsec(c*x)),x)
```

output
```
int(x/(a+b*arcsec(c*x)),x)
```

3.33.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a+b\sec^{-1}(cx)}\,dx = \int \frac{x}{b\operatorname{arcsec}(cx)+a}\,dx$$

input
```
integrate(x/(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral(x/(b*arcsec(c*x) + a), x)
```

3.33.6 Sympy [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{a+b\sec^{-1}(cx)}\,dx = \int \frac{x}{a+b\operatorname{asec}(cx)}\,dx$$

input
```
integrate(x/(a+b*asec(c*x)),x)
```

output
```
Integral(x/(a + b*asec(c*x)), x)
```

3.33.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a+b\sec^{-1}(cx)}\,dx = \int \frac{x}{b\operatorname{arcsec}(cx)+a}\,dx$$

input
```
integrate(x/(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
integrate(x/(b*arcsec(c*x) + a), x)
```

3.33.8 Giac [N/A]

Not integrable

Time = 33.76 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a + b \sec^{-1}(cx)}\,dx = \int \frac{x}{b \operatorname{arcsec}(cx) + a}\,dx$$

input
```
integrate(x/(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate(x/(b*arcsec(c*x) + a), x)
```

3.33.9 Mupad [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{x}{a + b \sec^{-1}(cx)}\,dx = \int \frac{x}{a + b \operatorname{acos}\left(\frac{1}{c\,x}\right)}\,dx$$

input
```
int(x/(a + b*acos(1/(c*x))),x)
```

output
```
int(x/(a + b*acos(1/(c*x))), x)
```

3.34 $\int \frac{1}{a+b\sec^{-1}(cx)}\,dx$

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3.34.8 Giac [N/A] 298
3.34.9 Mupad [N/A] 298

3.34.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{a+b\sec^{-1}(cx)}\,dx = \text{Int}\left(\frac{1}{a+b\sec^{-1}(cx)},x\right)$$

output
```
Unintegrable(1/(a+b*arcsec(c*x)),x)
```

3.34.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\sec^{-1}(cx)}\,dx = \int \frac{1}{a+b\sec^{-1}(cx)}\,dx$$

input
```
Integrate[(a + b*ArcSec[c*x])^(-1),x]
```

output
```
Integrate[(a + b*ArcSec[c*x])^(-1), x]
```

3.34.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a + b \sec^{-1}(cx)}\,dx$$
$$\downarrow 5771$$
$$\int \frac{1}{a + b \sec^{-1}(cx)}dx$$

input
```
Int[(a + b*ArcSec[c*x])^(-1),x]
```

output
```
$Aborted
```

3.34.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrable[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.34.4 Maple [N/A] (verified)

Not integrable

Time = 0.41 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a + b \operatorname{arcsec}(cx)}dx$$

input
```
int(1/(a+b*arcsec(c*x)),x)
```

output
```
int(1/(a+b*arcsec(c*x)),x)
```

3.34.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a + b \sec^{-1}(cx)}\,dx = \int \frac{1}{b \operatorname{arcsec}(cx) + a}\,dx$$

input
```
integrate(1/(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*arcsec(c*x) + a), x)
```

3.34.6 Sympy [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a + b \sec^{-1}(cx)}\,dx = \int \frac{1}{a + b \operatorname{asec}(cx)}\,dx$$

input
```
integrate(1/(a+b*asec(c*x)),x)
```

output
```
Integral(1/(a + b*asec(c*x)), x)
```

3.34.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a + b \sec^{-1}(cx)}\,dx = \int \frac{1}{b \operatorname{arcsec}(cx) + a}\,dx$$

input
```
integrate(1/(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/(b*arcsec(c*x) + a), x)
```

3.34.8 Giac [N/A]

Not integrable

Time = 11.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a + b \sec^{-1}(cx)}\,dx = \int \frac{1}{b \operatorname{arcsec}(cx) + a}\,dx$$

input
```
integrate(1/(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate(1/(b*arcsec(c*x) + a), x)
```

3.34.9 Mupad [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{1}{a + b \sec^{-1}(cx)}\,dx = \int \frac{1}{a + b \operatorname{acos}\left(\frac{1}{c\,x}\right)}\,dx$$

input
```
int(1/(a + b*acos(1/(c*x))),x)
```

output
```
int(1/(a + b*acos(1/(c*x))), x)
```

3.35 $\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)}\,dx$

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3.35.7 Maxima [N/A] 301
3.35.8 Giac [N/A] 302
3.35.9 Mupad [N/A] 302

3.35.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)}\,dx = \text{Int}\left(\frac{1}{x\left(a+b\sec^{-1}(cx)\right)},x\right)$$

output
```
Unintegrable(1/x/(a+b*arcsec(c*x)),x)
```

3.35.2 Mathematica [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)}\,dx = \int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)}\,dx$$

input
```
Integrate[1/(x*(a + b*ArcSec[c*x])),x]
```

output
```
Integrate[1/(x*(a + b*ArcSec[c*x])), x]
```

3.35.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)}\,dx$$
$$\downarrow 5771$$
$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)}dx$$

input
```
Int[1/(x*(a + b*ArcSec[c*x])),x]
```

output
```
$Aborted
```

3.35.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.35.4 Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\ \text{arcsec}\left(cx\right)\right)}dx$$

input
```
int(1/x/(a+b*arcsec(c*x)),x)
```

output
```
int(1/x/(a+b*arcsec(c*x)),x)
```

3.35.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)}\,dx = \int \frac{1}{(b\operatorname{arcsec}(cx)+a)x}\,dx$$

input
```
integrate(1/x/(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*x*arcsec(c*x) + a*x), x)
```

3.35.6 Sympy [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)}\,dx = \int \frac{1}{x\left(a+b\operatorname{asec}(cx)\right)}\,dx$$

input
```
integrate(1/x/(a+b*asec(c*x)),x)
```

output
```
Integral(1/(x*(a + b*asec(c*x))), x)
```

3.35.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)}\,dx = \int \frac{1}{(b\operatorname{arcsec}(cx)+a)x}\,dx$$

input
```
integrate(1/x/(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((b*arcsec(c*x) + a)*x), x)
```

3.35.8 Giac [N/A]

Not integrable

Time = 1.69 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)}\,dx = \int \frac{1}{(b\operatorname{arcsec}(cx)+a)x}\,dx$$

input
```
integrate(1/x/(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((b*arcsec(c*x) + a)*x), x)
```

3.35.9 Mupad [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)}\,dx = \int \frac{1}{x\,\left(a+b\operatorname{acos}\left(\frac{1}{c\,x}\right)\right)}\,dx$$

input
```
int(1/(x*(a + b*acos(1/(c*x)))),x)
```

output
```
int(1/(x*(a + b*acos(1/(c*x)))), x)
```

3.36 $\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)}\,dx$

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3.36.1 Optimal result

Integrand size = 14, antiderivative size = 46

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)}\,dx = -\frac{c\,\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\sin\left(\frac{a}{b}\right)}{b} + \frac{c\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b}$$

output
```
c*cos(a/b)*Si(a/b+arcsec(c*x))/b-c*Ci(a/b+arcsec(c*x))*sin(a/b)/b
```

3.36.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)}\,dx = \frac{c\left(-\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\sin\left(\frac{a}{b}\right)+\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\right)}{b}$$

input
```
Integrate[1/(x^2*(a + b*ArcSec[c*x])),x]
```

output
```
(c*(-(CosIntegral[a/b + ArcSec[c*x]]*Sin[a/b]) + Cos[a/b]*SinIntegral[a/b
+ ArcSec[c*x]]))/b
```

3.36.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5745, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)}\,dx$$

$$\downarrow 5745$$

$$c\int \frac{\sqrt{1-\frac{1}{c^2x^2}}}{a+b\sec^{-1}(cx)}\,d\sec^{-1}(cx)$$

$$\downarrow 3042$$

$$c\int \frac{\sin\left(\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}\,d\sec^{-1}(cx)$$

$$\downarrow 3784$$

$$c\left(\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}\,d\sec^{-1}(cx) - \sin\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}\,d\sec^{-1}(cx)\right)$$

$$\downarrow 3042$$

$$c\left(\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}\,d\sec^{-1}(cx) - \sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a}{b}+\sec^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\sec^{-1}(cx)}\,d\sec^{-1}(cx)\right)$$

$$\downarrow 3780$$

$$c\left(\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b} - \sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a}{b}+\sec^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\sec^{-1}(cx)}\,d\sec^{-1}(cx)\right)$$

$$\downarrow 3783$$

$$c\left(\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b} - \frac{\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b}\right)$$

input
```
Int[1/(x^2*(a + b*ArcSec[c*x])),x]
```

output
```
c*(-((CosIntegral[a/b + ArcSec[c*x]]*Sin[a/b])/b) + (Cos[a/b]*SinIntegral[
a/b + ArcSec[c*x]])/b)
```

3.36.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

3.36.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$c\left(\frac{\mathrm{Si}\left(\frac{a}{b}+\operatorname{arcsec}(cx)\right)\cos\left(\frac{a}{b}\right)}{b}-\frac{\mathrm{Ci}\left(\frac{a}{b}+\operatorname{arcsec}(cx)\right)\sin\left(\frac{a}{b}\right)}{b}\right)$	47
default	$c\left(\frac{\mathrm{Si}\left(\frac{a}{b}+\operatorname{arcsec}(cx)\right)\cos\left(\frac{a}{b}\right)}{b}-\frac{\mathrm{Ci}\left(\frac{a}{b}+\operatorname{arcsec}(cx)\right)\sin\left(\frac{a}{b}\right)}{b}\right)$	47

input
```
int(1/x^2/(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
c*(Si(a/b+arcsec(c*x))*cos(a/b)/b-Ci(a/b+arcsec(c*x))*sin(a/b)/b)
```

3.36.5 Fricas [F]

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)}\,dx = \int \frac{1}{(b\operatorname{arcsec}(cx)+a)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*x^2*arcsec(c*x) + a*x^2), x)
```

3.36.6 Sympy [F]

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)}\,dx = \int \frac{1}{x^2\left(a+b\operatorname{asec}(cx)\right)}\,dx$$

input
```
integrate(1/x**2/(a+b*asec(c*x)),x)
```

output
```
Integral(1/(x**2*(a + b*asec(c*x))), x)
```

3.36.7 Maxima [F]

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)}\,dx = \int \frac{1}{(b\operatorname{arcsec}(cx)+a)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((b*arcsec(c*x) + a)*x^2), x)
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)}\,dx = -c\left(\frac{\operatorname{Ci}\left(\frac{a}{b}+\arccos\left(\frac{1}{cx}\right)\right)\sin\left(\frac{a}{b}\right)}{b} - \frac{\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos\left(\frac{1}{cx}\right)\right)}{b}\right)$$

input
```
integrate(1/x^2/(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
-c*(cos_integral(a/b + arccos(1/(c*x)))*sin(a/b)/b - cos(a/b)*sin_integral
(a/b + arccos(1/(c*x)))/b)
```

3.36.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)}\,dx = \int \frac{1}{x^2\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)}\,dx$$

input
```
int(1/(x^2*(a + b*acos(1/(c*x)))),x)
```

output
```
int(1/(x^2*(a + b*acos(1/(c*x)))), x)
```

3.37 $\int \frac{1}{x^3\left(a+b\sec^{-1}(cx)\right)}\,dx$

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3.37.1 Optimal result

Integrand size = 14, antiderivative size = 63

$$\int \frac{1}{x^3\left(a+b\sec^{-1}(cx)\right)}\,dx = -\frac{c^2\,\text{CosIntegral}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)\sin\left(\frac{2a}{b}\right)}{2b} + \frac{c^2\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{2b}$$

output
```
1/2*c^2*cos(2*a/b)*Si(2*a/b+2*arcsec(c*x))/b-1/2*c^2*Ci(2*a/b+2*arcsec(c*x
))*sin(2*a/b)/b
```

3.37.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^3\left(a+b\sec^{-1}(cx)\right)}\,dx = \frac{c^2\left(-\text{CosIntegral}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)\sin\left(\frac{2a}{b}\right)+\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)\right)}{2b}$$

input
```
Integrate[1/(x^3*(a + b*ArcSec[c*x])),x]
```

output
```
(c^2*(-(CosIntegral[(2*a)/b + 2*ArcSec[c*x]]*Sin[(2*a)/b]) + Cos[(2*a)/b]*
SinIntegral[(2*a)/b + 2*ArcSec[c*x]]))/(2*b)
```

3.37.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5745, 4906, 27, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(a+b\sec^{-1}(cx)\right)}\,dx$$

$$\downarrow 5745$$

$$c^2\int \frac{\sqrt{1-\frac{1}{c^2x^2}}}{cx\left(a+b\sec^{-1}(cx)\right)}\,d\sec^{-1}(cx)$$

$$\downarrow 4906$$

$$c^2\int \frac{\sin\left(2\sec^{-1}(cx)\right)}{2\left(a+b\sec^{-1}(cx)\right)}\,d\sec^{-1}(cx)$$

$$\downarrow 27$$

$$\frac{1}{2}c^2\int \frac{\sin\left(2\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}\,d\sec^{-1}(cx)$$

$$\downarrow 3042$$

$$\frac{1}{2}c^2\int \frac{\sin\left(2\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}\,d\sec^{-1}(cx)$$

$$\downarrow 3784$$

$$\frac{1}{2}c^2\left(\cos\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}\,d\sec^{-1}(cx)-\sin\left(\frac{2a}{b}\right)\int \frac{\cos\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}\,d\sec^{-1}(cx)\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}c^2\left(\cos\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}\,d\sec^{-1}(cx)-\sin\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2a}{b}+2\sec^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\sec^{-1}(cx)}\,d\sec^{-1}(cx)\right)$$

$$\downarrow 3780$$

$$\frac{1}{2}c^2\left(\frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b}-\sin\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2a}{b}+2\sec^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\sec^{-1}(cx)}\,d\sec^{-1}(cx)\right)$$

$$\downarrow 3783$$

$$\frac{1}{2}c^2\left(\frac{\cos\left(\frac{2a}{b}\right)\operatorname{Si}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b}-\frac{\sin\left(\frac{2a}{b}\right)\operatorname{CosIntegral}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b}\right)$$

input
```
Int[1/(x^3*(a + b*ArcSec[c*x])),x]
```

output
```
(c^2*(-((CosIntegral[(2*a)/b + 2*ArcSec[c*x]]*Sin[(2*a)/b])/b) + (Cos[(2*a
)/b]*SinIntegral[(2*a)/b + 2*ArcSec[c*x]])/b))/2
```

3.37.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

3.37.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$c^2\left(\frac{\mathrm{Si}\left(\frac{2a}{b}+2\,\mathrm{arcsec}(cx)\right)\cos\left(\frac{2a}{b}\right)}{2b}-\frac{\mathrm{Ci}\left(\frac{2a}{b}+2\,\mathrm{arcsec}(cx)\right)\sin\left(\frac{2a}{b}\right)}{2b}\right)$	58
default	$c^2\left(\frac{\mathrm{Si}\left(\frac{2a}{b}+2\,\mathrm{arcsec}(cx)\right)\cos\left(\frac{2a}{b}\right)}{2b}-\frac{\mathrm{Ci}\left(\frac{2a}{b}+2\,\mathrm{arcsec}(cx)\right)\sin\left(\frac{2a}{b}\right)}{2b}\right)$	58

input
```
int(1/x^3/(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
c^2*(1/2*Si(2*a/b+2*arcsec(c*x))*cos(2*a/b)/b-1/2*Ci(2*a/b+2*arcsec(c*x))*
sin(2*a/b)/b)
```

3.37.5 Fricas [F]

$$\int \frac{1}{x^3\left(a+b\,\mathrm{sec}^{-1}(cx)\right)}\,dx = \int \frac{1}{(b\,\mathrm{arcsec}\,(cx)+a)x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*x^3*arcsec(c*x) + a*x^3), x)
```

3.37.6 Sympy [F]

$$\int \frac{1}{x^3 \left(a + b \sec^{-1}(cx)\right)} \, dx = \int \frac{1}{x^3 \left(a + b \operatorname{asec}(cx)\right)} \, dx$$

input
```
integrate(1/x**3/(a+b*asec(c*x)),x)
```

output
```
Integral(1/(x**3*(a + b*asec(c*x))), x)
```

3.37.7 Maxima [F]

$$\int \frac{1}{x^3 \left(a + b \sec^{-1}(cx)\right)} \, dx = \int \frac{1}{(b \operatorname{arcsec}(cx) + a) x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((b*arcsec(c*x) + a)*x^3), x)
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.51

$$\int \frac{1}{x^3 \left(a + b \sec^{-1}(cx)\right)} \, dx =$$
$$-\frac{1}{2}\left(\frac{2\,c\cos\left(\frac{a}{b}\right)\operatorname{Ci}\left(\frac{2a}{b} + 2\arccos\left(\frac{1}{cx}\right)\right)\sin\left(\frac{a}{b}\right)}{b} - \frac{2\,c\cos\left(\frac{a}{b}\right)^2\operatorname{Si}\left(\frac{2a}{b} + 2\arccos\left(\frac{1}{cx}\right)\right)}{b} + \frac{c\operatorname{Si}\left(\frac{2a}{b} + 2\arc\right.}{b}$$

input
```
integrate(1/x^3/(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
-1/2*(2*c*cos(a/b)*cos_integral(2*a/b + 2*arccos(1/(c*x)))*sin(a/b)/b - 2*
c*cos(a/b)^2*sin_integral(2*a/b + 2*arccos(1/(c*x)))/b + c*sin_integral(2*
a/b + 2*arccos(1/(c*x)))/b)*c
```

3.37.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(a+b\sec^{-1}(cx)\right)}\,dx = \int \frac{1}{x^3\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)}\,dx$$

input
```
int(1/(x^3*(a + b*acos(1/(c*x)))),x)
```

output
```
int(1/(x^3*(a + b*acos(1/(c*x)))), x)
```

3.38 $\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)}\,dx$

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3.38.1 Optimal result

Integrand size = 14, antiderivative size = 117

$$\begin{aligned}\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)}\,dx = &-\frac{c^3\,\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\sin\left(\frac{a}{b}\right)}{4b}\\ &-\frac{c^3\,\text{CosIntegral}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)\sin\left(\frac{3a}{b}\right)}{4b}\\ &+\frac{c^3\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{4b}\\ &+\frac{c^3\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)}{4b}\end{aligned}$$

output
```
1/4*c^3*cos(a/b)*Si(a/b+arcsec(c*x))/b+1/4*c^3*cos(3*a/b)*Si(3*a/b+3*arcse
c(c*x))/b-1/4*c^3*Ci(a/b+arcsec(c*x))*sin(a/b)/b-1/4*c^3*Ci(3*a/b+3*arcsec
(c*x))*sin(3*a/b)/b
```

3.38.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.78

$$\begin{aligned}&\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)}\,dx\\ &=\frac{c^3\left(-\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\sin\left(\frac{a}{b}\right)-\text{CosIntegral}\left(3\left(\frac{a}{b}+\sec^{-1}(cx)\right)\right)\sin\left(\frac{3a}{b}\right)+\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec\right.\right.}{4b}\end{aligned}$$

input
```
Integrate[1/(x^4*(a + b*ArcSec[c*x])),x]
```

output
```
(c^3*(-(CosIntegral[a/b + ArcSec[c*x]]*Sin[a/b]) - CosIntegral[3*(a/b + Ar
cSec[c*x])]*Sin[(3*a)/b] + Cos[a/b]*SinIntegral[a/b + ArcSec[c*x]] + Cos[(
3*a)/b]*SinIntegral[3*(a/b + ArcSec[c*x])]))/(4*b)
```

3.38.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5745, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)}\,dx$$

$$\downarrow 5745$$

$$c^3\int \frac{\sqrt{1-\frac{1}{c^2x^2}}}{c^2x^2\left(a+b\sec^{-1}(cx)\right)}\,d\sec^{-1}(cx)$$

$$\downarrow 4906$$

$$c^3\int \left(\frac{\sin\left(3\sec^{-1}(cx)\right)}{4\left(a+b\sec^{-1}(cx)\right)}+\frac{\sqrt{1-\frac{1}{c^2x^2}}}{4\left(a+b\sec^{-1}(cx)\right)}\right)d\sec^{-1}(cx)$$

$$\downarrow 2009$$

$$c^3\left(-\frac{\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{4b}-\frac{\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)}{4b}+\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{4b}\right.$$

input
```
Int[1/(x^4*(a + b*ArcSec[c*x])),x]
```

output
```
c^3*(-1/4*(CosIntegral[a/b + ArcSec[c*x]]*Sin[a/b])/b - (CosIntegral[(3*a)
/b + 3*ArcSec[c*x]]*Sin[(3*a)/b])/(4*b) + (Cos[a/b]*SinIntegral[a/b + ArcS
ec[c*x]])/(4*b) + (Cos[(3*a)/b]*SinIntegral[(3*a)/b + 3*ArcSec[c*x]])/(4*b
))
```

3.38.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

3.38.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.87

method	result
derivativedivides	$c^3\left(\frac{\mathrm{Si}\left(\frac{3a}{b}+3\,\mathrm{arcsec}(cx)\right)\cos\left(\frac{3a}{b}\right)}{4b}-\frac{\mathrm{Ci}\left(\frac{3a}{b}+3\,\mathrm{arcsec}(cx)\right)\sin\left(\frac{3a}{b}\right)}{4b}+\frac{\mathrm{Si}\left(\frac{a}{b}+\mathrm{arcsec}(cx)\right)\cos\left(\frac{a}{b}\right)}{4b}-\frac{\mathrm{Ci}\left(\frac{a}{b}+\mathrm{arcsec}(c\right.}{4b}$
default	$c^3\left(\frac{\mathrm{Si}\left(\frac{3a}{b}+3\,\mathrm{arcsec}(cx)\right)\cos\left(\frac{3a}{b}\right)}{4b}-\frac{\mathrm{Ci}\left(\frac{3a}{b}+3\,\mathrm{arcsec}(cx)\right)\sin\left(\frac{3a}{b}\right)}{4b}+\frac{\mathrm{Si}\left(\frac{a}{b}+\mathrm{arcsec}(cx)\right)\cos\left(\frac{a}{b}\right)}{4b}-\frac{\mathrm{Ci}\left(\frac{a}{b}+\mathrm{arcsec}(c\right.}{4b}$

input
```
int(1/x^4/(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
c^3*(1/4*Si(3*a/b+3*arcsec(c*x))*cos(3*a/b)/b-1/4*Ci(3*a/b+3*arcsec(c*x))*
sin(3*a/b)/b+1/4*Si(a/b+arcsec(c*x))*cos(a/b)/b-1/4*Ci(a/b+arcsec(c*x))*si
n(a/b)/b)
```

3.38.5 Fricas [F]

$$\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)}\,dx = \int \frac{1}{(b\operatorname{arcsec}(cx)+a)x^4}\,dx$$

input
```
integrate(1/x^4/(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*x^4*arcsec(c*x) + a*x^4), x)
```

3.38.6 Sympy [F]

$$\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)}\,dx = \int \frac{1}{x^4\left(a+b\operatorname{asec}(cx)\right)}\,dx$$

input
```
integrate(1/x**4/(a+b*asec(c*x)),x)
```

output
```
Integral(1/(x**4*(a + b*asec(c*x))), x)
```

3.38.7 Maxima [F]

$$\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)}\,dx = \int \frac{1}{(b\operatorname{arcsec}(cx)+a)x^4}\,dx$$

input
```
integrate(1/x^4/(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((b*arcsec(c*x) + a)*x^4), x)
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.70

$$\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)}\,dx =$$
$$-\frac{1}{4}\left(\frac{4\,c^2\cos\left(\frac{a}{b}\right)^2\mathrm{Ci}\left(\frac{3\,a}{b}+3\arccos\left(\frac{1}{cx}\right)\right)\sin\left(\frac{a}{b}\right)}{b} - \frac{4\,c^2\cos\left(\frac{a}{b}\right)^3\mathrm{Si}\left(\frac{3\,a}{b}+3\arccos\left(\frac{1}{cx}\right)\right)}{b} - \frac{c^2\,\mathrm{Ci}\left(\frac{3\,a}{b}+3\right.}{}$$

input
```
integrate(1/x^4/(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
-1/4*(4*c^2*cos(a/b)^2*cos_integral(3*a/b + 3*arccos(1/(c*x)))*sin(a/b)/b
- 4*c^2*cos(a/b)^3*sin_integral(3*a/b + 3*arccos(1/(c*x)))/b - c^2*cos_int
egral(3*a/b + 3*arccos(1/(c*x)))*sin(a/b)/b + c^2*cos_integral(a/b + arcco
s(1/(c*x)))*sin(a/b)/b + 3*c^2*cos(a/b)*sin_integral(3*a/b + 3*arccos(1/(c
*x)))/b - c^2*cos(a/b)*sin_integral(a/b + arccos(1/(c*x)))/b)*c
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)}\,dx = \int \frac{1}{x^4\left(a+b\,\mathrm{acos}\left(\frac{1}{cx}\right)\right)}\,dx$$

input
```
int(1/(x^4*(a + b*acos(1/(c*x)))),x)
```

output
```
int(1/(x^4*(a + b*acos(1/(c*x)))), x)
```

3.39 $\int \frac{x}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx$

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3.39.9 Mupad [N/A] . . . 322

3.39.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{x}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \text{Int}\left(\frac{x}{\left(a+b\sec^{-1}(cx)\right)^2}, x\right)$$

output
```
Unintegrable(x/(a+b*arcsec(c*x))^2,x)
```

3.39.2 Mathematica [N/A]

Not integrable

Time = 11.91 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{x}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx$$

input
```
Integrate[x/(a + b*ArcSec[c*x])^2,x]
```

output
```
Integrate[x/(a + b*ArcSec[c*x])^2, x]
```

3.39.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a + b \sec^{-1}(cx)\right)^2} dx$$
$$\downarrow 5771$$
$$\int \frac{x}{\left(a + b \sec^{-1}(cx)\right)^2} dx$$

input
```
Int[x/(a + b*ArcSec[c*x])^2,x]
```

output
```
$Aborted
```

3.39.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.39.4 Maple [N/A] (verified)

Not integrable

Time = 0.56 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{\left(a + b \operatorname{arcsec}(cx)\right)^2} dx$$

input
```
int(x/(a+b*arcsec(c*x))^2,x)
```

output
```
int(x/(a+b*arcsec(c*x))^2,x)
```

3.39.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.33

$$\int \frac{x}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{x}{\left(b\,\text{arcsec}\,(cx)+a\right)^2}\,dx$$

input
```
integrate(x/(a+b*arcsec(c*x))^2,x, algorithm="fricas")
```

output
```
integral(x/(b^2*arcsec(c*x)^2 + 2*a*b*arcsec(c*x) + a^2), x)
```

3.39.6 Sympy [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{x}{\left(a+b\,\text{asec}\,(cx)\right)^2}\,dx$$

input
```
integrate(x/(a+b*asec(c*x))**2,x)
```

output
```
Integral(x/(a + b*asec(c*x))**2, x)
```

3.39.7 Maxima [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 583, normalized size of antiderivative = 48.58

$$\int \frac{x}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{x}{\left(b\,\text{arcsec}\,(cx)+a\right)^2}\,dx$$

input
```
integrate(x/(a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

output
```
-(4*(b*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + a*x^2)*sqrt(c*x + 1)*sqrt
(c*x - 1) - (4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*log(c^2*x^2
)^2 + 4*b^3*log(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*log(x)^2 + 8*a*b^2*arct
an(sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*log(c) + b^3*log(x))*lo
g(c^2*x^2))*integrate(-4*(3*a*c^2*x^3 - 2*a*x + (3*b*c^2*x^3 - 2*b*x)*arct
an(sqrt(c*x + 1)*sqrt(c*x - 1)))*sqrt(c*x + 1)*sqrt(c*x - 1)/(4*b^3*log(c)
^2 + 4*a^2*b - 4*(b^3*c^2*log(c)^2 + a^2*b*c^2)*x^2 - 4*(b^3*c^2*x^2 - b^3
)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - (b^3*c^2*x^2 - b^3)*log(c^2*x^2)
^2 - 4*(b^3*c^2*x^2 - b^3)*log(x)^2 - 8*(a*b^2*c^2*x^2 - a*b^2)*arctan(sqr
t(c*x + 1)*sqrt(c*x - 1)) + 4*(b^3*c^2*x^2*log(c) - b^3*log(c) + (b^3*c^2*
x^2 - b^3)*log(x))*log(c^2*x^2) - 8*(b^3*c^2*x^2*log(c) - b^3*log(c))*log(
x)), x))/(4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*log(c^2*x^2)^2
 + 4*b^3*log(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*log(x)^2 + 8*a*b^2*arctan(
sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*log(c) + b^3*log(x))*log(c
^2*x^2))
```

3.39.8 Giac [N/A]

Not integrable

Time = 105.84 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{x}{\left(b\operatorname{arcsec}(cx)+a\right)^2}\,dx$$

input
```
integrate(x/(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

output
```
integrate(x/(b*arcsec(c*x) + a)^2, x)
```

3.39.9 Mupad [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{x}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{x}{\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)^2}\,dx$$

input
```
int(x/(a + b*acos(1/(c*x)))^2,x)
```

output
```
int(x/(a + b*acos(1/(c*x)))^2, x)
```

3.40 $\int \frac{1}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx$

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3.40.7 Maxima [N/A] . . . 326
3.40.8 Giac [N/A] . . . 327
3.40.9 Mupad [N/A] . . . 327

3.40.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \text{Int}\left(\frac{1}{\left(a+b\sec^{-1}(cx)\right)^2}, x\right)$$

output
```
Unintegrable(1/(a+b*arcsec(c*x))^2,x)
```

3.40.2 Mathematica [N/A]

Not integrable

Time = 23.96 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx$$

input
```
Integrate[(a + b*ArcSec[c*x])^(-2),x]
```

output
```
Integrate[(a + b*ArcSec[c*x])^(-2), x]
```

3.40.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a + b \sec^{-1}(cx)\right)^2} dx$$
$$\downarrow 5771$$
$$\int \frac{1}{\left(a + b \sec^{-1}(cx)\right)^2} dx$$

input
```
Int[(a + b*ArcSec[c*x])^(-2),x]
```

output
```
$Aborted
```

3.40.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.40.4 Maple [N/A] (verified)

Not integrable

Time = 0.50 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a + b \operatorname{arcsec}(cx)\right)^2} dx$$

input
```
int(1/(a+b*arcsec(c*x))^2,x)
```

output
```
int(1/(a+b*arcsec(c*x))^2,x)
```

3.40.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.60

$$\int \frac{1}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\left(b\operatorname{arcsec}(cx)+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*arcsec(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*arcsec(c*x)^2 + 2*a*b*arcsec(c*x) + a^2), x)
```

3.40.6 Sympy [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\left(a+b\operatorname{asec}(cx)\right)^2}\,dx$$

input
```
integrate(1/(a+b*asec(c*x))**2,x)
```

output
```
Integral((a + b*asec(c*x))**(-2), x)
```

3.40.7 Maxima [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 577, normalized size of antiderivative = 57.70

$$\int \frac{1}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\left(b\operatorname{arcsec}(cx)+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

output
```
-(4*(b*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + a*x)*sqrt(c*x + 1)*sqrt(c*x
 - 1) - (4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*log(c^2*x^2)^2
+ 4*b^3*log(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*log(x)^2 + 8*a*b^2*arctan(s
qrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*log(c) + b^3*log(x))*log(c^
2*x^2))*integrate(-4*(2*a*c^2*x^2 + (2*b*c^2*x^2 - b)*arctan(sqrt(c*x + 1)
*sqrt(c*x - 1)) - a)*sqrt(c*x + 1)*sqrt(c*x - 1)/(4*b^3*log(c)^2 + 4*a^2*b
 - 4*(b^3*c^2*log(c)^2 + a^2*b*c^2)*x^2 - 4*(b^3*c^2*x^2 - b^3)*arctan(sqr
t(c*x + 1)*sqrt(c*x - 1))^2 - (b^3*c^2*x^2 - b^3)*log(c^2*x^2)^2 - 4*(b^3*
c^2*x^2 - b^3)*log(x)^2 - 8*(a*b^2*c^2*x^2 - a*b^2)*arctan(sqrt(c*x + 1)*s
qrt(c*x - 1)) + 4*(b^3*c^2*x^2*log(c) - b^3*log(c) + (b^3*c^2*x^2 - b^3)*l
og(x))*log(c^2*x^2) - 8*(b^3*c^2*x^2*log(c) - b^3*log(c))*log(x)), x))/(4*
b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*log(c^2*x^2)^2 + 4*b^3*log
(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*log(x)^2 + 8*a*b^2*arctan(sqrt(c*x + 1
)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*log(c) + b^3*log(x))*log(c^2*x^2))
```

3.40.8 Giac [N/A]

Not integrable

Time = 23.49 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\left(b\operatorname{arcsec}(cx)+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)^(-2), x)
```

3.40.9 Mupad [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{1}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)^2}\,dx$$

input
```
int(1/(a + b*acos(1/(c*x)))^2,x)
```

output
```
int(1/(a + b*acos(1/(c*x)))^2, x)
```

3.41 $\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2}\,dx$

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3.41.3 Rubi [N/A] . . . 329
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3.41.7 Maxima [N/A] . . . 330
3.41.8 Giac [N/A] . . . 331
3.41.9 Mupad [N/A] . . . 331

3.41.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \text{Int}\left(\frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2},x\right)$$

output
```
Unintegrable(1/x/(a+b*arcsec(c*x))^2,x)
```

3.41.2 Mathematica [N/A]

Not integrable

Time = 3.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2}\,dx$$

input
```
Integrate[1/(x*(a + b*ArcSec[c*x])^2),x]
```

output
```
Integrate[1/(x*(a + b*ArcSec[c*x])^2), x]
```

3.41.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2}\,dx$$
$$\downarrow 5771$$
$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2}dx$$

input
```
Int[1/(x*(a + b*ArcSec[c*x])^2),x]
```

output
```
$Aborted
```

3.41.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.41.4 Maple [N/A] (verified)

Not integrable

Time = 0.40 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\ \text{arcsec}\left(cx\right)\right)^2}dx$$

input
```
int(1/x/(a+b*arcsec(c*x))^2,x)
```

output
```
int(1/x/(a+b*arcsec(c*x))^2,x)
```

3.41.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.14

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\left(b\operatorname{arcsec}(cx)+a\right)^2 x}\,dx$$

input
```
integrate(1/x/(a+b*arcsec(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*x*arcsec(c*x)^2 + 2*a*b*x*arcsec(c*x) + a^2*x), x)
```

3.41.6 Sympy [N/A]

Not integrable

Time = 1.84 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{1}{x\left(a+b\operatorname{asec}(cx)\right)^2}\,dx$$

input
```
integrate(1/x/(a+b*asec(c*x))**2,x)
```

output
```
Integral(1/(x*(a + b*asec(c*x))**2), x)
```

3.41.7 Maxima [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 560, normalized size of antiderivative = 40.00

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\left(b\operatorname{arcsec}(cx)+a\right)^2 x}\,dx$$

input
```
integrate(1/x/(a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

output
```
-(4*sqrt(c*x + 1)*sqrt(c*x - 1)*(b*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + a
) - (4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*log(c^2*x^2)^2 + 4*
b^3*log(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*log(x)^2 + 8*a*b^2*arctan(sqrt(
c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*log(c) + b^3*log(x))*log(c^2*x^
2))*integrate(-4*(b*c^2*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + a*c^2*x)*s
qrt(c*x + 1)*sqrt(c*x - 1)/(4*b^3*log(c)^2 + 4*a^2*b - 4*(b^3*c^2*log(c)^2
 + a^2*b*c^2)*x^2 - 4*(b^3*c^2*x^2 - b^3)*arctan(sqrt(c*x + 1)*sqrt(c*x -
1))^2 - (b^3*c^2*x^2 - b^3)*log(c^2*x^2)^2 - 4*(b^3*c^2*x^2 - b^3)*log(x)^
2 - 8*(a*b^2*c^2*x^2 - a*b^2)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*(b^3
*c^2*x^2*log(c) - b^3*log(c) + (b^3*c^2*x^2 - b^3)*log(x))*log(c^2*x^2) -
8*(b^3*c^2*x^2*log(c) - b^3*log(c))*log(x)), x))/(4*b^3*arctan(sqrt(c*x +
1)*sqrt(c*x - 1))^2 + b^3*log(c^2*x^2)^2 + 4*b^3*log(c)^2 + 8*b^3*log(c)*l
og(x) + 4*b^3*log(x)^2 + 8*a*b^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a
^2*b - 4*(b^3*log(c) + b^3*log(x))*log(c^2*x^2))
```

3.41.8 Giac [N/A]

Not integrable

Time = 4.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\left(b\operatorname{arcsec}(cx)+a\right)^2 x}\,dx$$

input
```
integrate(1/x/(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*arcsec(c*x) + a)^2*x), x)
```

3.41.9 Mupad [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{1}{x\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)^2}\,dx$$

input
```
int(1/(x*(a + b*acos(1/(c*x)))^2),x)
```

output
```
int(1/(x*(a + b*acos(1/(c*x)))^2), x)
```

3.42 $\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)^2}\,dx$

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3.42.8 Giac [B] (verification not implemented) . . . 337
3.42.9 Mupad [F(-1)] . . . 337

3.42.1 Optimal result

Integrand size = 14, antiderivative size = 75

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)^2}\,dx = -\frac{c\sqrt{1-\frac{1}{c^2x^2}}}{b\left(a+b\sec^{-1}(cx)\right)} + \frac{c\cos\left(\frac{a}{b}\right)\operatorname{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b^2} + \frac{c\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b^2}$$

output
```
c*Ci(a/b+arcsec(c*x))*cos(a/b)/b^2+c*Si(a/b+arcsec(c*x))*sin(a/b)/b^2-c*(1
-1/c^2/x^2)^(1/2)/b/(a+b*arcsec(c*x))
```

3.42.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \frac{c\left(-\frac{b\sqrt{1-\frac{1}{c^2x^2}}}{a+b\sec^{-1}(cx)} + \cos\left(\frac{a}{b}\right)\operatorname{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right) + \sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\right)}{b^2}$$

input
```
Integrate[1/(x^2*(a + b*ArcSec[c*x])^2),x]
```

output
```
(c*(-((b*Sqrt[1 - 1/(c^2*x^2)])/(a + b*ArcSec[c*x])) + Cos[a/b]*CosIntegra
l[a/b + ArcSec[c*x]] + Sin[a/b]*SinIntegral[a/b + ArcSec[c*x]]))/b^2
```

3.42.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5745, 3042, 3778, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+b \sec^{-1}(cx)\right)^2} dx$$

$$\downarrow 5745$$

$$c \int \frac{\sqrt{1-\frac{1}{c^2 x^2}}}{\left(a+b \sec^{-1}(cx)\right)^2} d \sec^{-1}(cx)$$

$$\downarrow 3042$$

$$c \int \frac{\sin\left(\sec^{-1}(cx)\right)}{\left(a+b \sec^{-1}(cx)\right)^2} d \sec^{-1}(cx)$$

$$\downarrow 3778$$

$$c \left(\frac{\int \frac{1}{cx\left(a+b \sec^{-1}(cx)\right)} d \sec^{-1}(cx)}{b} - \frac{\sqrt{1-\frac{1}{c^2 x^2}}}{b\left(a+b \sec^{-1}(cx)\right)} \right)$$

$$\downarrow 3042$$

$$c \left(\frac{\int \frac{\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)}{a+b \sec^{-1}(cx)} d \sec^{-1}(cx)}{b} - \frac{\sqrt{1-\frac{1}{c^2 x^2}}}{b\left(a+b \sec^{-1}(cx)\right)} \right)$$

$$\downarrow 3784$$

$$c \left(\frac{\cos\left(\frac{a}{b}\right) \int \frac{\cos\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{a+b \sec^{-1}(cx)} d \sec^{-1}(cx) + \sin\left(\frac{a}{b}\right) \int \frac{\sin\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{a+b \sec^{-1}(cx)} d \sec^{-1}(cx)}{b} - \frac{\sqrt{1-\frac{1}{c^2 x^2}}}{b\left(a+b \sec^{-1}(cx)\right)} \right)$$

$$\downarrow 3042$$

$$c \left(\frac{\sin\left(\frac{a}{b}\right) \int \frac{\sin\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{a+b \sec^{-1}(cx)} d \sec^{-1}(cx) + \cos\left(\frac{a}{b}\right) \int \frac{\sin\left(\frac{a}{b}+\sec^{-1}(cx)+\frac{\pi}{2}\right)}{a+b \sec^{-1}(cx)} d \sec^{-1}(cx)}{b} - \frac{\sqrt{1-\frac{1}{c^2 x^2}}}{b\left(a+b \sec^{-1}(cx)\right)} \right)$$

$$\downarrow 3780$$

$$c\left(\frac{\cos\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a}{b}+\sec^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)+\frac{\sin\left(\frac{a}{b}\right)\mathrm{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b}}{b}-\frac{\sqrt{1-\frac{1}{c^2x^2}}}{b\left(a+b\sec^{-1}(cx)\right)}\right)$$

$$\downarrow 3783$$

$$c\left(\frac{\frac{\cos\left(\frac{a}{b}\right)\mathrm{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b}+\frac{\sin\left(\frac{a}{b}\right)\mathrm{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b}}{b}-\frac{\sqrt{1-\frac{1}{c^2x^2}}}{b\left(a+b\sec^{-1}(cx)\right)}\right)$$

input
```
Int[1/(x^2*(a + b*ArcSec[c*x])^2),x]
```

output
```
c*(-(Sqrt[1 - 1/(c^2*x^2)]/(b*(a + b*ArcSec[c*x]))) + ((Cos[a/b]*CosIntegr
al[a/b + ArcSec[c*x]])/b + (Sin[a/b]*SinIntegral[a/b + ArcSec[c*x]])/b)/b)
```

3.42.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

3.42.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$c\left(-\frac{\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{(a+b\,\mathrm{arcsec}(cx))b}+\frac{\mathrm{Si}\left(\frac{a}{b}+\mathrm{arcsec}(cx)\right)\sin\left(\frac{a}{b}\right)+\mathrm{Ci}\left(\frac{a}{b}+\mathrm{arcsec}(cx)\right)\cos\left(\frac{a}{b}\right)}{b^2}\right)$	78
default	$c\left(-\frac{\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{(a+b\,\mathrm{arcsec}(cx))b}+\frac{\mathrm{Si}\left(\frac{a}{b}+\mathrm{arcsec}(cx)\right)\sin\left(\frac{a}{b}\right)+\mathrm{Ci}\left(\frac{a}{b}+\mathrm{arcsec}(cx)\right)\cos\left(\frac{a}{b}\right)}{b^2}\right)$	78

input
```
int(1/x^2/(a+b*arcsec(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
c*(-((c^2*x^2-1)/c^2/x^2)^(1/2)/(a+b*arcsec(c*x))/b+(Si(a/b+arcsec(c*x))*s
in(a/b)+Ci(a/b+arcsec(c*x))*cos(a/b))/b^2)
```

3.42.5 Fricas [F]

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\left(b\,\mathrm{arcsec}\left(cx\right)+a\right)^2x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arcsec(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*x^2*arcsec(c*x)^2 + 2*a*b*x^2*arcsec(c*x) + a^2*x^2), x)
```

3.42.6 Sympy [F]

$$\int \frac{1}{x^2 \left(a + b \sec^{-1}(cx)\right)^2} dx = \int \frac{1}{x^2 \left(a + b \operatorname{asec}(cx)\right)^2} dx$$

input
```
integrate(1/x**2/(a+b*asec(c*x))**2,x)
```

output
```
Integral(1/(x**2*(a + b*asec(c*x))**2), x)
```

3.42.7 Maxima [F]

$$\int \frac{1}{x^2 \left(a + b \sec^{-1}(cx)\right)^2} dx = \int \frac{1}{\left(b \operatorname{arcsec}(cx) + a\right)^2 x^2} dx$$

input
```
integrate(1/x^2/(a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

output
```
-(4*sqrt(c*x + 1)*sqrt(c*x - 1)*(b*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + a
) - (4*b^3*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*x*log(c^2*x^2)^2
+ 8*b^3*x*log(c)*log(x) + 4*b^3*x*log(x)^2 + 8*a*b^2*x*arctan(sqrt(c*x + 1
)*sqrt(c*x - 1)) + 4*(b^3*log(c)^2 + a^2*b)*x - 4*(b^3*x*log(c) + b^3*x*lo
g(x))*log(c^2*x^2))*integrate(4*sqrt(c*x + 1)*sqrt(c*x - 1)*(b*arctan(sqrt
(c*x + 1)*sqrt(c*x - 1)) + a)/(4*(b^3*c^2*log(c)^2 + a^2*b*c^2)*x^4 - 4*(b
^3*log(c)^2 + a^2*b)*x^2 + 4*(b^3*c^2*x^4 - b^3*x^2)*arctan(sqrt(c*x + 1)*
sqrt(c*x - 1))^2 + (b^3*c^2*x^4 - b^3*x^2)*log(c^2*x^2)^2 + 4*(b^3*c^2*x^4
 - b^3*x^2)*log(x)^2 + 8*(a*b^2*c^2*x^4 - a*b^2*x^2)*arctan(sqrt(c*x + 1)*
sqrt(c*x - 1)) - 4*(b^3*c^2*x^4*log(c) - b^3*x^2*log(c) + (b^3*c^2*x^4 - b
^3*x^2)*log(x))*log(c^2*x^2) + 8*(b^3*c^2*x^4*log(c) - b^3*x^2*log(c))*log
(x)), x))/(4*b^3*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*x*log(c^2*x
^2)^2 + 8*b^3*x*log(c)*log(x) + 4*b^3*x*log(x)^2 + 8*a*b^2*x*arctan(sqrt(c
*x + 1)*sqrt(c*x - 1)) + 4*(b^3*log(c)^2 + a^2*b)*x - 4*(b^3*x*log(c) + b^
3*x*log(x))*log(c^2*x^2))
```

3.42.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(73) = 146.

Time = 0.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.01

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)^2}\,dx$$

$$= \left(\frac{b\arccos\left(\frac{1}{cx}\right)\cos\left(\frac{a}{b}\right)\mathrm{Ci}\left(\frac{a}{b}+\arccos\left(\frac{1}{cx}\right)\right)}{b^3\arccos\left(\frac{1}{cx}\right)+ab^2} + \frac{b\arccos\left(\frac{1}{cx}\right)\sin\left(\frac{a}{b}\right)\mathrm{Si}\left(\frac{a}{b}+\arccos\left(\frac{1}{cx}\right)\right)}{b^3\arccos\left(\frac{1}{cx}\right)+ab^2} + \frac{a\cos\left(\frac{a}{b}\right)\mathrm{Ci}\left(\frac{a}{b}\right.}{b^3\arccos\left(\right.}$$

input
```
integrate(1/x^2/(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

output
```
(b*arccos(1/(c*x))*cos(a/b)*cos_integral(a/b + arccos(1/(c*x)))/(b^3*arcco
s(1/(c*x)) + a*b^2) + b*arccos(1/(c*x))*sin(a/b)*sin_integral(a/b + arccos
(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) + a*cos(a/b)*cos_integral(a/b + a
rccos(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) + a*sin(a/b)*sin_integral(a/
b + arccos(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) - b*sqrt(-1/(c^2*x^2) +
 1)/(b^3*arccos(1/(c*x)) + a*b^2))*c
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{1}{x^2\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^2}\,dx$$

input
```
int(1/(x^2*(a + b*acos(1/(c*x)))^2),x)
```

output
```
int(1/(x^2*(a + b*acos(1/(c*x)))^2), x)
```

3.43 $\int \frac{1}{x^3\left(a+b\sec^{-1}(cx)\right)^2}\,dx$

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3.43.1 Optimal result

Integrand size = 14, antiderivative size = 84

$$\int \frac{1}{x^3\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \frac{c^2\cos\left(\frac{2a}{b}\right)\operatorname{CosIntegral}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b^2} - \frac{c^2\sin\left(2\sec^{-1}(cx)\right)}{2b\left(a+b\sec^{-1}(cx)\right)} + \frac{c^2\sin\left(\frac{2a}{b}\right)\operatorname{Si}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b^2}$$

output
```
c^2*Ci(2*a/b+2*arcsec(c*x))*cos(2*a/b)/b^2+c^2*Si(2*a/b+2*arcsec(c*x))*sin
(2*a/b)/b^2-1/2*c^2*sin(2*arcsec(c*x))/b/(a+b*arcsec(c*x))
```

3.43.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^3\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \frac{c\left(-\frac{b\sqrt{1-\frac{1}{c^2x^2}}}{ax+bx\sec^{-1}(cx)} + c\cos\left(\frac{2a}{b}\right)\operatorname{CosIntegral}\left(2\left(\frac{a}{b}+\sec^{-1}(cx)\right)\right) + c\sin\left(\frac{2a}{b}\right)\operatorname{Si}\left(2\left(\frac{a}{b}+\sec^{-1}(cx)\right)\right)\right)}{b^2}$$

input
```
Integrate[1/(x^3*(a + b*ArcSec[c*x])^2),x]
```

output
```
(c*(-((b*Sqrt[1 - 1/(c^2*x^2)])/(a*x + b*x*ArcSec[c*x])) + c*Cos[(2*a)/b]*
CosIntegral[2*(a/b + ArcSec[c*x])] + c*Sin[(2*a)/b]*SinIntegral[2*(a/b + A
rcSec[c*x])]))/b^2
```

3.43.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5745, 4906, 27, 3042, 3778, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+b \sec^{-1}(cx)\right)^2} dx$$

$$\downarrow 5745$$

$$c^2 \int \frac{\sqrt{1-\frac{1}{c^2x^2}}}{cx \left(a+b \sec^{-1}(cx)\right)^2} d\sec^{-1}(cx)$$

$$\downarrow 4906$$

$$c^2 \int \frac{\sin\left(2 \sec^{-1}(cx)\right)}{2 \left(a+b \sec^{-1}(cx)\right)^2} d\sec^{-1}(cx)$$

$$\downarrow 27$$

$$\frac{1}{2} c^2 \int \frac{\sin\left(2 \sec^{-1}(cx)\right)}{\left(a+b \sec^{-1}(cx)\right)^2} d\sec^{-1}(cx)$$

$$\downarrow 3042$$

$$\frac{1}{2} c^2 \int \frac{\sin\left(2 \sec^{-1}(cx)\right)}{\left(a+b \sec^{-1}(cx)\right)^2} d\sec^{-1}(cx)$$

$$\downarrow 3778$$

$$\frac{1}{2} c^2 \left(\frac{2 \int \frac{\cos\left(2 \sec^{-1}(cx)\right)}{a+b \sec^{-1}(cx)} d\sec^{-1}(cx)}{b} - \frac{\sin\left(2 \sec^{-1}(cx)\right)}{b \left(a+b \sec^{-1}(cx)\right)} \right)$$

$$\downarrow 3042$$

$$\frac{1}{2} c^2 \left(\frac{2 \int \frac{\sin\left(2 \sec^{-1}(cx)+\frac{\pi}{2}\right)}{a+b \sec^{-1}(cx)} d\sec^{-1}(cx)}{b} - \frac{\sin\left(2 \sec^{-1}(cx)\right)}{b \left(a+b \sec^{-1}(cx)\right)} \right)$$

$$\downarrow 3784$$

$$\frac{1}{2}c^2\left(\frac{2\left(\cos\left(\frac{2a}{b}\right)\int\frac{\cos\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)+\sin\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)\right)}{b}-\frac{\sin\left(2\sec^{-1}(cx)\right)}{b\left(a+b\sec^{-1}(cx)\right)}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}c^2\left(\frac{2\left(\sin\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)+\cos\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2a}{b}+2\sec^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)\right)}{b}-\frac{\sin\left(2\sec^{-1}(cx)\right)}{b\left(a+b\sec^{-1}(cx)\right)}\right)$$

$$\downarrow 3780$$

$$\frac{1}{2}c^2\left(\frac{2\left(\cos\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2a}{b}+2\sec^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)+\frac{\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b}\right)}{b}-\frac{\sin\left(2\sec^{-1}(cx)\right)}{b\left(a+b\sec^{-1}(cx)\right)}\right)$$

$$\downarrow 3783$$

$$\frac{1}{2}c^2\left(\frac{2\left(\frac{\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b}+\frac{\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b}\right)}{b}-\frac{\sin\left(2\sec^{-1}(cx)\right)}{b\left(a+b\sec^{-1}(cx)\right)}\right)$$

input
```
Int[1/(x^3*(a + b*ArcSec[c*x])^2),x]
```

output
```
(c^2*(-(Sin[2*ArcSec[c*x]]/(b*(a + b*ArcSec[c*x]))) + (2*((Cos[(2*a)/b]*Co
sIntegral[(2*a)/b + 2*ArcSec[c*x]])/b + (Sin[(2*a)/b]*SinIntegral[(2*a)/b
+ 2*ArcSec[c*x]])/b))/b))/2
```

3.43.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

3.43.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$c^2\left(-\frac{\sin(2\operatorname{arcsec}(cx))}{2(a+b\operatorname{arcsec}(cx))b}+\frac{\operatorname{Si}(\frac{2a}{b}+2\operatorname{arcsec}(cx))\sin(\frac{2a}{b})+\operatorname{Ci}(\frac{2a}{b}+2\operatorname{arcsec}(cx))\cos(\frac{2a}{b})}{b^2}\right)$	77
default	$c^2\left(-\frac{\sin(2\operatorname{arcsec}(cx))}{2(a+b\operatorname{arcsec}(cx))b}+\frac{\operatorname{Si}(\frac{2a}{b}+2\operatorname{arcsec}(cx))\sin(\frac{2a}{b})+\operatorname{Ci}(\frac{2a}{b}+2\operatorname{arcsec}(cx))\cos(\frac{2a}{b})}{b^2}\right)$	77

input
```
int(1/x^3/(a+b*arcsec(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
c^2*(-1/2*sin(2*arcsec(c*x))/(a+b*arcsec(c*x))/b+(Si(2*a/b+2*arcsec(c*x))*
sin(2*a/b)+Ci(2*a/b+2*arcsec(c*x))*cos(2*a/b))/b^2)
```

3.43.5 Fricas [F]

$$\int \frac{1}{x^3 \left(a + b \sec^{-1}(cx)\right)^2} \, dx = \int \frac{1}{\left(b \operatorname{arcsec}(cx) + a\right)^2 x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*arcsec(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*x^3*arcsec(c*x)^2 + 2*a*b*x^3*arcsec(c*x) + a^2*x^3), x)
```

3.43.6 Sympy [F]

$$\int \frac{1}{x^3 \left(a + b \sec^{-1}(cx)\right)^2} \, dx = \int \frac{1}{x^3 \left(a + b \operatorname{asec}(cx)\right)^2} \, dx$$

input
```
integrate(1/x**3/(a+b*asec(c*x))**2,x)
```

output
```
Integral(1/(x**3*(a + b*asec(c*x))**2), x)
```

3.43.7 Maxima [F]

$$\int \frac{1}{x^3 \left(a + b \sec^{-1}(cx)\right)^2} \, dx = \int \frac{1}{\left(b \operatorname{arcsec}(cx) + a\right)^2 x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

output
```
-(4*sqrt(c*x + 1)*sqrt(c*x - 1)*(b*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + a
) + (4*b^3*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*x^2*log(c^2*x^2
)^2 + 8*b^3*x^2*log(c)*log(x) + 4*b^3*x^2*log(x)^2 + 8*a*b^2*x^2*arctan(sq
rt(c*x + 1)*sqrt(c*x - 1)) + 4*(b^3*log(c)^2 + a^2*b)*x^2 - 4*(b^3*x^2*log
(c) + b^3*x^2*log(x))*log(c^2*x^2))*integrate(4*(a*c^2*x^2 + (b*c^2*x^2 -
2*b)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 2*a)*sqrt(c*x + 1)*sqrt(c*x - 1
)/(4*(b^3*c^2*log(c)^2 + a^2*b*c^2)*x^5 - 4*(b^3*log(c)^2 + a^2*b)*x^3 + 4
*(b^3*c^2*x^5 - b^3*x^3)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + (b^3*c^2*
x^5 - b^3*x^3)*log(c^2*x^2)^2 + 4*(b^3*c^2*x^5 - b^3*x^3)*log(x)^2 + 8*(a*
b^2*c^2*x^5 - a*b^2*x^3)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 4*(b^3*c^2*
x^5*log(c) - b^3*x^3*log(c) + (b^3*c^2*x^5 - b^3*x^3)*log(x))*log(c^2*x^2)
 + 8*(b^3*c^2*x^5*log(c) - b^3*x^3*log(c))*log(x)), x))/(4*b^3*x^2*arctan(
sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*x^2*log(c^2*x^2)^2 + 8*b^3*x^2*log(c)
*log(x) + 4*b^3*x^2*log(x)^2 + 8*a*b^2*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x -
 1)) + 4*(b^3*log(c)^2 + a^2*b)*x^2 - 4*(b^3*x^2*log(c) + b^3*x^2*log(x))*
log(c^2*x^2))
```

3.43.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(82) = 164$.

Time $= 0.27$ (sec) , antiderivative size $= 357$, normalized size of antiderivative $= 4.25$

$$\int \frac{1}{x^3\left(a+b\sec^{-1}(cx)\right)^2}\,dx$$

$$= \left(\frac{2\,bc\arccos\left(\frac{1}{cx}\right)\cos\left(\frac{a}{b}\right)^2 \mathrm{Ci}\left(\frac{2a}{b}+2\arccos\left(\frac{1}{cx}\right)\right)}{b^3\arccos\left(\frac{1}{cx}\right)+ab^2} + \frac{2\,bc\arccos\left(\frac{1}{cx}\right)\cos\left(\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)\mathrm{Si}\left(\frac{2a}{b}+2\arccos\left(\frac{1}{cx}\right)\right.}{b^3\arccos\left(\frac{1}{cx}\right)+ab^2}\right.$$

input
```
integrate(1/x^3/(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

output
```
(2*b*c*arccos(1/(c*x))*cos(a/b)^2*cos_integral(2*a/b + 2*arccos(1/(c*x)))/
(b^3*arccos(1/(c*x)) + a*b^2) + 2*b*c*arccos(1/(c*x))*cos(a/b)*sin(a/b)*si
n_integral(2*a/b + 2*arccos(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) + 2*a*
c*cos(a/b)^2*cos_integral(2*a/b + 2*arccos(1/(c*x)))/(b^3*arccos(1/(c*x))
+ a*b^2) + 2*a*c*cos(a/b)*sin(a/b)*sin_integral(2*a/b + 2*arccos(1/(c*x)))
/(b^3*arccos(1/(c*x)) + a*b^2) - b*c*arccos(1/(c*x))*cos_integral(2*a/b +
2*arccos(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) - a*c*cos_integral(2*a/b
+ 2*arccos(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) - b*sqrt(-1/(c^2*x^2) +
 1)/((b^3*arccos(1/(c*x)) + a*b^2)*x))*c
```

3.43.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{1}{x^3\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)^2}\,dx$$

input
```
int(1/(x^3*(a + b*acos(1/(c*x)))^2),x)
```

output
```
int(1/(x^3*(a + b*acos(1/(c*x)))^2), x)
```

3.44 $\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)^2}\,dx$

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3.44.8 Giac [B] (verification not implemented) . . . 349
3.44.9 Mupad [F(-1)] . . . 350

3.44.1 Optimal result

Integrand size = 14, antiderivative size = 178

$$\begin{aligned}\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)^2}\,dx = &-\frac{c^3\sqrt{1-\frac{1}{c^2x^2}}}{4b\left(a+b\sec^{-1}(cx)\right)}+\frac{c^3\cos\left(\frac{a}{b}\right)\operatorname{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{4b^2}\\&+\frac{3c^3\cos\left(\frac{3a}{b}\right)\operatorname{CosIntegral}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)}{4b^2}\\&-\frac{c^3\sin\left(3\sec^{-1}(cx)\right)}{4b\left(a+b\sec^{-1}(cx)\right)}+\frac{c^3\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{4b^2}\\&+\frac{3c^3\sin\left(\frac{3a}{b}\right)\operatorname{Si}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)}{4b^2}\end{aligned}$$

output

```
1/4*c^3*Ci(a/b+arcsec(c*x))*cos(a/b)/b^2+3/4*c^3*Ci(3*a/b+3*arcsec(c*x))*c
os(3*a/b)/b^2+1/4*c^3*Si(a/b+arcsec(c*x))*sin(a/b)/b^2+3/4*c^3*Si(3*a/b+3*
arcsec(c*x))*sin(3*a/b)/b^2-1/4*c^3*sin(3*arcsec(c*x))/b/(a+b*arcsec(c*x))
-1/4*c^3*(1-1/c^2/x^2)^(1/2)/b/(a+b*arcsec(c*x))
```

3.44.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)^2}\,dx$$
$$= \frac{-4bc\sqrt{1-\frac{1}{c^2x^2}}+c^3x^2(a+b\sec^{-1}(cx))\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)+3c^3x^2(a+b\sec^{-1}(cx))\cos$$

input
```
Integrate[1/(x^4*(a + b*ArcSec[c*x])^2),x]
```

output
```
(-4*b*c*Sqrt[1 - 1/(c^2*x^2)] + c^3*x^2*(a + b*ArcSec[c*x])*Cos[a/b]*CosIn
tegral[a/b + ArcSec[c*x]] + 3*c^3*x^2*(a + b*ArcSec[c*x])*Cos[(3*a)/b]*Cos
Integral[3*(a/b + ArcSec[c*x])] + a*c^3*x^2*Sin[a/b]*SinIntegral[a/b + Arc
Sec[c*x]] + b*c^3*x^2*ArcSec[c*x]*Sin[a/b]*SinIntegral[a/b + ArcSec[c*x]]
+ 3*a*c^3*x^2*Sin[(3*a)/b]*SinIntegral[3*(a/b + ArcSec[c*x])] + 3*b*c^3*x^
2*ArcSec[c*x]*Sin[(3*a)/b]*SinIntegral[3*(a/b + ArcSec[c*x])])/(4*b^2*x^2*
(a + b*ArcSec[c*x]))
```

3.44.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5745, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)^2}\,dx$$
$$\downarrow 5745$$
$$c^3\int \frac{\sqrt{1-\frac{1}{c^2x^2}}}{c^2x^2\left(a+b\sec^{-1}(cx)\right)^2}\,d\sec^{-1}(cx)$$
$$\downarrow 4906$$
$$c^3\int \left(\frac{\sin\left(3\sec^{-1}(cx)\right)}{4\left(a+b\sec^{-1}(cx)\right)^2}+\frac{\sqrt{1-\frac{1}{c^2x^2}}}{4\left(a+b\sec^{-1}(cx)\right)^2}\right)d\sec^{-1}(cx)$$

$$\downarrow 2009$$

$$c^3\left(\frac{\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{4b^2}+\frac{3\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)}{4b^2}+\frac{\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{4b^2}\right.$$

input
```
Int[1/(x^4*(a + b*ArcSec[c*x])^2),x]
```

output
```
c^3*(-1/4*Sqrt[1 - 1/(c^2*x^2)]/(b*(a + b*ArcSec[c*x])) + (Cos[a/b]*CosInt
egral[a/b + ArcSec[c*x]])/(4*b^2) + (3*Cos[(3*a)/b]*CosIntegral[(3*a)/b +
3*ArcSec[c*x]])/(4*b^2) - Sin[3*ArcSec[c*x]]/(4*b*(a + b*ArcSec[c*x])) + (
Sin[a/b]*SinIntegral[a/b + ArcSec[c*x]])/(4*b^2) + (3*Sin[(3*a)/b]*SinInte
gral[(3*a)/b + 3*ArcSec[c*x]])/(4*b^2))
```

3.44.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

3.44.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.86

method	result
derivativedivides	$c^3\left(-\frac{\sin(3\,\text{arcsec}(cx))}{4(a+b\,\text{arcsec}(cx))b}+\frac{\frac{3\,\text{Si}\left(\frac{3a}{b}+3\,\text{arcsec}(cx)\right)\sin\left(\frac{3a}{b}\right)}{4}+\frac{3\,\text{Ci}\left(\frac{3a}{b}+3\,\text{arcsec}(cx)\right)\cos\left(\frac{3a}{b}\right)}{4}}{b^2}-\frac{\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4(a+b\,\text{arcsec}(cx))b}$
default	$c^3\left(-\frac{\sin(3\,\text{arcsec}(cx))}{4(a+b\,\text{arcsec}(cx))b}+\frac{\frac{3\,\text{Si}\left(\frac{3a}{b}+3\,\text{arcsec}(cx)\right)\sin\left(\frac{3a}{b}\right)}{4}+\frac{3\,\text{Ci}\left(\frac{3a}{b}+3\,\text{arcsec}(cx)\right)\cos\left(\frac{3a}{b}\right)}{4}}{b^2}-\frac{\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4(a+b\,\text{arcsec}(cx))b}$

input
```
int(1/x^4/(a+b*arcsec(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
c^3*(-1/4*sin(3*arcsec(c*x))/(a+b*arcsec(c*x))/b+3/4*(Si(3*a/b+3*arcsec(c*
x))*sin(3*a/b)+Ci(3*a/b+3*arcsec(c*x))*cos(3*a/b))/b^2-1/4*((c^2*x^2-1)/c^
2/x^2)^(1/2)/(a+b*arcsec(c*x))/b+1/4*(Si(a/b+arcsec(c*x))*sin(a/b)+Ci(a/b+
arcsec(c*x))*cos(a/b))/b^2)
```

3.44.5 Fricas [F]

$$\int \frac{1}{x^4\left(a+b\,\text{sec}^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\left(b\,\text{arcsec}\left(cx\right)+a\right)^2x^4}\,dx$$

input
```
integrate(1/x^4/(a+b*arcsec(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*x^4*arcsec(c*x)^2 + 2*a*b*x^4*arcsec(c*x) + a^2*x^4), x)
```

3.44.6 Sympy [F]

$$\int \frac{1}{x^4\left(a+b\,\text{sec}^{-1}(cx)\right)^2}\,dx = \int \frac{1}{x^4\left(a+b\,\text{asec}\left(cx\right)\right)^2}\,dx$$

input
```
integrate(1/x**4/(a+b*asec(c*x))**2,x)
```

output
```
Integral(1/(x**4*(a + b*asec(c*x))**2), x)
```

3.44.7 Maxima [F]

$$\int \frac{1}{x^4 \left(a + b \sec^{-1}(cx)\right)^2} \, dx = \int \frac{1}{\left(b \operatorname{arcsec}(cx) + a\right)^2 x^4} \, dx$$

input
```
integrate(1/x^4/(a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

output
```
-(4*sqrt(c*x + 1)*sqrt(c*x - 1)*(b*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + a
) + (4*b^3*x^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*x^3*log(c^2*x^2
)^2 + 8*b^3*x^3*log(c)*log(x) + 4*b^3*x^3*log(x)^2 + 8*a*b^2*x^3*arctan(sq
rt(c*x + 1)*sqrt(c*x - 1)) + 4*(b^3*log(c)^2 + a^2*b)*x^3 - 4*(b^3*x^3*log
(c) + b^3*x^3*log(x))*log(c^2*x^2))*integrate(4*(2*a*c^2*x^2 + (2*b*c^2*x^
2 - 3*b)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 3*a)*sqrt(c*x + 1)*sqrt(c*x
 - 1)/(4*(b^3*c^2*log(c)^2 + a^2*b*c^2)*x^6 - 4*(b^3*log(c)^2 + a^2*b)*x^4
 + 4*(b^3*c^2*x^6 - b^3*x^4)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + (b^3*
c^2*x^6 - b^3*x^4)*log(c^2*x^2)^2 + 4*(b^3*c^2*x^6 - b^3*x^4)*log(x)^2 + 8
*(a*b^2*c^2*x^6 - a*b^2*x^4)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 4*(b^3*
c^2*x^6*log(c) - b^3*x^4*log(c) + (b^3*c^2*x^6 - b^3*x^4)*log(x))*log(c^2*
x^2) + 8*(b^3*c^2*x^6*log(c) - b^3*x^4*log(c))*log(x)), x))/(4*b^3*x^3*arc
tan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*x^3*log(c^2*x^2)^2 + 8*b^3*x^3*lo
g(c)*log(x) + 4*b^3*x^3*log(x)^2 + 8*a*b^2*x^3*arctan(sqrt(c*x + 1)*sqrt(c
*x - 1)) + 4*(b^3*log(c)^2 + a^2*b)*x^3 - 4*(b^3*x^3*log(c) + b^3*x^3*log(
x))*log(c^2*x^2))
```

3.44.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 694 vs. $2(164) = 328$.

Time $= 0.30$ (sec) , antiderivative size $= 694$, normalized size of antiderivative $= 3.90$

$$\int \frac{1}{x^4 \left(a + b \sec^{-1}(cx)\right)^2} \, dx$$

$$= \frac{1}{4} \left(\frac{12\, bc^2 \arccos\left(\frac{1}{cx}\right) \cos\left(\frac{a}{b}\right)^3 \operatorname{Ci}\left(\frac{3a}{b} + 3 \arccos\left(\frac{1}{cx}\right)\right)}{b^3 \arccos\left(\frac{1}{cx}\right) + ab^2} + \frac{12\, bc^2 \arccos\left(\frac{1}{cx}\right) \cos\left(\frac{a}{b}\right)^2 \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{3a}{b} + 3 \arc\right.}{b^3 \arccos\left(\frac{1}{cx}\right) + ab^2}$$

input
```
integrate(1/x^4/(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

output
```
1/4*(12*b*c^2*arccos(1/(c*x))*cos(a/b)^3*cos_integral(3*a/b + 3*arccos(1/(
c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) + 12*b*c^2*arccos(1/(c*x))*cos(a/b)^2
*sin(a/b)*sin_integral(3*a/b + 3*arccos(1/(c*x)))/(b^3*arccos(1/(c*x)) + a
*b^2) + 12*a*c^2*cos(a/b)^3*cos_integral(3*a/b + 3*arccos(1/(c*x)))/(b^3*a
rccos(1/(c*x)) + a*b^2) + 12*a*c^2*cos(a/b)^2*sin(a/b)*sin_integral(3*a/b
+ 3*arccos(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) - 9*b*c^2*arccos(1/(c*x
))*cos(a/b)*cos_integral(3*a/b + 3*arccos(1/(c*x)))/(b^3*arccos(1/(c*x)) +
 a*b^2) + b*c^2*arccos(1/(c*x))*cos(a/b)*cos_integral(a/b + arccos(1/(c*x)
))/(b^3*arccos(1/(c*x)) + a*b^2) - 3*b*c^2*arccos(1/(c*x))*sin(a/b)*sin_in
tegral(3*a/b + 3*arccos(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) + b*c^2*ar
ccos(1/(c*x))*sin(a/b)*sin_integral(a/b + arccos(1/(c*x)))/(b^3*arccos(1/(
c*x)) + a*b^2) - 9*a*c^2*cos(a/b)*cos_integral(3*a/b + 3*arccos(1/(c*x)))/
(b^3*arccos(1/(c*x)) + a*b^2) + a*c^2*cos(a/b)*cos_integral(a/b + arccos(1
/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) - 3*a*c^2*sin(a/b)*sin_integral(3*a
/b + 3*arccos(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) + a*c^2*sin(a/b)*sin
_integral(a/b + arccos(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) - 4*b*sqrt(
-1/(c^2*x^2) + 1)/((b^3*arccos(1/(c*x)) + a*b^2)*x^2))*c
```

3.44.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\left(a+b\,\mathrm{sec}^{-1}(cx)\right)^2}\,dx = \int \frac{1}{x^4\left(a+b\,\mathrm{acos}\left(\frac{1}{cx}\right)\right)^2}\,dx$$

input
```
int(1/(x^4*(a + b*acos(1/(c*x)))^2),x)
```

output
```
int(1/(x^4*(a + b*acos(1/(c*x)))^2), x)
```

3.45 $\int \frac{x}{\left(a+b\sec^{-1}(cx)\right)^3}\,dx$

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3.45.9 Mupad [N/A] . . . 355

3.45.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{x}{\left(a+b\sec^{-1}(cx)\right)^3}\,dx = \text{Int}\left(\frac{x}{\left(a+b\sec^{-1}(cx)\right)^3}, x\right)$$

output
```
Unintegrable(x/(a+b*arcsec(c*x))^3,x)
```

3.45.2 Mathematica [N/A]

Not integrable

Time = 2.74 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{\left(a+b\sec^{-1}(cx)\right)^3}\,dx = \int \frac{x}{\left(a+b\sec^{-1}(cx)\right)^3}\,dx$$

input
```
Integrate[x/(a + b*ArcSec[c*x])^3,x]
```

output
```
Integrate[x/(a + b*ArcSec[c*x])^3, x]
```

3.45.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+b\sec^{-1}(cx))^3}\,dx$$
$$\downarrow 5771$$
$$\int \frac{x}{(a+b\sec^{-1}(cx))^3}dx$$

input
```
Int[x/(a + b*ArcSec[c*x])^3,x]
```

output
```
$Aborted
```

3.45.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.45.4 Maple [N/A] (verified)

Not integrable

Time = 0.60 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a+b\,\text{arcsec}\,(cx))^3}dx$$

input
```
int(x/(a+b*arcsec(c*x))^3,x)
```

output
```
int(x/(a+b*arcsec(c*x))^3,x)
```

3.45.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.50

$$\int \frac{x}{(a+b\sec^{-1}(cx))^3}\,dx = \int \frac{x}{(b\operatorname{arcsec}(cx)+a)^3}\,dx$$

input
```
integrate(x/(a+b*arcsec(c*x))^3,x, algorithm="fricas")
```

output
```
integral(x/(b^3*arcsec(c*x)^3 + 3*a*b^2*arcsec(c*x)^2 + 3*a^2*b*arcsec(c*x
) + a^3), x)
```

3.45.6 Sympy [N/A]

Not integrable

Time = 1.94 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a+b\sec^{-1}(cx))^3}\,dx = \int \frac{x}{(a+b\operatorname{asec}(cx))^3}\,dx$$

input
```
integrate(x/(a+b*asec(c*x))**3,x)
```

output
```
Integral(x/(a + b*asec(c*x))**3, x)
```

3.45.7 Maxima [N/A]

Not integrable

Time = 29.31 (sec) , antiderivative size = 1790, normalized size of antiderivative = 149.17

$$\int \frac{x}{(a+b\sec^{-1}(cx))^3}\,dx = \int \frac{x}{(b\operatorname{arcsec}(cx)+a)^3}\,dx$$

input
```
integrate(x/(a+b*arcsec(c*x))^3,x, algorithm="maxima")
```

output
```
-(24*(a*b^2*c^2*log(c)^2 + a^3*c^2)*x^4 + 8*(3*b^3*c^2*x^4 - 2*b^3*x^2)*ar
ctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 16*(a*b^2*log(c)^2 + a^3)*x^2 + 24*(
3*a*b^2*c^2*x^4 - 2*a*b^2*x^2)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 2*(
3*a*b^2*c^2*x^4 - 2*a*b^2*x^2)*log(c^2*x^2)^2 + 8*(3*a*b^2*c^2*x^4 - 2*a*b
^2*x^2)*log(x)^2 + 2*(4*b^3*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - b^
3*x^2*log(c^2*x^2)^2 - 8*b^3*x^2*log(c)*log(x) - 4*b^3*x^2*log(x)^2 + 8*a*
b^2*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 4*(b^3*log(c)^2 - a^2*b)*x^2
 + 4*(b^3*x^2*log(c) + b^3*x^2*log(x))*log(c^2*x^2))*sqrt(c*x + 1)*sqrt(c*
x - 1) + 2*(12*(b^3*c^2*log(c)^2 + 3*a^2*b*c^2)*x^4 - 8*(b^3*log(c)^2 + 3*
a^2*b)*x^2 + (3*b^3*c^2*x^4 - 2*b^3*x^2)*log(c^2*x^2)^2 + 4*(3*b^3*c^2*x^4
 - 2*b^3*x^2)*log(x)^2 - 4*(3*b^3*c^2*x^4*log(c) - 2*b^3*x^2*log(c) + (3*b
^3*c^2*x^4 - 2*b^3*x^2)*log(x))*log(c^2*x^2) + 8*(3*b^3*c^2*x^4*log(c) - 2
*b^3*x^2*log(c))*log(x))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - (16*b^6*arc
tan(sqrt(c*x + 1)*sqrt(c*x - 1))^4 + b^6*log(c^2*x^2)^4 + 16*b^6*log(c)^4
+ 64*b^6*log(c)*log(x)^3 + 16*b^6*log(x)^4 + 64*a*b^5*arctan(sqrt(c*x + 1)
*sqrt(c*x - 1))^3 + 32*a^2*b^4*log(c)^2 + 16*a^4*b^2 - 8*(b^6*log(c) + b^6
*log(x))*log(c^2*x^2)^3 + 8*(b^6*log(c^2*x^2)^2 + 4*b^6*log(c)^2 + 8*b^6*l
og(c)*log(x) + 4*b^6*log(x)^2 + 12*a^2*b^4 - 4*(b^6*log(c) + b^6*log(x))*l
og(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 8*(3*b^6*log(c)^2 + 6
*b^6*log(c)*log(x) + 3*b^6*log(x)^2 + a^2*b^4)*log(c^2*x^2)^2 + 32*(3*b...
```

3.45.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\left(a + b \sec^{-1}(cx)\right)^3}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a+b*arcsec(c*x))^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Not invertible Error: Bad Argument Value
```

3.45.9 Mupad [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{x}{(a+b\sec^{-1}(cx))^3}\,dx = \int \frac{x}{\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)^3}\,dx$$

input
```
int(x/(a + b*acos(1/(c*x)))^3,x)
```

output
```
int(x/(a + b*acos(1/(c*x)))^3, x)
```

3.46 $\int \frac{1}{\left(a+b\sec^{-1}(cx)\right)^3}\,dx$

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3.46.3 Rubi [N/A] 357
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3.46.7 Maxima [N/A] 358
3.46.8 Giac [N/A] 359
3.46.9 Mupad [N/A] 360

3.46.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{\left(a+b\sec^{-1}(cx)\right)^3}\,dx = \text{Int}\left(\frac{1}{\left(a+b\sec^{-1}(cx)\right)^3}, x\right)$$

output
```
Unintegrable(1/(a+b*arcsec(c*x))^3,x)
```

3.46.2 Mathematica [N/A]

Not integrable

Time = 9.57 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(a+b\sec^{-1}(cx)\right)^3}\,dx = \int \frac{1}{\left(a+b\sec^{-1}(cx)\right)^3}\,dx$$

input
```
Integrate[(a + b*ArcSec[c*x])^(-3),x]
```

output
```
Integrate[(a + b*ArcSec[c*x])^(-3), x]
```

3.46.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec^{-1}(cx))^3}\,dx$$
$$\downarrow 5771$$
$$\int \frac{1}{(a+b\sec^{-1}(cx))^3}dx$$

input
```
Int[(a + b*ArcSec[c*x])^(-3),x]
```

output
```
$Aborted
```

3.46.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.46.4 Maple [N/A] (verified)

Not integrable

Time = 0.58 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+b\,\text{arcsec}\,(cx))^3}dx$$

input
```
int(1/(a+b*arcsec(c*x))^3,x)
```

output
```
int(1/(a+b*arcsec(c*x))^3,x)
```

3.46.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 4.00

$$\int \frac{1}{(a+b\sec^{-1}(cx))^3}\,dx = \int \frac{1}{(b\operatorname{arcsec}(cx)+a)^3}\,dx$$

input
```
integrate(1/(a+b*arcsec(c*x))^3,x, algorithm="fricas")
```

output
```
integral(1/(b^3*arcsec(c*x)^3 + 3*a*b^2*arcsec(c*x)^2 + 3*a^2*b*arcsec(c*x
) + a^3), x)
```

3.46.6 Sympy [N/A]

Not integrable

Time = 2.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{(a+b\sec^{-1}(cx))^3}\,dx = \int \frac{1}{(a+b\operatorname{asec}(cx))^3}\,dx$$

input
```
integrate(1/(a+b*asec(c*x))**3,x)
```

output
```
Integral((a + b*asec(c*x))**(-3), x)
```

3.46.7 Maxima [N/A]

Not integrable

Time = 28.50 (sec) , antiderivative size = 1744, normalized size of antiderivative = 174.40

$$\int \frac{1}{(a+b\sec^{-1}(cx))^3}\,dx = \int \frac{1}{(b\operatorname{arcsec}(cx)+a)^3}\,dx$$

input
```
integrate(1/(a+b*arcsec(c*x))^3,x, algorithm="maxima")
```

output
```
-(16*(a*b^2*c^2*log(c)^2 + a^3*c^2)*x^3 + 8*(2*b^3*c^2*x^3 - b^3*x)*arctan
(sqrt(c*x + 1)*sqrt(c*x - 1))^3 + 24*(2*a*b^2*c^2*x^3 - a*b^2*x)*arctan(sq
rt(c*x + 1)*sqrt(c*x - 1))^2 + 2*(2*a*b^2*c^2*x^3 - a*b^2*x)*log(c^2*x^2)^
2 + 8*(2*a*b^2*c^2*x^3 - a*b^2*x)*log(x)^2 + 2*(4*b^3*x*arctan(sqrt(c*x +
1)*sqrt(c*x - 1))^2 - b^3*x*log(c^2*x^2)^2 - 8*b^3*x*log(c)*log(x) - 4*b^3
*x*log(x)^2 + 8*a*b^2*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 4*(b^3*log(c
)^2 - a^2*b)*x + 4*(b^3*x*log(c) + b^3*x*log(x))*log(c^2*x^2))*sqrt(c*x +
1)*sqrt(c*x - 1) - 8*(a*b^2*log(c)^2 + a^3)*x + 2*(8*(b^3*c^2*log(c)^2 + 3
*a^2*b*c^2)*x^3 + (2*b^3*c^2*x^3 - b^3*x)*log(c^2*x^2)^2 + 4*(2*b^3*c^2*x^
3 - b^3*x)*log(x)^2 - 4*(b^3*log(c)^2 + 3*a^2*b)*x - 4*(2*b^3*c^2*x^3*log(
c) - b^3*x*log(c) + (2*b^3*c^2*x^3 - b^3*x)*log(x))*log(c^2*x^2) + 8*(2*b^
3*c^2*x^3*log(c) - b^3*x*log(c))*log(x))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1
)) - (16*b^6*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^4 + b^6*log(c^2*x^2)^4 +
16*b^6*log(c)^4 + 64*b^6*log(c)*log(x)^3 + 16*b^6*log(x)^4 + 64*a*b^5*arct
an(sqrt(c*x + 1)*sqrt(c*x - 1))^3 + 32*a^2*b^4*log(c)^2 + 16*a^4*b^2 - 8*(
b^6*log(c) + b^6*log(x))*log(c^2*x^2)^3 + 8*(b^6*log(c^2*x^2)^2 + 4*b^6*lo
g(c)^2 + 8*b^6*log(c)*log(x) + 4*b^6*log(x)^2 + 12*a^2*b^4 - 4*(b^6*log(c)
 + b^6*log(x))*log(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 8*(3*
b^6*log(c)^2 + 6*b^6*log(c)*log(x) + 3*b^6*log(x)^2 + a^2*b^4)*log(c^2*x^2
)^2 + 32*(3*b^6*log(c)^2 + a^2*b^4)*log(x)^2 + 16*(a*b^5*log(c^2*x^2)^2...
```

3.46.8 Giac [N/A]

Not integrable

Time = 52.48 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{(a+b\sec^{-1}(cx))^3}\,dx = \int \frac{1}{(b\operatorname{arcsec}(cx)+a)^3}\,dx$$

input
```
integrate(1/(a+b*arcsec(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)^(-3), x)
```

3.46.9 Mupad [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{1}{\left(a+b\sec^{-1}(cx)\right)^3}\,dx = \int \frac{1}{\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)^3}\,dx$$

input
```
int(1/(a + b*acos(1/(c*x)))^3,x)
```

output
```
int(1/(a + b*acos(1/(c*x)))^3, x)
```

3.47 $\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}\,dx$

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3.47.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}\,dx = \text{Int}\left(\frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}, x\right)$$

output
```
Unintegrable(1/x/(a+b*arcsec(c*x))^3,x)
```

3.47.2 Mathematica [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}\,dx = \int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}\,dx$$

input
```
Integrate[1/(x*(a + b*ArcSec[c*x])^3),x]
```

output
```
Integrate[1/(x*(a + b*ArcSec[c*x])^3), x]
```

3.47.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}\,dx$$
$$\downarrow 5771$$
$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}dx$$

input
```
Int[1/(x*(a + b*ArcSec[c*x])^3),x]
```

output
```
$Aborted
```

3.47.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.47.4 Maple [N/A] (verified)

Not integrable

Time = 0.51 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\ \text{arcsec}\left(cx\right)\right)^3}dx$$

input
```
int(1/x/(a+b*arcsec(c*x))^3,x)
```

output
```
int(1/x/(a+b*arcsec(c*x))^3,x)
```

3.47.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 3.21

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}\,dx = \int \frac{1}{\left(b\operatorname{arcsec}(cx)+a\right)^3 x}\,dx$$

input
```
integrate(1/x/(a+b*arcsec(c*x))^3,x, algorithm="fricas")
```

output
```
integral(1/(b^3*x*arcsec(c*x)^3 + 3*a*b^2*x*arcsec(c*x)^2 + 3*a^2*b*x*arcs
ec(c*x) + a^3*x), x)
```

3.47.6 Sympy [N/A]

Not integrable

Time = 3.58 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}\,dx = \int \frac{1}{x\left(a+b\operatorname{asec}(cx)\right)^3}\,dx$$

input
```
integrate(1/x/(a+b*asec(c*x))**3,x)
```

output
```
Integral(1/(x*(a + b*asec(c*x))**3), x)
```

3.47.7 Maxima [N/A]

Not integrable

Time = 21.87 (sec) , antiderivative size = 1568, normalized size of antiderivative = 112.00

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}\,dx = \int \frac{1}{\left(b\operatorname{arcsec}(cx)+a\right)^3 x}\,dx$$

input
```
integrate(1/x/(a+b*arcsec(c*x))^3,x, algorithm="maxima")
```

output
```
-(8*b^3*c^2*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 + 24*a*b^2*c^2*x^2*a
rctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 2*a*b^2*c^2*x^2*log(c^2*x^2)^2 + 16
*a*b^2*c^2*x^2*log(c)*log(x) + 8*a*b^2*c^2*x^2*log(x)^2 + 8*(a*b^2*c^2*log
(c)^2 + a^3*c^2)*x^2 + 2*(4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - b^
3*log(c^2*x^2)^2 - 4*b^3*log(c)^2 - 8*b^3*log(c)*log(x) - 4*b^3*log(x)^2 +
 8*a*b^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b + 4*(b^3*log(c) + b
^3*log(x))*log(c^2*x^2))*sqrt(c*x + 1)*sqrt(c*x - 1) + 2*(b^3*c^2*x^2*log(
c^2*x^2)^2 + 8*b^3*c^2*x^2*log(c)*log(x) + 4*b^3*c^2*x^2*log(x)^2 + 4*(b^3
*c^2*log(c)^2 + 3*a^2*b*c^2)*x^2 - 4*(b^3*c^2*x^2*log(c) + b^3*c^2*x^2*log
(x))*log(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - (16*b^6*arctan(sq
rt(c*x + 1)*sqrt(c*x - 1))^4 + b^6*log(c^2*x^2)^4 + 16*b^6*log(c)^4 + 64*b
^6*log(c)*log(x)^3 + 16*b^6*log(x)^4 + 64*a*b^5*arctan(sqrt(c*x + 1)*sqrt(
c*x - 1))^3 + 32*a^2*b^4*log(c)^2 + 16*a^4*b^2 - 8*(b^6*log(c) + b^6*log(x
))*log(c^2*x^2)^3 + 8*(b^6*log(c^2*x^2)^2 + 4*b^6*log(c)^2 + 8*b^6*log(c)*
log(x) + 4*b^6*log(x)^2 + 12*a^2*b^4 - 4*(b^6*log(c) + b^6*log(x))*log(c^2
*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 8*(3*b^6*log(c)^2 + 6*b^6*l
og(c)*log(x) + 3*b^6*log(x)^2 + a^2*b^4)*log(c^2*x^2)^2 + 32*(3*b^6*log(c)
^2 + a^2*b^4)*log(x)^2 + 16*(a*b^5*log(c^2*x^2)^2 + 4*a*b^5*log(c)^2 + 8*a
*b^5*log(c)*log(x) + 4*a*b^5*log(x)^2 + 4*a^3*b^3 - 4*(a*b^5*log(c) + a*b^
5*log(x))*log(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 32*(b^6*l...
```

3.47.8 Giac [N/A]

Not integrable

Time = 7.96 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}\,dx = \int \frac{1}{\left(b\operatorname{arcsec}(cx)+a\right)^3 x}\,dx$$

input
```
integrate(1/x/(a+b*arcsec(c*x))^3,x, algorithm="giac")
```

output
```
integrate(1/((b*arcsec(c*x) + a)^3*x), x)
```

3.47.9 Mupad [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}\,dx = \int \frac{1}{x\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)^3}\,dx$$

input
```
int(1/(x*(a + b*acos(1/(c*x)))^3),x)
```

output
```
int(1/(x*(a + b*acos(1/(c*x)))^3), x)
```

3.48 $\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)^3}\,dx$

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3.48.1 Optimal result

Integrand size = 14, antiderivative size = 103

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)^3}\,dx = -\frac{c\sqrt{1-\frac{1}{c^2x^2}}}{2b\left(a+b\sec^{-1}(cx)\right)^2} - \frac{1}{2b^2x\left(a+b\sec^{-1}(cx)\right)} + \frac{c\,\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\sin\left(\frac{a}{b}\right)}{2b^3} - \frac{c\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{2b^3}$$

output

```
-1/2/b^2/x/(a+b*arcsec(c*x))-1/2*c*cos(a/b)*Si(a/b+arcsec(c*x))/b^3+1/2*c*
Ci(a/b+arcsec(c*x))*sin(a/b)/b^3-1/2*c*(1-1/c^2/x^2)^(1/2)/b/(a+b*arcsec(c
*x))^2
```

3.48.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)^3}\,dx = -\frac{\frac{b\left(a+bc\sqrt{1-\frac{1}{c^2x^2}}x+b\sec^{-1}(cx)\right)}{x\left(a+b\sec^{-1}(cx)\right)^2} - c\,\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\sin\left(\frac{a}{b}\right) + c\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{2b^3}$$

input
```
Integrate[1/(x^2*(a + b*ArcSec[c*x])^3),x]
```

output
```
-1/2*((b*(a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x + b*ArcSec[c*x]))/(x*(a + b*ArcS
ec[c*x])^2) - c*CosIntegral[a/b + ArcSec[c*x]]*Sin[a/b] + c*Cos[a/b]*SinIn
tegral[a/b + ArcSec[c*x]])/b^3
```

3.48.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {5745, 3042, 3778, 3042, 3778, 25, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+b \sec^{-1}(cx)\right)^3} dx$$

$$\downarrow 5745$$

$$c \int \frac{\sqrt{1-\frac{1}{c^2 x^2}}}{\left(a+b \sec^{-1}(cx)\right)^3} d \sec^{-1}(cx)$$

$$\downarrow 3042$$

$$c \int \frac{\sin\left(\sec^{-1}(cx)\right)}{\left(a+b \sec^{-1}(cx)\right)^3} d \sec^{-1}(cx)$$

$$\downarrow 3778$$

$$c \left(\frac{\int \frac{1}{cx\left(a+b \sec^{-1}(cx)\right)^2} d \sec^{-1}(cx)}{2b} - \frac{\sqrt{1-\frac{1}{c^2 x^2}}}{2b \left(a+b \sec^{-1}(cx)\right)^2} \right)$$

$$\downarrow 3042$$

$$c \left(\frac{\int \frac{\sin\left(\sec^{-1}(cx)+\frac{\pi}{2}\right)}{\left(a+b \sec^{-1}(cx)\right)^2} d \sec^{-1}(cx)}{2b} - \frac{\sqrt{1-\frac{1}{c^2 x^2}}}{2b \left(a+b \sec^{-1}(cx)\right)^2} \right)$$

$$\downarrow 3778$$

$$c\left(\frac{\int -\frac{\sqrt{1-\frac{1}{c^2x^2}}}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)}{b}-\frac{1}{bcx(a+b\sec^{-1}(cx))}-\frac{\sqrt{1-\frac{1}{c^2x^2}}}{2b\left(a+b\sec^{-1}(cx)\right)^2}\right)$$

$$\downarrow 25$$

$$c\left(\frac{-\frac{\int \frac{\sqrt{1-\frac{1}{c^2x^2}}}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)}{b}-\frac{1}{bcx(a+b\sec^{-1}(cx))}}{2b}-\frac{\sqrt{1-\frac{1}{c^2x^2}}}{2b\left(a+b\sec^{-1}(cx)\right)^2}\right)$$

$$\downarrow 3042$$

$$c\left(\frac{-\frac{\int \frac{\sin\left(\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)}{b}-\frac{1}{bcx(a+b\sec^{-1}(cx))}}{2b}-\frac{\sqrt{1-\frac{1}{c^2x^2}}}{2b\left(a+b\sec^{-1}(cx)\right)^2}\right)$$

$$\downarrow 3784$$

$$c\left(\frac{-\frac{\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)-\sin\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)}{b}-\frac{1}{bcx(a+b\sec^{-1}(cx))}}{2b}-\frac{\sqrt{1-\frac{1}{c^2x^2}}}{2b\left(a+b\sec^{-1}(cx)\right)^2}\right)$$

$$\downarrow 3042$$

$$c\left(\frac{-\frac{\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)-\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a}{b}+\sec^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)}{b}-\frac{1}{bcx(a+b\sec^{-1}(cx))}}{2b}-\frac{\sqrt{1-\frac{1}{c^2x^2}}}{2b\left(a+b\sec^{-1}(cx)\right)^2}\right)$$

$$\downarrow 3780$$

$$c\left(\frac{-\frac{\cos\left(\frac{a}{b}\right)\mathrm{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b}-\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a}{b}+\sec^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)}{b}-\frac{1}{bcx(a+b\sec^{-1}(cx))}}{2b}-\frac{\sqrt{1-\frac{1}{c^2x^2}}}{2b\left(a+b\sec^{-1}(cx)\right)^2}\right)$$

$$\downarrow 3783$$

$$c\left(\frac{-\frac{\cos\left(\frac{a}{b}\right)\mathrm{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b}-\frac{\sin\left(\frac{a}{b}\right)\mathrm{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b}}{2b}-\frac{1}{bcx(a+b\sec^{-1}(cx))}-\frac{\sqrt{1-\frac{1}{c^2x^2}}}{2b\left(a+b\sec^{-1}(cx)\right)^2}\right)$$

input
```
Int[1/(x^2*(a + b*ArcSec[c*x])^3),x]
```

output
```
c*(-1/2*Sqrt[1 - 1/(c^2*x^2)]/(b*(a + b*ArcSec[c*x])^2) + (-(1/(b*c*x*(a +
 b*ArcSec[c*x]))) - (-((CosIntegral[a/b + ArcSec[c*x]]*Sin[a/b])/b) + (Cos
[a/b]*SinIntegral[a/b + ArcSec[c*x]])/b)/b)/(2*b))
```

3.48.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

3.48.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.50

method	result
derivativedivides	$c\left(-\frac{\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{2(a+b\,\mathrm{arcsec}(cx))^2b}-\frac{\mathrm{arcsec}(cx)\cos\left(\frac{a}{b}\right)\mathrm{Si}\left(\frac{a}{b}+\mathrm{arcsec}(cx)\right)bcx-\mathrm{arcsec}(cx)\sin\left(\frac{a}{b}\right)\mathrm{Ci}\left(\frac{a}{b}+\mathrm{arcsec}(cx)\right)bcx+\cos($
default	$c\left(-\frac{\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{2(a+b\,\mathrm{arcsec}(cx))^2b}-\frac{\mathrm{arcsec}(cx)\cos\left(\frac{a}{b}\right)\mathrm{Si}\left(\frac{a}{b}+\mathrm{arcsec}(cx)\right)bcx-\mathrm{arcsec}(cx)\sin\left(\frac{a}{b}\right)\mathrm{Ci}\left(\frac{a}{b}+\mathrm{arcsec}(cx)\right)bcx+\cos($

input
```
int(1/x^2/(a+b*arcsec(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
c*(-1/2*((c^2*x^2-1)/c^2/x^2)^(1/2)/(a+b*arcsec(c*x))^2/b-1/2*(arcsec(c*x)
*cos(a/b)*Si(a/b+arcsec(c*x))*b*c*x-arcsec(c*x)*sin(a/b)*Ci(a/b+arcsec(c*x
))*b*c*x+cos(a/b)*Si(a/b+arcsec(c*x))*a*c*x-sin(a/b)*Ci(a/b+arcsec(c*x))*a
*c*x+b)/c/x/(a+b*arcsec(c*x))/b^3)
```

3.48.5 Fricas [F]

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)^3}\,dx = \int \frac{1}{\left(b\,\mathrm{arcsec}\left(cx\right)+a\right)^3x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arcsec(c*x))^3,x, algorithm="fricas")
```

output
```
integral(1/(b^3*x^2*arcsec(c*x)^3 + 3*a*b^2*x^2*arcsec(c*x)^2 + 3*a^2*b*x^
2*arcsec(c*x) + a^3*x^2), x)
```

3.48.6 Sympy [F]

$$\int \frac{1}{x^2 \left(a + b \sec^{-1}(cx)\right)^3} \, dx = \int \frac{1}{x^2 \left(a + b \operatorname{asec}(cx)\right)^3} \, dx$$

input
```
integrate(1/x**2/(a+b*asec(c*x))**3,x)
```

output
```
Integral(1/(x**2*(a + b*asec(c*x))**3), x)
```

3.48.7 Maxima [F]

$$\int \frac{1}{x^2 \left(a + b \sec^{-1}(cx)\right)^3} \, dx = \int \frac{1}{\left(b \operatorname{arcsec}(cx) + a\right)^3 x^2} \, dx$$

input
```
integrate(1/x^2/(a+b*arcsec(c*x))^3,x, algorithm="maxima")
```

output
```
-(8*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 + 24*a*b^2*arctan(sqrt(c*x +
 1)*sqrt(c*x - 1))^2 + 2*a*b^2*log(c^2*x^2)^2 + 8*a*b^2*log(c)^2 + 16*a*b^
2*log(c)*log(x) + 8*a*b^2*log(x)^2 + 8*a^3 + 2*(4*b^3*arctan(sqrt(c*x + 1)
*sqrt(c*x - 1))^2 - b^3*log(c^2*x^2)^2 - 4*b^3*log(c)^2 - 8*b^3*log(c)*log
(x) - 4*b^3*log(x)^2 + 8*a*b^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2
*b + 4*(b^3*log(c) + b^3*log(x))*log(c^2*x^2))*sqrt(c*x + 1)*sqrt(c*x - 1)
 + 2*(b^3*log(c^2*x^2)^2 + 4*b^3*log(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*lo
g(x)^2 + 12*a^2*b - 4*(b^3*log(c) + b^3*log(x))*log(c^2*x^2))*arctan(sqrt(
c*x + 1)*sqrt(c*x - 1)) + (16*b^6*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^4
+ b^6*x*log(c^2*x^2)^4 + 64*b^6*x*log(c)*log(x)^3 + 16*b^6*x*log(x)^4 + 64
*a*b^5*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 8*(b^6*x*log(c) + b^6*x*l
og(x))*log(c^2*x^2)^3 + 32*(3*b^6*log(c)^2 + a^2*b^4)*x*log(x)^2 + 8*(b^6*
x*log(c^2*x^2)^2 + 8*b^6*x*log(c)*log(x) + 4*b^6*x*log(x)^2 + 4*(b^6*log(c
)^2 + 3*a^2*b^4)*x - 4*(b^6*x*log(c) + b^6*x*log(x))*log(c^2*x^2))*arctan(
sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 8*(6*b^6*x*log(c)*log(x) + 3*b^6*x*log(x)
^2 + (3*b^6*log(c)^2 + a^2*b^4)*x)*log(c^2*x^2)^2 + 64*(b^6*log(c)^3 + a^2
*b^4*log(c))*x*log(x) + 16*(b^6*log(c)^4 + 2*a^2*b^4*log(c)^2 + a^4*b^2)*x
 + 16*(a*b^5*x*log(c^2*x^2)^2 + 8*a*b^5*x*log(c)*log(x) + 4*a*b^5*x*log(x)
^2 + 4*(a*b^5*log(c)^2 + a^3*b^3)*x - 4*(a*b^5*x*log(c) + a*b^5*x*log(x))*
log(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 32*(3*b^6*x*log(c)*...
```

3.48.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(93) = 186.

Time = 0.31 (sec) , antiderivative size = 580, normalized size of antiderivative = 5.63

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)^3}\,dx$$

$$=\frac{1}{2}\left(\frac{b^2\arccos\left(\frac{1}{cx}\right)^2\operatorname{Ci}\left(\frac{a}{b}+\arccos\left(\frac{1}{cx}\right)\right)\sin\left(\frac{a}{b}\right)}{b^5\arccos\left(\frac{1}{cx}\right)^2+2\,ab^4\arccos\left(\frac{1}{cx}\right)+a^2b^3}-\frac{b^2\arccos\left(\frac{1}{cx}\right)^2\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos\left(\frac{1}{cx}\right)\right)}{b^5\arccos\left(\frac{1}{cx}\right)^2+2\,ab^4\arccos\left(\frac{1}{cx}\right)+a^2b^3}+\frac{2\,ab\arccos}{b^5\arccos}\right.$$

input
```
integrate(1/x^2/(a+b*arcsec(c*x))^3,x, algorithm="giac")
```

output
```
1/2*(b^2*arccos(1/(c*x))^2*cos_integral(a/b + arccos(1/(c*x)))*sin(a/b)/(b
^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) - b^2*arccos(1/(
c*x))^2*cos(a/b)*sin_integral(a/b + arccos(1/(c*x)))/(b^5*arccos(1/(c*x))^
2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) + 2*a*b*arccos(1/(c*x))*cos_integra
l(a/b + arccos(1/(c*x)))*sin(a/b)/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(
1/(c*x)) + a^2*b^3) - 2*a*b*arccos(1/(c*x))*cos(a/b)*sin_integral(a/b + ar
ccos(1/(c*x)))/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3)
 + a^2*cos_integral(a/b + arccos(1/(c*x)))*sin(a/b)/(b^5*arccos(1/(c*x))^2
 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) - a^2*cos(a/b)*sin_integral(a/b + ar
ccos(1/(c*x)))/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3)
 - b^2*sqrt(-1/(c^2*x^2) + 1)/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c
*x)) + a^2*b^3) - b^2*arccos(1/(c*x))/((b^5*arccos(1/(c*x))^2 + 2*a*b^4*ar
ccos(1/(c*x)) + a^2*b^3)*c*x) - a*b/((b^5*arccos(1/(c*x))^2 + 2*a*b^4*arcc
os(1/(c*x)) + a^2*b^3)*c*x))*c
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)^3}\,dx=\int \frac{1}{x^2\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)^3}\,dx$$

input
```
int(1/(x^2*(a + b*acos(1/(c*x)))^3),x)
```

output
```
int(1/(x^2*(a + b*acos(1/(c*x)))^3), x)
```

3.49 $\int \frac{1}{x^3\left(a+b\sec^{-1}(cx)\right)^3}\,dx$

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3.49.1 Optimal result

Integrand size = 14, antiderivative size = 112

$$\begin{aligned}\int \frac{1}{x^3\left(a+b\sec^{-1}(cx)\right)^3}\,dx &= -\frac{c^2\cos\left(2\sec^{-1}(cx)\right)}{2b^2\left(a+b\sec^{-1}(cx)\right)}\\ &\quad+\frac{c^2\,\text{CosIntegral}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)\sin\left(\frac{2a}{b}\right)}{b^3}\\ &\quad-\frac{c^2\sin\left(2\sec^{-1}(cx)\right)}{4b\left(a+b\sec^{-1}(cx)\right)^2}-\frac{c^2\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b^3}\end{aligned}$$

output

```
-1/2*c^2*cos(2*arcsec(c*x))/b^2/(a+b*arcsec(c*x))-c^2*cos(2*a/b)*Si(2*a/b+
2*arcsec(c*x))/b^3+c^2*Ci(2*a/b+2*arcsec(c*x))*sin(2*a/b)/b^3-1/4*c^2*sin(
2*arcsec(c*x))/b/(a+b*arcsec(c*x))^2
```

3.49.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.02

$$\begin{aligned}&\int \frac{1}{x^3\left(a+b\sec^{-1}(cx)\right)^3}\,dx\\ &=\frac{-\frac{b^2c\sqrt{1-\frac{1}{c^2x^2}}}{x\left(a+b\sec^{-1}(cx)\right)^2}+\frac{b\left(-2+c^2x^2\right)}{x^2\left(a+b\sec^{-1}(cx)\right)}+2c^2\left(\text{CosIntegral}\left(2\left(\frac{a}{b}+\sec^{-1}(cx)\right)\right)\sin\left(\frac{2a}{b}\right)-\cos\left(\frac{2a}{b}\right)\text{Si}\left(2\left(\frac{a}{b}+\sec^{-}\right.\right.\right.}{2b^3}\end{aligned}$$

input
```
Integrate[1/(x^3*(a + b*ArcSec[c*x])^3),x]
```

output
```
(-((b^2*c*Sqrt[1 - 1/(c^2*x^2)])/(x*(a + b*ArcSec[c*x])^2)) + (b*(-2 + c^2
*x^2))/(x^2*(a + b*ArcSec[c*x])) + 2*c^2*(CosIntegral[2*(a/b + ArcSec[c*x]
)]*Sin[(2*a)/b] - Cos[(2*a)/b]*SinIntegral[2*(a/b + ArcSec[c*x])]))/(2*b^3
)
```

3.49.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {5745, 4906, 27, 3042, 3778, 3042, 3778, 25, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+b \sec^{-1}(cx)\right)^3} dx$$

$$\downarrow 5745$$

$$c^2 \int \frac{\sqrt{1-\frac{1}{c^2x^2}}}{cx \left(a+b \sec^{-1}(cx)\right)^3} d\sec^{-1}(cx)$$

$$\downarrow 4906$$

$$c^2 \int \frac{\sin\left(2 \sec^{-1}(cx)\right)}{2 \left(a+b \sec^{-1}(cx)\right)^3} d\sec^{-1}(cx)$$

$$\downarrow 27$$

$$\frac{1}{2} c^2 \int \frac{\sin\left(2 \sec^{-1}(cx)\right)}{\left(a+b \sec^{-1}(cx)\right)^3} d\sec^{-1}(cx)$$

$$\downarrow 3042$$

$$\frac{1}{2} c^2 \int \frac{\sin\left(2 \sec^{-1}(cx)\right)}{\left(a+b \sec^{-1}(cx)\right)^3} d\sec^{-1}(cx)$$

$$\downarrow 3778$$

$$\frac{1}{2} c^2 \left(\frac{\int \frac{\cos\left(2 \sec^{-1}(cx)\right)}{\left(a+b \sec^{-1}(cx)\right)^2} d\sec^{-1}(cx)}{b} - \frac{\sin\left(2 \sec^{-1}(cx)\right)}{2b \left(a+b \sec^{-1}(cx)\right)^2} \right)$$

$$\downarrow 3042$$

$$\frac{1}{2}c^2\left(\frac{\int \frac{\sin\left(2\sec^{-1}(cx)+\frac{\pi}{2}\right)}{\left(a+b\sec^{-1}(cx)\right)^2}d\sec^{-1}(cx)}{b}-\frac{\sin\left(2\sec^{-1}(cx)\right)}{2b\left(a+b\sec^{-1}(cx)\right)^2}\right)$$

$$\downarrow 3778$$

$$\frac{1}{2}c^2\left(\frac{\frac{2\int -\frac{\sin\left(2\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)}{b}-\frac{\cos\left(2\sec^{-1}(cx)\right)}{b\left(a+b\sec^{-1}(cx)\right)}}{b}-\frac{\sin\left(2\sec^{-1}(cx)\right)}{2b\left(a+b\sec^{-1}(cx)\right)^2}\right)$$

$$\downarrow 25$$

$$\frac{1}{2}c^2\left(\frac{-\frac{2\int \frac{\sin\left(2\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)}{b}-\frac{\cos\left(2\sec^{-1}(cx)\right)}{b\left(a+b\sec^{-1}(cx)\right)}}{b}-\frac{\sin\left(2\sec^{-1}(cx)\right)}{2b\left(a+b\sec^{-1}(cx)\right)^2}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}c^2\left(\frac{-\frac{2\int \frac{\sin\left(2\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)}{b}-\frac{\cos\left(2\sec^{-1}(cx)\right)}{b\left(a+b\sec^{-1}(cx)\right)}}{b}-\frac{\sin\left(2\sec^{-1}(cx)\right)}{2b\left(a+b\sec^{-1}(cx)\right)^2}\right)$$

$$\downarrow 3784$$

$$\frac{1}{2}c^2\left(\frac{-\frac{2\left(\cos\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)-\sin\left(\frac{2a}{b}\right)\int \frac{\cos\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)\right)}{b}-\frac{\cos\left(2\sec^{-1}(cx)\right)}{b\left(a+b\sec^{-1}(cx)\right)}}{b}-\frac{\sin\left(2\sec^{-1}(\right.}{2b\left(a+b\sec^{-1}\right.}\right.$$

$$\downarrow 3042$$

$$\frac{1}{2}c^2\left(\frac{-\frac{2\left(\cos\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)-\sin\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2a}{b}+2\sec^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)\right)}{b}-\frac{\cos\left(2\sec^{-1}(cx)\right)}{b\left(a+b\sec^{-1}(cx)\right)}}{b}-\frac{\sin\left(2\sec^{-}\right.}{2b\left(a+b\sec\right.}\right.$$

$$\downarrow 3780$$

$$\frac{1}{2}c^2\left(\frac{-\frac{2\left(\frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b}-\sin\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2a}{b}+2\sec^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\sec^{-1}(cx)}d\sec^{-1}(cx)\right)}{b}-\frac{\cos\left(2\sec^{-1}(cx)\right)}{b(a+b\sec^{-1}(cx))}}{b}-\frac{\sin\left(2\sec^{-1}(cx)\right)}{2b\left(a+b\sec^{-1}(cx)\right)^2}\right)$$

$$\downarrow 3783$$

$$\frac{1}{2}c^2\left(\frac{-\frac{2\left(\frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b}-\frac{\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b}\right)}{b}-\frac{\cos\left(2\sec^{-1}(cx)\right)}{b(a+b\sec^{-1}(cx))}}{b}-\frac{\sin\left(2\sec^{-1}(cx)\right)}{2b\left(a+b\sec^{-1}(cx)\right)^2}\right)$$

input
```
Int[1/(x^3*(a + b*ArcSec[c*x])^3),x]
```

output
```
(c^2*(-1/2*Sin[2*ArcSec[c*x]]/(b*(a + b*ArcSec[c*x])^2) + (-(Cos[2*ArcSec[
c*x]]/(b*(a + b*ArcSec[c*x]))) - (2*(-((CosIntegral[(2*a)/b + 2*ArcSec[c*x
]]*Sin[(2*a)/b])/b) + (Cos[(2*a)/b]*SinIntegral[(2*a)/b + 2*ArcSec[c*x]])/
b))/b)/b))/2
```

3.49.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

3.49.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.40

method	result
derivativedivides	$c^2\left(-\frac{\sin(2\operatorname{arcsec}(cx))}{4(a+b\operatorname{arcsec}(cx))^2 b}-\frac{2\cos\left(\frac{2a}{b}\right)\operatorname{Si}\left(\frac{2a}{b}+2\operatorname{arcsec}(cx)\right)\operatorname{arcsec}(cx)b-2\sin\left(\frac{2a}{b}\right)\operatorname{Ci}\left(\frac{2a}{b}+2\operatorname{arcsec}(cx)\right)\operatorname{arcsec}(cx)}{2(a+b\,\mathrm{ar}}\right.$
default	$c^2\left(-\frac{\sin(2\operatorname{arcsec}(cx))}{4(a+b\operatorname{arcsec}(cx))^2 b}-\frac{2\cos\left(\frac{2a}{b}\right)\operatorname{Si}\left(\frac{2a}{b}+2\operatorname{arcsec}(cx)\right)\operatorname{arcsec}(cx)b-2\sin\left(\frac{2a}{b}\right)\operatorname{Ci}\left(\frac{2a}{b}+2\operatorname{arcsec}(cx)\right)\operatorname{arcsec}(cx)}{2(a+b\,\mathrm{ar}}\right.$

input
```
int(1/x^3/(a+b*arcsec(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(-1/4*sin(2*arcsec(c*x))/(a+b*arcsec(c*x))^2/b-1/2*(2*cos(2*a/b)*Si(2*
a/b+2*arcsec(c*x))*arcsec(c*x)*b-2*sin(2*a/b)*Ci(2*a/b+2*arcsec(c*x))*arcs
ec(c*x)*b+2*cos(2*a/b)*Si(2*a/b+2*arcsec(c*x))*a-2*sin(2*a/b)*Ci(2*a/b+2*a
rcsec(c*x))*a+cos(2*arcsec(c*x))*b)/(a+b*arcsec(c*x))/b^3)
```

3.49.5 Fricas [F]

$$\int \frac{1}{x^3 \left(a + b \sec^{-1}(cx)\right)^3} \, dx = \int \frac{1}{(b \operatorname{arcsec}(cx) + a)^3 x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*arcsec(c*x))^3,x, algorithm="fricas")
```

output
```
integral(1/(b^3*x^3*arcsec(c*x)^3 + 3*a*b^2*x^3*arcsec(c*x)^2 + 3*a^2*b*x^
3*arcsec(c*x) + a^3*x^3), x)
```

3.49.6 Sympy [F]

$$\int \frac{1}{x^3 \left(a + b \sec^{-1}(cx)\right)^3} \, dx = \int \frac{1}{x^3 \left(a + b \operatorname{asec}(cx)\right)^3} \, dx$$

input
```
integrate(1/x**3/(a+b*asec(c*x))**3,x)
```

output
```
Integral(1/(x**3*(a + b*asec(c*x))**3), x)
```

3.49.7 Maxima [F]

$$\int \frac{1}{x^3 \left(a + b \sec^{-1}(cx)\right)^3} \, dx = \int \frac{1}{(b \operatorname{arcsec}(cx) + a)^3 x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*arcsec(c*x))^3,x, algorithm="maxima")
```

output
```
-(16*a*b^2*log(c)^2 - 8*(b^3*c^2*x^2 - 2*b^3)*arctan(sqrt(c*x + 1)*sqrt(c*
x - 1))^3 + 16*a^3 - 8*(a*b^2*c^2*log(c)^2 + a^3*c^2)*x^2 - 24*(a*b^2*c^2*
x^2 - 2*a*b^2)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - 2*(a*b^2*c^2*x^2 -
2*a*b^2)*log(c^2*x^2)^2 - 8*(a*b^2*c^2*x^2 - 2*a*b^2)*log(x)^2 + 2*(4*b^3*
arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - b^3*log(c^2*x^2)^2 - 4*b^3*log(c)^
2 - 8*b^3*log(c)*log(x) - 4*b^3*log(x)^2 + 8*a*b^2*arctan(sqrt(c*x + 1)*sq
rt(c*x - 1)) + 4*a^2*b + 4*(b^3*log(c) + b^3*log(x))*log(c^2*x^2))*sqrt(c*
x + 1)*sqrt(c*x - 1) + 2*(8*b^3*log(c)^2 + 24*a^2*b - 4*(b^3*c^2*log(c)^2
+ 3*a^2*b*c^2)*x^2 - (b^3*c^2*x^2 - 2*b^3)*log(c^2*x^2)^2 - 4*(b^3*c^2*x^2
 - 2*b^3)*log(x)^2 + 4*(b^3*c^2*x^2*log(c) - 2*b^3*log(c) + (b^3*c^2*x^2 -
 2*b^3)*log(x))*log(c^2*x^2) - 8*(b^3*c^2*x^2*log(c) - 2*b^3*log(c))*log(x
))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + (16*b^6*x^2*arctan(sqrt(c*x + 1)*
sqrt(c*x - 1))^4 + b^6*x^2*log(c^2*x^2)^4 + 64*b^6*x^2*log(c)*log(x)^3 + 1
6*b^6*x^2*log(x)^4 + 64*a*b^5*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 +
32*(3*b^6*log(c)^2 + a^2*b^4)*x^2*log(x)^2 - 8*(b^6*x^2*log(c) + b^6*x^2*l
og(x))*log(c^2*x^2)^3 + 64*(b^6*log(c)^3 + a^2*b^4*log(c))*x^2*log(x) + 16
*(b^6*log(c)^4 + 2*a^2*b^4*log(c)^2 + a^4*b^2)*x^2 + 8*(b^6*x^2*log(c^2*x^
2)^2 + 8*b^6*x^2*log(c)*log(x) + 4*b^6*x^2*log(x)^2 + 4*(b^6*log(c)^2 + 3*
a^2*b^4)*x^2 - 4*(b^6*x^2*log(c) + b^6*x^2*log(x))*log(c^2*x^2))*arctan(sq
rt(c*x + 1)*sqrt(c*x - 1))^2 + 8*(6*b^6*x^2*log(c)*log(x) + 3*b^6*x^2*l...
```

3.49.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 929 vs. $2(108) = 216$.

Time $= 0.30$ (sec) , antiderivative size $= 929$, normalized size of antiderivative $= 8.29$

$$\int \frac{1}{x^3 \left(a + b \sec^{-1}(cx)\right)^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(a+b*arcsec(c*x))^3,x, algorithm="giac")
```

output
```
1/2*(4*b^2*c*arccos(1/(c*x))^2*cos(a/b)*cos_integral(2*a/b + 2*arccos(1/(c
*x)))*sin(a/b)/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3)
 - 4*b^2*c*arccos(1/(c*x))^2*cos(a/b)^2*sin_integral(2*a/b + 2*arccos(1/(c
*x)))/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) + 8*a*b*
c*arccos(1/(c*x))*cos(a/b)*cos_integral(2*a/b + 2*arccos(1/(c*x)))*sin(a/b
)/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) - 8*a*b*c*ar
ccos(1/(c*x))*cos(a/b)^2*sin_integral(2*a/b + 2*arccos(1/(c*x)))/(b^5*arcc
os(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) + 4*a^2*c*cos(a/b)*cos_
integral(2*a/b + 2*arccos(1/(c*x)))*sin(a/b)/(b^5*arccos(1/(c*x))^2 + 2*a*
b^4*arccos(1/(c*x)) + a^2*b^3) + 2*b^2*c*arccos(1/(c*x))^2*sin_integral(2*
a/b + 2*arccos(1/(c*x)))/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x))
+ a^2*b^3) - 4*a^2*c*cos(a/b)^2*sin_integral(2*a/b + 2*arccos(1/(c*x)))/(b
^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) + 4*a*b*c*arccos
(1/(c*x))*sin_integral(2*a/b + 2*arccos(1/(c*x)))/(b^5*arccos(1/(c*x))^2 +
 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) + b^2*c*arccos(1/(c*x))/(b^5*arccos(1/
(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) + 2*a^2*c*sin_integral(2*a/b
 + 2*arccos(1/(c*x)))/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a
^2*b^3) + a*b*c/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3
) - b^2*sqrt(-1/(c^2*x^2) + 1)/((b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/
(c*x)) + a^2*b^3)*x) - 2*b^2*arccos(1/(c*x))/((b^5*arccos(1/(c*x))^2 + ...
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(a + b \sec^{-1}(cx)\right)^3} \, dx = \int \frac{1}{x^3 \left(a + b \operatorname{acos}\left(\frac{1}{cx}\right)\right)^3} \, dx$$

input
```
int(1/(x^3*(a + b*acos(1/(c*x)))^3),x)
```

output
```
int(1/(x^3*(a + b*acos(1/(c*x)))^3), x)
```

3.50 $\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)^3}\,dx$

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3.50.1 Optimal result

Integrand size = 14, antiderivative size = 228

$$\begin{aligned}\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)^3}\,dx = &-\frac{c^3\sqrt{1-\frac{1}{c^2x^2}}}{8b\left(a+b\sec^{-1}(cx)\right)^2}-\frac{c^2}{8b^2x\left(a+b\sec^{-1}(cx)\right)}\\ &-\frac{3c^3\cos\left(3\sec^{-1}(cx)\right)}{8b^2\left(a+b\sec^{-1}(cx)\right)}\\ &+\frac{c^3\,\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\sin\left(\frac{a}{b}\right)}{8b^3}\\ &+\frac{9c^3\,\text{CosIntegral}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)\sin\left(\frac{3a}{b}\right)}{8b^3}\\ &-\frac{c^3\sin\left(3\sec^{-1}(cx)\right)}{8b\left(a+b\sec^{-1}(cx)\right)^2}-\frac{c^3\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{8b^3}\\ &-\frac{9c^3\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)}{8b^3}\end{aligned}$$

output

```
-1/8*c^2/b^2/x/(a+b*arcsec(c*x))-3/8*c^3*cos(3*arcsec(c*x))/b^2/(a+b*arcse
c(c*x))-1/8*c^3*cos(a/b)*Si(a/b+arcsec(c*x))/b^3-9/8*c^3*cos(3*a/b)*Si(3*a
/b+3*arcsec(c*x))/b^3+1/8*c^3*Ci(a/b+arcsec(c*x))*sin(a/b)/b^3+9/8*c^3*Ci(
3*a/b+3*arcsec(c*x))*sin(3*a/b)/b^3-1/8*c^3*sin(3*arcsec(c*x))/b/(a+b*arcs
ec(c*x))^2-1/8*c^3*(1-1/c^2/x^2)^(1/2)/b/(a+b*arcsec(c*x))^2
```

3.50.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)^3}\,dx$$

$$= \frac{-\frac{4b^2c\sqrt{1-\frac{1}{c^2x^2}}}{x^2\left(a+b\sec^{-1}(cx)\right)^2} - \frac{12b}{x^3\left(a+b\sec^{-1}(cx)\right)} + \frac{8bc^2}{ax+bx\sec^{-1}(cx)} + c^3\,\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\sin\left(\frac{a}{b}\right) + 9c^3\,\text{CosInteg}}{8b^3}$$

input
```
Integrate[1/(x^4*(a + b*ArcSec[c*x])^3),x]
```

output
```
((-4*b^2*c*Sqrt[1 - 1/(c^2*x^2)])/(x^2*(a + b*ArcSec[c*x])^2) - (12*b)/(x^
3*(a + b*ArcSec[c*x])) + (8*b*c^2)/(a*x + b*x*ArcSec[c*x]) + c^3*CosIntegr
al[a/b + ArcSec[c*x]]*Sin[a/b] + 9*c^3*CosIntegral[3*(a/b + ArcSec[c*x])]*
Sin[(3*a)/b] - c^3*Cos[a/b]*SinIntegral[a/b + ArcSec[c*x]] - 9*c^3*Cos[(3*
a)/b]*SinIntegral[3*(a/b + ArcSec[c*x])])/(8*b^3)
```

3.50.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5745, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)^3}\,dx$$

$$\downarrow 5745$$

$$c^3\int \frac{\sqrt{1-\frac{1}{c^2x^2}}}{c^2x^2\left(a+b\sec^{-1}(cx)\right)^3}\,d\sec^{-1}(cx)$$

$$\downarrow 4906$$

$$c^3\int \left(\frac{\sin\left(3\sec^{-1}(cx)\right)}{4\left(a+b\sec^{-1}(cx)\right)^3} + \frac{\sqrt{1-\frac{1}{c^2x^2}}}{4\left(a+b\sec^{-1}(cx)\right)^3}\right)d\sec^{-1}(cx)$$

$$\downarrow 2009$$

$$c^3\left(\frac{\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{8b^3}+\frac{9\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)}{8b^3}-\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{8b^3}\right.$$

input
```
Int[1/(x^4*(a + b*ArcSec[c*x])^3),x]
```

output
```
c^3*(-1/8*Sqrt[1 - 1/(c^2*x^2)]/(b*(a + b*ArcSec[c*x])^2) - 1/(8*b^2*c*x*(
a + b*ArcSec[c*x])) - (3*Cos[3*ArcSec[c*x]])/(8*b^2*(a + b*ArcSec[c*x])) +
 (CosIntegral[a/b + ArcSec[c*x]]*Sin[a/b])/(8*b^3) + (9*CosIntegral[(3*a)/
b + 3*ArcSec[c*x]]*Sin[(3*a)/b])/(8*b^3) - Sin[3*ArcSec[c*x]]/(8*b*(a + b*
ArcSec[c*x])^2) - (Cos[a/b]*SinIntegral[a/b + ArcSec[c*x]])/(8*b^3) - (9*C
os[(3*a)/b]*SinIntegral[(3*a)/b + 3*ArcSec[c*x]])/(8*b^3))
```

3.50.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

3.50.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.35

method	result
derivativedivides	$c^3\left(-\frac{\sin(3\,\text{arcsec}(cx))}{8(a+b\,\text{arcsec}(cx))^2 b} - \frac{3(3\,\text{arcsec}(cx)\cos(\frac{3a}{b})\,\text{Si}(\frac{3a}{b}+3\,\text{arcsec}(cx))b-3\,\text{arcsec}(cx)\sin(\frac{3a}{b})\,\text{Ci}(\frac{3a}{b}+3\,\text{arcsec}(c$ $8(a+b$
default	$c^3\left(-\frac{\sin(3\,\text{arcsec}(cx))}{8(a+b\,\text{arcsec}(cx))^2 b} - \frac{3(3\,\text{arcsec}(cx)\cos(\frac{3a}{b})\,\text{Si}(\frac{3a}{b}+3\,\text{arcsec}(cx))b-3\,\text{arcsec}(cx)\sin(\frac{3a}{b})\,\text{Ci}(\frac{3a}{b}+3\,\text{arcsec}(c$ $8(a+b$

input
```
int(1/x^4/(a+b*arcsec(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
c^3*(-1/8*sin(3*arcsec(c*x))/(a+b*arcsec(c*x))^2/b-3/8*(3*arcsec(c*x)*cos(
3*a/b)*Si(3*a/b+3*arcsec(c*x))*b-3*arcsec(c*x)*sin(3*a/b)*Ci(3*a/b+3*arcse
c(c*x))*b+3*cos(3*a/b)*Si(3*a/b+3*arcsec(c*x))*a-3*sin(3*a/b)*Ci(3*a/b+3*a
rcsec(c*x))*a+cos(3*arcsec(c*x))*b)/(a+b*arcsec(c*x))/b^3-1/8*((c^2*x^2-1)
/c^2/x^2)^(1/2)/(a+b*arcsec(c*x))^2/b-1/8*(arcsec(c*x)*cos(a/b)*Si(a/b+arc
sec(c*x))*b*c*x-arcsec(c*x)*sin(a/b)*Ci(a/b+arcsec(c*x))*b*c*x+cos(a/b)*Si
(a/b+arcsec(c*x))*a*c*x-sin(a/b)*Ci(a/b+arcsec(c*x))*a*c*x+b)/c/x/(a+b*arc
sec(c*x))/b^3)
```

3.50.5 Fricas [F]

$$\int \frac{1}{x^4\left(a+b\,\text{sec}^{-1}(cx)\right)^3}\,dx = \int \frac{1}{\left(b\,\text{arcsec}\left(cx\right)+a\right)^3 x^4}\,dx$$

input
```
integrate(1/x^4/(a+b*arcsec(c*x))^3,x, algorithm="fricas")
```

output
```
integral(1/(b^3*x^4*arcsec(c*x)^3 + 3*a*b^2*x^4*arcsec(c*x)^2 + 3*a^2*b*x^
4*arcsec(c*x) + a^3*x^4), x)
```

3.50.6 Sympy [F]

$$\int \frac{1}{x^4 \left(a + b \sec^{-1}(cx)\right)^3} \, dx = \int \frac{1}{x^4 \left(a + b \operatorname{asec}(cx)\right)^3} \, dx$$

input
```
integrate(1/x**4/(a+b*asec(c*x))**3,x)
```

output
```
Integral(1/(x**4*(a + b*asec(c*x))**3), x)
```

3.50.7 Maxima [F]

$$\int \frac{1}{x^4 \left(a + b \sec^{-1}(cx)\right)^3} \, dx = \int \frac{1}{\left(b \operatorname{arcsec}(cx) + a\right)^3 x^4} \, dx$$

input
```
integrate(1/x^4/(a+b*arcsec(c*x))^3,x, algorithm="maxima")
```

output
```
-(24*a*b^2*log(c)^2 - 8*(2*b^3*c^2*x^2 - 3*b^3)*arctan(sqrt(c*x + 1)*sqrt(
c*x - 1))^3 + 24*a^3 - 16*(a*b^2*c^2*log(c)^2 + a^3*c^2)*x^2 - 24*(2*a*b^2
*c^2*x^2 - 3*a*b^2)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - 2*(2*a*b^2*c^2
*x^2 - 3*a*b^2)*log(c^2*x^2)^2 - 8*(2*a*b^2*c^2*x^2 - 3*a*b^2)*log(x)^2 +
2*(4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - b^3*log(c^2*x^2)^2 - 4*b^
3*log(c)^2 - 8*b^3*log(c)*log(x) - 4*b^3*log(x)^2 + 8*a*b^2*arctan(sqrt(c*
x + 1)*sqrt(c*x - 1)) + 4*a^2*b + 4*(b^3*log(c) + b^3*log(x))*log(c^2*x^2)
)*sqrt(c*x + 1)*sqrt(c*x - 1) + 2*(12*b^3*log(c)^2 + 36*a^2*b - 8*(b^3*c^2
*log(c)^2 + 3*a^2*b*c^2)*x^2 - (2*b^3*c^2*x^2 - 3*b^3)*log(c^2*x^2)^2 - 4*
(2*b^3*c^2*x^2 - 3*b^3)*log(x)^2 + 4*(2*b^3*c^2*x^2*log(c) - 3*b^3*log(c)
+ (2*b^3*c^2*x^2 - 3*b^3)*log(x))*log(c^2*x^2) - 8*(2*b^3*c^2*x^2*log(c) -
 3*b^3*log(c))*log(x))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - (16*b^6*x^3*a
rctan(sqrt(c*x + 1)*sqrt(c*x - 1))^4 + b^6*x^3*log(c^2*x^2)^4 + 64*b^6*x^3
*log(c)*log(x)^3 + 16*b^6*x^3*log(x)^4 + 64*a*b^5*x^3*arctan(sqrt(c*x + 1)
*sqrt(c*x - 1))^3 + 32*(3*b^6*log(c)^2 + a^2*b^4)*x^3*log(x)^2 + 64*(b^6*l
og(c)^3 + a^2*b^4*log(c))*x^3*log(x) + 16*(b^6*log(c)^4 + 2*a^2*b^4*log(c)
^2 + a^4*b^2)*x^3 - 8*(b^6*x^3*log(c) + b^6*x^3*log(x))*log(c^2*x^2)^3 + 8
*(b^6*x^3*log(c^2*x^2)^2 + 8*b^6*x^3*log(c)*log(x) + 4*b^6*x^3*log(x)^2 +
4*(b^6*log(c)^2 + 3*a^2*b^4)*x^3 - 4*(b^6*x^3*log(c) + b^6*x^3*log(x))*log
(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 8*(6*b^6*x^3*log(c)*...
```

3.50.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1640 vs. 2(210) = 420.

Time = 0.31 (sec) , antiderivative size = 1640, normalized size of antiderivative = 7.19

$$\int \frac{1}{x^4 \left(a + b \sec^{-1}(cx)\right)^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(a+b*arcsec(c*x))^3,x, algorithm="giac")
```

output
```
1/8*(36*b^2*c^2*arccos(1/(c*x))^2*cos(a/b)^2*cos_integral(3*a/b + 3*arccos
(1/(c*x)))*sin(a/b)/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2
*b^3) - 36*b^2*c^2*arccos(1/(c*x))^2*cos(a/b)^3*sin_integral(3*a/b + 3*arc
cos(1/(c*x)))/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3)
+ 72*a*b*c^2*arccos(1/(c*x))*cos(a/b)^2*cos_integral(3*a/b + 3*arccos(1/(c
*x)))*sin(a/b)/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3)
 - 72*a*b*c^2*arccos(1/(c*x))*cos(a/b)^3*sin_integral(3*a/b + 3*arccos(1/(
c*x)))/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) - 9*b^2
*c^2*arccos(1/(c*x))^2*cos_integral(3*a/b + 3*arccos(1/(c*x)))*sin(a/b)/(b
^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) + 36*a^2*c^2*cos
(a/b)^2*cos_integral(3*a/b + 3*arccos(1/(c*x)))*sin(a/b)/(b^5*arccos(1/(c*
x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) + b^2*c^2*arccos(1/(c*x))^2*cos
_integral(a/b + arccos(1/(c*x)))*sin(a/b)/(b^5*arccos(1/(c*x))^2 + 2*a*b^4
*arccos(1/(c*x)) + a^2*b^3) + 27*b^2*c^2*arccos(1/(c*x))^2*cos(a/b)*sin_in
tegral(3*a/b + 3*arccos(1/(c*x)))/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(
1/(c*x)) + a^2*b^3) - 36*a^2*c^2*cos(a/b)^3*sin_integral(3*a/b + 3*arccos(
1/(c*x)))/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) - b^
2*c^2*arccos(1/(c*x))^2*cos(a/b)*sin_integral(a/b + arccos(1/(c*x)))/(b^5*
arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) - 18*a*b*c^2*arccos
(1/(c*x))*cos_integral(3*a/b + 3*arccos(1/(c*x)))*sin(a/b)/(b^5*arccos(...
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(a + b \sec^{-1}(cx)\right)^3} \, dx = \int \frac{1}{x^4 \left(a + b \operatorname{acos}\left(\frac{1}{cx}\right)\right)^3} \, dx$$

input
```
int(1/(x^4*(a + b*acos(1/(c*x)))^3),x)
```

output
```
int(1/(x^4*(a + b*acos(1/(c*x)))^3), x)
```

3.51 $\int (dx)^m \left(a + b \sec^{-1}(cx)\right)^3 \, dx$

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3.51.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (dx)^m \left(a + b \sec^{-1}(cx)\right)^3 \, dx = \text{Int}\left((dx)^m \left(a + b \sec^{-1}(cx)\right)^3, x\right)$$

output
```
Unintegrable((d*x)^m*(a+b*arcsec(c*x))^3,x)
```

3.51.2 Mathematica [N/A]

Not integrable

Time = 4.37 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a + b \sec^{-1}(cx)\right)^3 \, dx = \int (dx)^m \left(a + b \sec^{-1}(cx)\right)^3 \, dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcSec[c*x])^3,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcSec[c*x])^3, x]
```

3.51.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a + b \sec^{-1}(cx)\right)^3 \, dx$$

$$\downarrow 5771$$

$$\int (dx)^m \left(a + b \sec^{-1}(cx)\right)^3 \, dx$$

input
```
Int[(d*x)^m*(a + b*ArcSec[c*x])^3,x]
```

output
```
$Aborted
```

3.51.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrable[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.51.4 Maple [N/A] (verified)

Not integrable

Time = 0.76 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a + b \operatorname{arcsec}(cx)\right)^3 \, dx$$

input
```
int((d*x)^m*(a+b*arcsec(c*x))^3,x)
```

output
```
int((d*x)^m*(a+b*arcsec(c*x))^3,x)
```

3.51.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.75

$$\int (dx)^m \left(a + b \sec^{-1}(cx)\right)^3 \, dx = \int (b \operatorname{arcsec}(cx) + a)^3 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arcsec(c*x))^3,x, algorithm="fricas")
```

output
```
integral((b^3*arcsec(c*x)^3 + 3*a*b^2*arcsec(c*x)^2 + 3*a^2*b*arcsec(c*x)
+ a^3)*(d*x)^m, x)
```

3.51.6 Sympy [N/A]

Not integrable

Time = 22.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (dx)^m \left(a + b \sec^{-1}(cx)\right)^3 \, dx = \int (dx)^m \left(a + b \operatorname{asec}(cx)\right)^3 \, dx$$

input
```
integrate((d*x)**m*(a+b*asec(c*x))**3,x)
```

output
```
Integral((d*x)**m*(a + b*asec(c*x))**3, x)
```

3.51.7 Maxima [N/A]

Not integrable

Time = 14.78 (sec) , antiderivative size = 991, normalized size of antiderivative = 61.94

$$\int (dx)^m \left(a + b \sec^{-1}(cx)\right)^3 \, dx = \int (b \operatorname{arcsec}(cx) + a)^3 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arcsec(c*x))^3,x, algorithm="maxima")
```

output
```
(d*x)^(m + 1)*a^3/(d*(m + 1)) + 1/4*(4*b^3*d^m*x*x^m*arctan(sqrt(c*x + 1)*
sqrt(c*x - 1))^3 - 3*b^3*d^m*x*x^m*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log
(c^2*x^2)^2 - 4*(m + 1)*integrate(3/4*(4*(a*b^2*d^m*m + a*b^2*d^m - (a*b^2
*c^2*d^m*m + a*b^2*c^2*d^m)*x^2)*x^m*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2
 - (a*b^2*d^m*m + a*b^2*d^m - (a*b^2*c^2*d^m*m + a*b^2*c^2*d^m)*x^2)*x^m*l
og(c^2*x^2)^2 - 4*(a*b^2*d^m*m + a*b^2*d^m - (a*b^2*c^2*d^m*m + a*b^2*c^2*
d^m)*x^2)*x^m*log(x)^2 - 8*(a*b^2*d^m*m*log(c) + a*b^2*d^m*log(c) - (a*b^2
*c^2*d^m*m*log(c) + a*b^2*c^2*d^m*log(c))*x^2)*x^m*log(x) + (4*b^3*d^m*x^m
*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - b^3*d^m*x^m*log(c^2*x^2)^2)*sqrt(
c*x + 1)*sqrt(c*x - 1) - 4*(a*b^2*d^m*m*log(c)^2 + a*b^2*d^m*log(c)^2 - (a
*b^2*c^2*d^m*m*log(c)^2 + a*b^2*c^2*d^m*log(c)^2)*x^2)*x^m - 4*((b^3*d^m*m
 + b^3*d^m - (b^3*c^2*d^m*m + b^3*c^2*d^m)*x^2)*x^m*log(x)^2 + 2*(b^3*d^m*
m*log(c) + b^3*d^m*log(c) - (b^3*c^2*d^m*m*log(c) + b^3*c^2*d^m*log(c))*x^
2)*x^m*log(x) + ((b^3*log(c)^2 - a^2*b)*d^m*m - ((b^3*c^2*log(c)^2 - a^2*b
*c^2)*d^m*m + (b^3*c^2*log(c)^2 - a^2*b*c^2)*d^m)*x^2 + (b^3*log(c)^2 - a^
2*b)*d^m)*x^m - ((b^3*d^m*m + b^3*d^m - (b^3*c^2*d^m*m + b^3*c^2*d^m)*x^2)
*x^m*log(x) + (b^3*d^m*m*log(c) - (b^3*c^2*d^m*m*log(c) + (b^3*c^2*log(c)
+ b^3*c^2)*d^m)*x^2 + (b^3*log(c) + b^3)*d^m)*x^m)*log(c^2*x^2))*arctan(sq
rt(c*x + 1)*sqrt(c*x - 1)) + 4*((a*b^2*d^m*m + a*b^2*d^m - (a*b^2*c^2*d^m*
m + a*b^2*c^2*d^m)*x^2)*x^m*log(x) + (a*b^2*d^m*m*log(c) + a*b^2*d^m*lo...
```

3.51.8 Giac [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a + b \sec^{-1}(cx)\right)^3 \, dx = \int (b \operatorname{arcsec}(cx) + a)^3 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arcsec(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)^3*(d*x)^m, x)
```

3.51.9 Mupad [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int (dx)^m \left(a + b \sec^{-1}(cx)\right)^3 \, dx = \int (d\,x)^m \left(a + b \operatorname{acos}\left(\frac{1}{c\,x}\right)\right)^3 \, dx$$

input
```
int((d*x)^m*(a + b*acos(1/(c*x)))^3,x)
```

output
```
int((d*x)^m*(a + b*acos(1/(c*x)))^3, x)
```

3.52 $\int (dx)^m \left(a + b \sec^{-1}(cx)\right)^2 \, dx$

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3.52.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (dx)^m \left(a + b \sec^{-1}(cx)\right)^2 \, dx = \text{Int}\left((dx)^m \left(a + b \sec^{-1}(cx)\right)^2, x\right)$$

output
```
Unintegrable((d*x)^m*(a+b*arcsec(c*x))^2,x)
```

3.52.2 Mathematica [N/A]

Not integrable

Time = 2.77 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a + b \sec^{-1}(cx)\right)^2 \, dx = \int (dx)^m \left(a + b \sec^{-1}(cx)\right)^2 \, dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcSec[c*x])^2,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcSec[c*x])^2, x]
```

3.52.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a + b \sec^{-1}(cx)\right)^2 dx$$

$$\downarrow 5771$$

$$\int (dx)^m \left(a + b \sec^{-1}(cx)\right)^2 dx$$

input
```
Int[(d*x)^m*(a + b*ArcSec[c*x])^2,x]
```

output
```
$Aborted
```

3.52.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrable[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.52.4 Maple [N/A] (verified)

Not integrable

Time = 0.61 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a + b \operatorname{arcsec}(cx)\right)^2 dx$$

input
```
int((d*x)^m*(a+b*arcsec(c*x))^2,x)
```

output
```
int((d*x)^m*(a+b*arcsec(c*x))^2,x)
```

3.52.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int (dx)^m \left(a + b \sec^{-1}(cx)\right)^2 \, dx = \int (b \operatorname{arcsec}(cx) + a)^2 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arcsec(c*x))^2,x, algorithm="fricas")
```

output
```
integral((b^2*arcsec(c*x)^2 + 2*a*b*arcsec(c*x) + a^2)*(d*x)^m, x)
```

3.52.6 Sympy [N/A]

Not integrable

Time = 10.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (dx)^m \left(a + b \sec^{-1}(cx)\right)^2 \, dx = \int (dx)^m \left(a + b \operatorname{asec}(cx)\right)^2 \, dx$$

input
```
integrate((d*x)**m*(a+b*asec(c*x))**2,x)
```

output
```
Integral((d*x)**m*(a + b*asec(c*x))**2, x)
```

3.52.7 Maxima [N/A]

Not integrable

Time = 6.85 (sec) , antiderivative size = 512, normalized size of antiderivative = 32.00

$$\int (dx)^m \left(a + b \sec^{-1}(cx)\right)^2 \, dx = \int (b \operatorname{arcsec}(cx) + a)^2 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

output
```
(d*x)^(m + 1)*a^2/(d*(m + 1)) + 1/4*(4*b^2*d^m*x*x^m*arctan(sqrt(c*x + 1)*
sqrt(c*x - 1))^2 - b^2*d^m*x*x^m*log(c^2*x^2)^2 - 4*(m + 1)*integrate((2*s
qrt(c*x + 1)*sqrt(c*x - 1)*b^2*d^m*x^m*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))
 - (b^2*d^m*m + b^2*d^m - (b^2*c^2*d^m*m + b^2*c^2*d^m)*x^2)*x^m*log(x)^2
+ 2*(a*b*d^m*m + a*b*d^m - (a*b*c^2*d^m*m + a*b*c^2*d^m)*x^2)*x^m*arctan(s
qrt(c*x + 1)*sqrt(c*x - 1)) - 2*(b^2*d^m*m*log(c) + b^2*d^m*log(c) - (b^2*
c^2*d^m*m*log(c) + b^2*c^2*d^m*log(c))*x^2)*x^m*log(x) - (b^2*d^m*m*log(c)
^2 + b^2*d^m*log(c)^2 - (b^2*c^2*d^m*m*log(c)^2 + b^2*c^2*d^m*log(c)^2)*x^
2)*x^m + ((b^2*d^m*m + b^2*d^m - (b^2*c^2*d^m*m + b^2*c^2*d^m)*x^2)*x^m*lo
g(x) + (b^2*d^m*m*log(c) - (b^2*c^2*d^m*m*log(c) + (b^2*c^2*log(c) + b^2*c
^2)*d^m)*x^2 + (b^2*log(c) + b^2)*d^m)*x^m)*log(c^2*x^2))/((c^2*m + c^2)*x
^2 - m - 1), x))/(m + 1)
```

3.52.8 Giac [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a + b \sec^{-1}(cx)\right)^2 \, dx = \int (b \operatorname{arcsec}(cx) + a)^2 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)^2*(d*x)^m, x)
```

3.52.9 Mupad [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int (dx)^m \left(a + b \sec^{-1}(cx)\right)^2 \, dx = \int (d\,x)^m \left(a + b \operatorname{acos}\left(\frac{1}{c\,x}\right)\right)^2 dx$$

input
```
int((d*x)^m*(a + b*acos(1/(c*x)))^2,x)
```

output
```
int((d*x)^m*(a + b*acos(1/(c*x)))^2, x)
```

3.53 $\int (dx)^m \left(a + b \sec^{-1}(cx)\right)\, dx$

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3.53.1 Optimal result

Integrand size = 14, antiderivative size = 67

$$\int (dx)^m \left(a + b \sec^{-1}(cx)\right)\, dx = \frac{(dx)^{1+m}\left(a + b \sec^{-1}(cx)\right)}{d(1+m)} - \frac{b(dx)^m \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, -\frac{m}{2}, 1 - \frac{m}{2}, \frac{1}{c^2x^2}\right)}{cm(1+m)}$$

output
```
(d*x)^(1+m)*(a+b*arcsec(c*x))/d/(1+m)-b*(d*x)^m*hypergeom([1/2, -1/2*m],[1
-1/2*m],1/c^2/x^2)/c/m/(1+m)
```

3.53.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.22

$$\int (dx)^m \left(a + b \sec^{-1}(cx)\right)\, dx = \frac{x(dx)^m \left((1+m)\left(a + b \sec^{-1}(cx)\right) + \frac{bc\sqrt{1-\frac{1}{c^2x^2}}\, x \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2x^2\right)}{\sqrt{1-c^2x^2}}\right)}{(1+m)^2}$$

input
```
Integrate[(d*x)^m*(a + b*ArcSec[c*x]),x]
```

output
```
(x*(d*x)^m*((1 + m)*(a + b*ArcSec[c*x]) + (b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Hyp
ergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/Sqrt[1 - c^2*x^2]))/(1
 + m)^2
```

3.53.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5743, 862, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a + b \sec^{-1}(cx)\right) \, dx$$

$$\downarrow 5743$$

$$\frac{(dx)^{m+1} \left(a + b \sec^{-1}(cx)\right)}{d(m+1)} - \frac{bd \int \frac{(dx)^{m-1}}{\sqrt{1-\frac{1}{c^2x^2}}} dx}{c(m+1)}$$

$$\downarrow 862$$

$$\frac{b\left(\frac{1}{x}\right)^m (dx)^m \int \frac{\left(\frac{1}{x}\right)^{-m-1}}{\sqrt{1-\frac{1}{c^2x^2}}} d\frac{1}{x}}{c(m+1)} + \frac{(dx)^{m+1} \left(a + b \sec^{-1}(cx)\right)}{d(m+1)}$$

$$\downarrow 278$$

$$\frac{(dx)^{m+1} \left(a + b \sec^{-1}(cx)\right)}{d(m+1)} - \frac{b(dx)^m \, \text{Hypergeometric2F1}\left(\frac{1}{2}, -\frac{m}{2}, 1 - \frac{m}{2}, \frac{1}{c^2x^2}\right)}{cm(m+1)}$$

input
```
Int[(d*x)^m*(a + b*ArcSec[c*x]),x]
```

output
```
((d*x)^(1 + m)*(a + b*ArcSec[c*x]))/(d*(1 + m)) - (b*(d*x)^m*Hypergeometri
c2F1[1/2, -1/2*m, 1 - m/2, 1/(c^2*x^2)])/(c*m*(1 + m))
```

3.53.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 862
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1)   Subst[Int[(a + b/x^n)^p/x^(m + 2), x],
x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

rule 5743
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

3.53.4 Maple [F]

$$\int (dx)^m (a + b \operatorname{arcsec}(cx))\, dx$$

input
```
int((d*x)^m*(a+b*arcsec(c*x)),x)
```

output
```
int((d*x)^m*(a+b*arcsec(c*x)),x)
```

3.53.5 Fricas [F]

$$\int (dx)^m \left(a + b \sec^{-1}(cx)\right)\, dx = \int (b \operatorname{arcsec}(cx) + a)(dx)^m\, dx$$

input
```
integrate((d*x)^m*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral((b*arcsec(c*x) + a)*(d*x)^m, x)
```

3.53.6 Sympy [F]

$$\int (dx)^m \left(a + b \sec^{-1}(cx)\right)\, dx = \int (dx)^m (a + b \operatorname{asec}(cx))\, dx$$

input
```
integrate((d*x)**m*(a+b*asec(c*x)),x)
```

output
```
Integral((d*x)**m*(a + b*asec(c*x)), x)
```

3.53.7 Maxima [F]

$$\int (dx)^m \left(a + b \sec^{-1}(cx)\right)\, dx = \int (b \operatorname{arcsec}(cx) + a)(dx)^m\, dx$$

input
```
integrate((d*x)^m*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
(d^m*x*x^m*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - (c^2*d^m*m + c^2*d^m)*int
egrate(-sqrt(c*x + 1)*sqrt(c*x - 1)*x^m/(c^2*m - (c^4*m + c^4)*x^2 + c^2),
 x))*b/(m + 1) + (d*x)^(m + 1)*a/(d*(m + 1))
```

3.53.8 Giac [F]

$$\int (dx)^m \left(a + b \sec^{-1}(cx)\right)\, dx = \int (b \operatorname{arcsec}(cx) + a)(dx)^m\, dx$$

input
```
integrate((d*x)^m*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*(d*x)^m, x)
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a + b \sec^{-1}(cx)\right)\, dx = \int (d\,x)^m \left(a + b \operatorname{acos}\left(\frac{1}{c\,x}\right)\right)\, dx$$

input
```
int((d*x)^m*(a + b*acos(1/(c*x))),x)
```

output
```
int((d*x)^m*(a + b*acos(1/(c*x))), x)
```

3.54 $\int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx$

3.54.1 Optimal result 400
3.54.2 Mathematica [N/A] 400
3.54.3 Rubi [N/A] 401
3.54.4 Maple [N/A] (verified) 401
3.54.5 Fricas [N/A] 402
3.54.6 Sympy [N/A] 402
3.54.7 Maxima [N/A] 402
3.54.8 Giac [N/A] 403
3.54.9 Mupad [N/A] 403

3.54.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx = \text{Int}\left(\frac{(dx)^m}{a+b\sec^{-1}(cx)},x\right)$$

output
```
Unintegrable((d*x)^m/(a+b*arcsec(c*x)),x)
```

3.54.2 Mathematica [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx = \int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx$$

input
```
Integrate[(d*x)^m/(a + b*ArcSec[c*x]),x]
```

output
```
Integrate[(d*x)^m/(a + b*ArcSec[c*x]), x]
```

3.54.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{a + b \sec^{-1}(cx)}\, dx$$
$$\downarrow 5771$$
$$\int \frac{(dx)^m}{a + b \sec^{-1}(cx)} dx$$

input
```
Int[(d*x)^m/(a + b*ArcSec[c*x]),x]
```

output
```
$Aborted
```

3.54.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrable[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.54.4 Maple [N/A] (verified)

Not integrable

Time = 1.94 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{a + b \operatorname{arcsec}(cx)} dx$$

input
```
int((d*x)^m/(a+b*arcsec(c*x)),x)
```

output
```
int((d*x)^m/(a+b*arcsec(c*x)),x)
```

3.54.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx = \int \frac{(dx)^m}{b\operatorname{arcsec}(cx)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b*arcsec(c*x) + a), x)
```

3.54.6 Sympy [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx = \int \frac{(dx)^m}{a+b\operatorname{asec}(cx)}\,dx$$

input
```
integrate((d*x)**m/(a+b*asec(c*x)),x)
```

output
```
Integral((d*x)**m/(a + b*asec(c*x)), x)
```

3.54.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx = \int \frac{(dx)^m}{b\operatorname{arcsec}(cx)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(b*arcsec(c*x) + a), x)
```

3.54.8 Giac [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx = \int \frac{(dx)^m}{b\operatorname{arcsec}(cx)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b*arcsec(c*x) + a), x)
```

3.54.9 Mupad [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx = \int \frac{(d\,x)^m}{a+b\operatorname{acos}\left(\frac{1}{c\,x}\right)}\,dx$$

input
```
int((d*x)^m/(a + b*acos(1/(c*x))),x)
```

output
```
int((d*x)^m/(a + b*acos(1/(c*x))), x)
```

3.55 $\int \frac{(dx)^m}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx$

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3.55.2 Mathematica [N/A] 404
3.55.3 Rubi [N/A] 405
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3.55.6 Sympy [N/A] 406
3.55.7 Maxima [N/A] 406
3.55.8 Giac [N/A] 407
3.55.9 Mupad [N/A] 407

3.55.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(dx)^m}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \text{Int}\left(\frac{(dx)^m}{\left(a+b\sec^{-1}(cx)\right)^2}, x\right)$$

output
```
Unintegrable((d*x)^m/(a+b*arcsec(c*x))^2,x)
```

3.55.2 Mathematica [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{(dx)^m}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx$$

input
```
Integrate[(d*x)^m/(a + b*ArcSec[c*x])^2,x]
```

output
```
Integrate[(d*x)^m/(a + b*ArcSec[c*x])^2, x]
```

3.55.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{\left(a + b\sec^{-1}(cx)\right)^2}\,dx$$
$$\downarrow 5771$$
$$\int \frac{(dx)^m}{\left(a + b\sec^{-1}(cx)\right)^2}dx$$

input
```
Int[(d*x)^m/(a + b*ArcSec[c*x])^2,x]
```

output
```
$Aborted
```

3.55.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.55.4 Maple [N/A] (verified)

Not integrable

Time = 1.37 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{\left(a + b\ \text{arcsec}\left(cx\right)\right)^2}dx$$

input
```
int((d*x)^m/(a+b*arcsec(c*x))^2,x)
```

output
```
int((d*x)^m/(a+b*arcsec(c*x))^2,x)
```

3.55.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.00

$$\int \frac{(dx)^m}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{(dx)^m}{\left(b\operatorname{arcsec}(cx)+a\right)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arcsec(c*x))^2,x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b^2*arcsec(c*x)^2 + 2*a*b*arcsec(c*x) + a^2), x)
```

3.55.6 Sympy [N/A]

Not integrable

Time = 5.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{(dx)^m}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{(dx)^m}{\left(a+b\operatorname{asec}(cx)\right)^2}\,dx$$

input
```
integrate((d*x)**m/(a+b*asec(c*x))**2,x)
```

output
```
Integral((d*x)**m/(a + b*asec(c*x))**2, x)
```

3.55.7 Maxima [N/A]

Not integrable

Time = 1.90 (sec) , antiderivative size = 643, normalized size of antiderivative = 40.19

$$\int \frac{(dx)^m}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{(dx)^m}{\left(b\operatorname{arcsec}(cx)+a\right)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

output
```
-(4*(b*d^m*x*x^m*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + a*d^m*x*x^m)*sqrt(c
*x + 1)*sqrt(c*x - 1) - (4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3
*log(c^2*x^2)^2 + 4*b^3*log(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*log(x)^2 +
8*a*b^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*log(c) + b^
3*log(x))*log(c^2*x^2))*integrate(4*((b*d^m*m - (b*c^2*d^m*m + 2*b*c^2*d^m
)*x^2 + b*d^m)*x^m*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + (a*d^m*m - (a*c^2
*d^m*m + 2*a*c^2*d^m)*x^2 + a*d^m)*x^m)*sqrt(c*x + 1)*sqrt(c*x - 1)/(4*b^3
*log(c)^2 + 4*a^2*b - 4*(b^3*c^2*log(c)^2 + a^2*b*c^2)*x^2 - 4*(b^3*c^2*x^
2 - b^3)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - (b^3*c^2*x^2 - b^3)*log(c
^2*x^2)^2 - 4*(b^3*c^2*x^2 - b^3)*log(x)^2 - 8*(a*b^2*c^2*x^2 - a*b^2)*arc
tan(sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*(b^3*c^2*x^2*log(c) - b^3*log(c) + (b
^3*c^2*x^2 - b^3)*log(x))*log(c^2*x^2) - 8*(b^3*c^2*x^2*log(c) - b^3*log(c
))*log(x)), x))/(4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*log(c^2
*x^2)^2 + 4*b^3*log(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*log(x)^2 + 8*a*b^2*
arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*log(c) + b^3*log(x)
)*log(c^2*x^2))
```

3.55.8 Giac [N/A]

Not integrable

Time = 1.65 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{(dx)^m}{\left(b\operatorname{arcsec}(cx)+a\right)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b*arcsec(c*x) + a)^2, x)
```

3.55.9 Mupad [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{(dx)^m}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{(d\,x)^m}{\left(a+b\operatorname{acos}\left(\frac{1}{c\,x}\right)\right)^2}\,dx$$

input
```
int((d*x)^m/(a + b*acos(1/(c*x)))^2,x)
```

output
```
int((d*x)^m/(a + b*acos(1/(c*x)))^2, x)
```

3.56 $\int (d+ex)^3 \left(a+b\sec^{-1}(cx)\right)\, dx$

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3.56.2 Mathematica [A] (verified) 410
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3.56.5 Fricas [A] (verification not implemented) 415
3.56.6 Sympy [A] (verification not implemented) 416
3.56.7 Maxima [A] (verification not implemented) 417
3.56.8 Giac [B] (verification not implemented) 418
3.56.9 Mupad [F(-1)] 418

3.56.1 Optimal result

Integrand size = 16, antiderivative size = 167

$$\begin{aligned}\int (d+ex)^3 \left(a+b\sec^{-1}(cx)\right)\, dx = &-\frac{be(9c^2d^2+e^2)\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3} - \frac{bde^2\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{2c} \\ &- \frac{be^3\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c} + \frac{bd^4\csc^{-1}(cx)}{4e} \\ &+ \frac{(d+ex)^4\left(a+b\sec^{-1}(cx)\right)}{4e} \\ &- \frac{bd(2c^2d^2+e^2)\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{2c^3}\end{aligned}$$

output

```
1/4*b*d^4*arccsc(c*x)/e+1/4*(e*x+d)^4*(a+b*arcsec(c*x))/e-1/2*b*d*(2*c^2*d
^2+e^2)*arctanh((1-1/c^2/x^2)^(1/2))/c^3-1/6*b*e*(9*c^2*d^2+e^2)*x*(1-1/c^
2/x^2)^(1/2)/c^3-1/2*b*d*e^2*x^2*(1-1/c^2/x^2)^(1/2)/c-1/12*b*e^3*x^3*(1-1
/c^2/x^2)^(1/2)/c
```

3.56.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.99

$$\int (d+ex)^3\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{3ac^3x(4d^3+6d^2ex+4de^2x^2+e^3x^3)-be\sqrt{1-\frac{1}{c^2x^2}}x(2e^2+c^2(18d^2+6dex+e^2x^2))+3bc^3x(4d^3+6d^2e}{12c^3}$$

input
```
Integrate[(d + e*x)^3*(a + b*ArcSec[c*x]),x]
```

output
```
(3*a*c^3*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) - b*e*Sqrt[1 - 1/(c
^2*x^2)]*x*(2*e^2 + c^2*(18*d^2 + 6*d*e*x + e^2*x^2)) + 3*b*c^3*x*(4*d^3 +
 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)*ArcSec[c*x] - 6*b*d*(2*c^2*d^2 + e^2)*
Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/(12*c^3)
```

3.56.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {5749, 1892, 1803, 540, 25, 2338, 27, 2338, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$\downarrow 5749$$
$$\frac{(d+ex)^4\left(a+b\sec^{-1}(cx)\right)}{4e}-\frac{b\int\frac{(d+ex)^4}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{4ce}$$
$$\downarrow 1892$$
$$\frac{(d+ex)^4\left(a+b\sec^{-1}(cx)\right)}{4e}-\frac{b\int\frac{\left(\frac{d}{x}+e\right)^4x^2}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{4ce}$$
$$\downarrow 1803$$

$$\frac{b\int \frac{\left(\frac{d}{x}+e\right)^4 x^4}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{4ce}+\frac{(d+ex)^4\left(a+b\sec^{-1}(cx)\right)}{4e}$$

$$\downarrow 540$$

$$\frac{b\left(-\frac{1}{3}\int -\frac{\left(\frac{3d^4}{x^3}+\frac{12ed^3}{x^2}+12e^3d+\frac{2e^2\left(9d^2+\frac{e^2}{c^2}\right)}{x}\right)x^3}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-\frac{1}{3}e^4x^3\sqrt{1-\frac{1}{c^2x^2}}\right)}{4ce}+\frac{(d+ex)^4\left(a+b\sec^{-1}(cx)\right)}{4e}$$

$$\downarrow 25$$

$$\frac{b\left(\frac{1}{3}\int \frac{\left(\frac{3d^4}{x^3}+\frac{12ed^3}{x^2}+12e^3d+\frac{2e^2\left(9d^2+\frac{e^2}{c^2}\right)}{x}\right)x^3}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-\frac{1}{3}e^4x^3\sqrt{1-\frac{1}{c^2x^2}}\right)}{4ce}+\frac{(d+ex)^4\left(a+b\sec^{-1}(cx)\right)}{4e}$$

$$\downarrow 2338$$

$$\frac{b\left(\frac{1}{3}\left(-\frac{1}{2}\int -\frac{2\left(\frac{3d^4}{x^2}+\frac{6e\left(2d^2+\frac{e^2}{c^2}\right)d}{x}+2e^2\left(9d^2+\frac{e^2}{c^2}\right)\right)x^2}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-6de^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{3}e^4x^3\sqrt{1-\frac{1}{c^2x^2}}\right)}{4ce}+\frac{(d+ex)^4\left(a+b\sec^{-1}(cx)\right)}{4e}$$

$$\downarrow 27$$

$$\frac{b\left(\frac{1}{3}\left(\int \frac{\left(\frac{3d^4}{x^2}+\frac{6e\left(2d^2+\frac{e^2}{c^2}\right)d}{x}+2e^2\left(9d^2+\frac{e^2}{c^2}\right)\right)x^2}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-6de^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{3}e^4x^3\sqrt{1-\frac{1}{c^2x^2}}\right)}{4ce}+\frac{(d+ex)^4\left(a+b\sec^{-1}(cx)\right)}{4e}$$

$$\downarrow 2338$$

$$\frac{b\left(\frac{1}{3}\left(-\int -\frac{3d\left(\frac{d^3}{x}+2e\left(2d^2+\frac{e^2}{c^2}\right)\right)x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-2e^2x\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{e^2}{c^2}+9d^2\right)-6de^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{3}e^4x^3\sqrt{1-\frac{1}{c^2x^2}}\right)}{4ce}+\frac{(d+ex)^4\left(a+b\sec^{-1}(cx)\right)}{4e}$$

$$\downarrow 27$$

$$\frac{b\left(\frac{1}{3}\left(3d\int\frac{\left(\frac{d^3}{x}+2e\left(2d^2+\frac{e^2}{c^2}\right)\right)x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-2e^2x\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{e^2}{c^2}+9d^2\right)-6de^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{3}e^4x^3\sqrt{1-\frac{1}{c^2x^2}}\right)}{4ce}+$$
$$\frac{(d+ex)^4\left(a+b\sec^{-1}(cx)\right)}{4e}$$

$$\downarrow 538$$

$$\frac{b\left(\frac{1}{3}\left(3d\left(d^3\int\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}+2e\left(\frac{e^2}{c^2}+2d^2\right)\int\frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}\right)-2e^2x\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{e^2}{c^2}+9d^2\right)-6de^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)-\right.}{4ce}$$
$$\frac{(d+ex)^4\left(a+b\sec^{-1}(cx)\right)}{4e}$$

$$\downarrow 223$$

$$\frac{b\left(\frac{1}{3}\left(3d\left(2e\left(\frac{e^2}{c^2}+2d^2\right)\int\frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}+cd^3\arcsin\left(\frac{1}{cx}\right)\right)-2e^2x\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{e^2}{c^2}+9d^2\right)-6de^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{3}e\right.}{4ce}$$
$$\frac{(d+ex)^4\left(a+b\sec^{-1}(cx)\right)}{4e}$$

$$\downarrow 243$$

$$\frac{b\left(\frac{1}{3}\left(3d\left(e\left(\frac{e^2}{c^2}+2d^2\right)\int\frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}+cd^3\arcsin\left(\frac{1}{cx}\right)\right)-2e^2x\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{e^2}{c^2}+9d^2\right)-6de^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{3}e\right.}{4ce}$$
$$\frac{(d+ex)^4\left(a+b\sec^{-1}(cx)\right)}{4e}$$

$$\downarrow 73$$

$$\frac{b\left(\frac{1}{3}\left(3d\left(cd^3\arcsin\left(\frac{1}{cx}\right)-2c^2e\left(\frac{e^2}{c^2}+2d^2\right)\int\frac{1}{c^2-c^2\sqrt{1-\frac{1}{c^2x^2}}}d\sqrt{1-\frac{1}{c^2x^2}}\right)-2e^2x\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{e^2}{c^2}+9d^2\right)-6de^3x^2\right.\right.}{4ce}$$
$$\frac{(d+ex)^4\left(a+b\sec^{-1}(cx)\right)}{4e}$$

$$\downarrow 221$$

$$\frac{(d+ex)^4\left(a+b\sec^{-1}(cx)\right)}{4e}+\frac{b\left(\frac{1}{3}\left(3d\left(cd^3\arcsin\left(\frac{1}{cx}\right)-2e\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)\left(\frac{e^2}{c^2}+2d^2\right)\right)-2e^2x\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{e^2}{c^2}+9d^2\right)-6de^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right.\right.}{4ce}$$

input
```
Int[(d + e*x)^3*(a + b*ArcSec[c*x]),x]
```

output
```
((d + e*x)^4*(a + b*ArcSec[c*x]))/(4*e) + (b*(-1/3*(e^4*Sqrt[1 - 1/(c^2*x^
2)]*x^3) + (-2*e^2*(9*d^2 + e^2/c^2)*Sqrt[1 - 1/(c^2*x^2)]*x - 6*d*e^3*Sqr
t[1 - 1/(c^2*x^2)]*x^2 + 3*d*(c*d^3*ArcSin[1/(c*x)] - 2*e*(2*d^2 + e^2/c^2
)*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]]))/3))/(4*c*e)
```

3.56.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1892
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 5749
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.56.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(147) = 294$.

Time $= 0.39$ (sec) , antiderivative size $= 402$, normalized size of antiderivative $= 2.41$

method	result
parts	$\frac{a(ex+d)^4}{4e} + \frac{b\,e^3 \operatorname{arcsec}(cx) x^4}{4} + b\,e^2 \operatorname{arcsec}(cx)\, x^3 d + \frac{3be \operatorname{arcsec}(cx) x^2 d^2}{2} + b \operatorname{arcsec}(cx)\, x\, d^3 + \frac{b}{}$
derivativedivides	$\frac{a(cex+cd)^4}{4c^3 e} + \frac{bc \operatorname{arcsec}(cx) d^4}{4e} + b \operatorname{arcsec}(cx) d^3 cx + \frac{3bce \operatorname{arcsec}(cx) d^2 x^2}{2} + bc\,e^2 \operatorname{arcsec}(cx) d\, x^3 + \frac{bc\,e^3 \operatorname{arcsec}(cx) x^4}{4} + \frac{b\sqrt{c^2x^2-1}}{4}$
default	$\frac{a(cex+cd)^4}{4c^3 e} + \frac{bc \operatorname{arcsec}(cx) d^4}{4e} + b \operatorname{arcsec}(cx) d^3 cx + \frac{3bce \operatorname{arcsec}(cx) d^2 x^2}{2} + bc\,e^2 \operatorname{arcsec}(cx) d\, x^3 + \frac{bc\,e^3 \operatorname{arcsec}(cx) x^4}{4} + \frac{b\sqrt{c^2x^2-1}}{4}$

input
```
int((e*x+d)^3*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*(e*x+d)^4/e+1/4*b*e^3*arcsec(c*x)*x^4+b*e^2*arcsec(c*x)*x^3*d+3/2*b*
e*arcsec(c*x)*x^2*d^2+b*arcsec(c*x)*x*d^3+1/4*b/e*arcsec(c*x)*d^4-1/12*b/c
^3*e^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x+1/4*b/c/e*(c^2*x^2-1)^(1/
2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^4*arctan(1/(c^2*x^2-1)^(1/2))-1/2*b/c^3
*e^2*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*d-b/c^2*(c^2*x^2-1)^(1/2)/((c
^2*x^2-1)/c^2/x^2)^(1/2)/x*d^3*ln(c*x+(c^2*x^2-1)^(1/2))-3/2*b/c^3*e*(c^2*
x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^2-1/2*b/c^4*e^2*(c^2*x^2-1)^(1/2)/(
(c^2*x^2-1)/c^2/x^2)^(1/2)/x*d*ln(c*x+(c^2*x^2-1)^(1/2))-1/6*b/c^5*e^3*(c^
2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x
```

3.56.5 Fricas [A] (verification not implemented)

Time $= 0.38$ (sec) , antiderivative size $= 291$, normalized size of antiderivative $= 1.74$

$$\int (d+ex)^3 \left(a + b \sec^{-1}(cx)\right)\, dx$$
$$= \frac{3\,ac^4e^3x^4 + 12\,ac^4de^2x^3 + 18\,ac^4d^2ex^2 + 12\,ac^4d^3x + 3\,(bc^4e^3x^4 + 4\,bc^4de^2x^3 + 6\,bc^4d^2ex^2 + 4\,bc^4d^3x - 4\,}{}$$

input
```
integrate((e*x+d)^3*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/12*(3*a*c^4*e^3*x^4 + 12*a*c^4*d*e^2*x^3 + 18*a*c^4*d^2*e*x^2 + 12*a*c^4
*d^3*x + 3*(b*c^4*e^3*x^4 + 4*b*c^4*d*e^2*x^3 + 6*b*c^4*d^2*e*x^2 + 4*b*c^
4*d^3*x - 4*b*c^4*d^3 - 6*b*c^4*d^2*e - 4*b*c^4*d*e^2 - b*c^4*e^3)*arcsec(
c*x) + 6*(4*b*c^4*d^3 + 6*b*c^4*d^2*e + 4*b*c^4*d*e^2 + b*c^4*e^3)*arctan(
-c*x + sqrt(c^2*x^2 - 1)) + 6*(2*b*c^3*d^3 + b*c*d*e^2)*log(-c*x + sqrt(c^
2*x^2 - 1)) - (b*c^2*e^3*x^2 + 6*b*c^2*d*e^2*x + 18*b*c^2*d^2*e + 2*b*e^3)
*sqrt(c^2*x^2 - 1))/c^4
```

3.56.6 Sympy [A] (verification not implemented)

Time = 3.94 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.17

$$
\begin{aligned}
&\int (d+ex)^3\left(a+b\operatorname{sec}^{-1}(cx)\right)\,dx \\
&= ad^3x + \frac{3ad^2ex^2}{2} + ade^2x^3 + \frac{ae^3x^4}{4} + bd^3x\operatorname{asec}(cx) \\
&\quad + \frac{3bd^2ex^2\operatorname{asec}(cx)}{2} + bde^2x^3\operatorname{asec}(cx) + \frac{be^3x^4\operatorname{asec}(cx)}{4} \\
&\quad - \frac{bd^3\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c} - \frac{3bd^2e\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c} \\
&\quad - \frac{bde^2\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{c} \\
&\quad - \frac{be^3\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{4c}
\end{aligned}
$$

input
```
integrate((e*x+d)**3*(a+b*asec(c*x)),x)
```

output
```
a*d**3*x + 3*a*d**2*e*x**2/2 + a*d*e**2*x**3 + a*e**3*x**4/4 + b*d**3*x*as
ec(c*x) + 3*b*d**2*e*x**2*asec(c*x)/2 + b*d*e**2*x**3*asec(c*x) + b*e**3*x
**4*asec(c*x)/4 - b*d**3*Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*a
sin(c*x), True))/c - 3*b*d**2*e*Piecewise((sqrt(c**2*x**2 - 1)/c, Abs(c**2
*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, True))/(2*c) - b*d*e**2*Piecewise(
(x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (
-I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*as
in(c*x)/(2*c**2), True))/c - b*e**3*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3
*c) + 2*sqrt(c**2*x**2 - 1)/(3*c**3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c
**2*x**2 + 1)/(3*c) + 2*I*sqrt(-c**2*x**2 + 1)/(3*c**3), True))/(4*c)
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.63

$$
\begin{aligned}
&\int (d+ex)^3\left(a+b\operatorname{sec}^{-1}(cx)\right)\,dx \\
&= \frac{1}{4}ae^3x^4 + ade^2x^3 + \frac{3}{2}ad^2ex^2 + \frac{3}{2}\left(x^2\operatorname{arcsec}(cx) - \frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)bd^2e \\
&\quad + \frac{1}{4}\left(4\,x^3\operatorname{arcsec}(cx) - \frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bde^2 \\
&\quad + \frac{1}{12}\left(3\,x^4\operatorname{arcsec}(cx) - \frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)be^3 + ad^3x \\
&\quad + \frac{\left(2\,cx\operatorname{arcsec}(cx) - \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) + \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bd^3}{2\,c}
\end{aligned}
$$

input
```
integrate((e*x+d)^3*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/4*a*e^3*x^4 + a*d*e^2*x^3 + 3/2*a*d^2*e*x^2 + 3/2*(x^2*arcsec(c*x) - x*s
qrt(-1/(c^2*x^2) + 1)/c)*b*d^2*e + 1/4*(4*x^3*arcsec(c*x) - (2*sqrt(-1/(c^
2*x^2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1
)/c^2 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d*e^2 + 1/12*(3*x^4*arcs
ec(c*x) - (c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/
c^3)*b*e^3 + a*d^3*x + 1/2*(2*c*x*arcsec(c*x) - log(sqrt(-1/(c^2*x^2) + 1)
 + 1) + log(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*d^3/c
```

3.56.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9430 vs. 2(147) = 294.

Time = 3.07 (sec) , antiderivative size = 9430, normalized size of antiderivative = 56.47

$$\int (d+ex)^3 \left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
1/12*(12*b*c^3*d^3*arccos(1/(c*x))/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x)
 + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) -
 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) - 12*b*c^
3*d^3*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(c^5 + 4*c^5*(1/(c^2*
x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*
c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x)
 + 1)^8) + 12*b*c^3*d^3*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(c^
5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1
/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^
2) - 1)^4/(1/(c*x) + 1)^8) + 12*a*c^3*d^3/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(
1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2
*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) +
18*b*c^2*d^2*e*arccos(1/(c*x))/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1
)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^
3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) + 24*b*c^3*d^
3*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*
x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2)
 - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x
) + 1)^2) - 48*b*c^3*d^3*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1)
+ 1/(c*x) + 1))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5...
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3 \left(a+b\sec^{-1}(cx)\right)\,dx = \int \left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,(d+e\,x)^3\,dx$$

input
```
int((a + b*acos(1/(c*x)))*(d + e*x)^3,x)
```

output
```
int((a + b*acos(1/(c*x)))*(d + e*x)^3, x)
```

3.57 $\int (d+ex)^2 \left(a+b\sec^{-1}(cx)\right)\, dx$

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3.57.1 Optimal result

Integrand size = 16, antiderivative size = 124

$$\int (d+ex)^2 \left(a+b\sec^{-1}(cx)\right)\, dx = -\frac{bde\sqrt{1-\frac{1}{c^2x^2}}\,x}{c} - \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{6c} + \frac{bd^3\csc^{-1}(cx)}{3e} + \frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e} - \frac{b(6c^2d^2+e^2)\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^3}$$

output
```
1/3*b*d^3*arccsc(c*x)/e+1/3*(e*x+d)^3*(a+b*arcsec(c*x))/e-1/6*b*(6*c^2*d^2
+e^2)*arctanh((1-1/c^2/x^2)^(1/2))/c^3-b*d*e*x*(1-1/c^2/x^2)^(1/2)/c-1/6*b
*e^2*x^2*(1-1/c^2/x^2)^(1/2)/c
```

3.57.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00

$$\int (d+ex)^2 \left(a+b\sec^{-1}(cx)\right)\, dx = \frac{c^2x\left(-be\sqrt{1-\frac{1}{c^2x^2}}(6d+ex)+2ac(3d^2+3dex+e^2x^2)\right)+2bc^3x(3d^2+3dex+e^2x^2)\sec^{-1}(cx)-b(6c^2d^2$$

input
```
Integrate[(d + e*x)^2*(a + b*ArcSec[c*x]),x]
```

output
```
(c^2*x*(-(b*e*Sqrt[1 - 1/(c^2*x^2)]*(6*d + e*x)) + 2*a*c*(3*d^2 + 3*d*e*x
+ e^2*x^2)) + 2*b*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2)*ArcSec[c*x] - b*(6*c^2
*d^2 + e^2)*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/(6*c^3)
```

3.57.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5749, 1892, 1803, 540, 25, 2338, 25, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2 \left(a+b\sec^{-1}(cx)\right)\, dx$$

$$\downarrow 5749$$

$$\frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e} - \frac{b\int \frac{(d+ex)^3}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{3ce}$$

$$\downarrow 1892$$

$$\frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e} - \frac{b\int \frac{\left(\frac{d}{x}+e\right)^3 x}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{3ce}$$

$$\downarrow 1803$$

$$\frac{b\int \frac{\left(\frac{d}{x}+e\right)^3 x^3}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{3ce} + \frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e}$$

$$\downarrow 540$$

$$\frac{b\left(-\frac{1}{2}\int -\frac{\left(\frac{2d^3}{x^2}+6e^2d+\frac{e\left(6d^2+\frac{e^2}{c^2}\right)}{x}\right)x^2}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x} - \frac{1}{2}e^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{3ce} + \frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e}$$

$$\downarrow 25$$

$$\frac{b\left(\frac{1}{2}\int\frac{\left(\frac{2d^3}{x^2}+6e^2d+\frac{e\left(6d^2+\frac{e^2}{c^2}\right)}{x}\right)x^2}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-\frac{1}{2}e^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{3ce}+\frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e}$$

$$\downarrow 2338$$

$$\frac{b\left(\frac{1}{2}\left(-\int-\frac{\left(\frac{2d^3}{x}+e\left(6d^2+\frac{e^2}{c^2}\right)\right)x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-6de^2x\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{2}e^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{3ce}+\frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e}$$

$$\downarrow 25$$

$$\frac{b\left(\frac{1}{2}\left(\int\frac{\left(\frac{2d^3}{x}+e\left(6d^2+\frac{e^2}{c^2}\right)\right)x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-6de^2x\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{2}e^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{3ce}+\frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e}$$

$$\downarrow 538$$

$$\frac{b\left(\frac{1}{2}\left(2d^3\int\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}+e\left(\frac{e^2}{c^2}+6d^2\right)\int\frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-6de^2x\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{2}e^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{3ce}+\frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e}$$

$$\downarrow 223$$

$$\frac{b\left(\frac{1}{2}\left(e\left(\frac{e^2}{c^2}+6d^2\right)\int\frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}+2cd^3\arcsin\left(\frac{1}{cx}\right)-6de^2x\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{2}e^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{3ce}+\frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e}$$

$$\downarrow 243$$

$$\frac{b\left(\frac{1}{2}\left(\frac{1}{2}e\left(\frac{e^2}{c^2}+6d^2\right)\int\frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}+2cd^3\arcsin\left(\frac{1}{cx}\right)-6de^2x\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{2}e^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{3ce}+\frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e}$$

$$\downarrow 73$$

$$\frac{b\left(\frac{1}{2}\left(-c^2e\left(\frac{e^2}{c^2}+6d^2\right)\int\frac{1}{c^2-c^2\sqrt{1-\frac{1}{c^2x^2}}}d\sqrt{1-\frac{1}{c^2x^2}}+2cd^3\arcsin\left(\frac{1}{cx}\right)-6de^2x\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{2}e^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{3ce}+\frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e}$$

$$\downarrow 221$$

$$\frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e}+\frac{b\left(\frac{1}{2}\left(2cd^3\arcsin\left(\frac{1}{cx}\right)-e\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)\left(\frac{e^2}{c^2}+6d^2\right)-6de^2x\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{2}e^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{3ce}$$

input
```
Int[(d + e*x)^2*(a + b*ArcSec[c*x]),x]
```

output
```
((d + e*x)^3*(a + b*ArcSec[c*x]))/(3*e) + (b*(-1/2*(e^3*Sqrt[1 - 1/(c^2*x^
2)]*x^2) + (-6*d*e^2*Sqrt[1 - 1/(c^2*x^2)]*x + 2*c*d^3*ArcSin[1/(c*x)] - e
*(6*d^2 + e^2/c^2)*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/2))/(3*c*e)
```

3.57.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1892
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 5749
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.57.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(110) = 220$.

Time $= 0.38$ (sec) , antiderivative size $= 306$, normalized size of antiderivative $= 2.47$

method	result
parts	$\frac{a(ex+d)^3}{3e} + \frac{b\,e^2 \operatorname{arcsec}(cx) x^3}{3} + be \operatorname{arcsec}(cx) x^2 d + b \operatorname{arcsec}(cx) x\, d^2 + \frac{b \operatorname{arcsec}(cx) d^3}{3e} + \frac{b\sqrt{c^2x^2-}}{}$
derivativedivides	$\frac{\frac{a(cex+cd)^3}{3c^2e} + \frac{bc \operatorname{arcsec}(cx) d^3}{3e} + b \operatorname{arcsec}(cx) d^2 cx + bce \operatorname{arcsec}(cx) d x^2 + \frac{bc\,e^2 \operatorname{arcsec}(cx) x^3}{3} + \frac{b\sqrt{c^2x^2-1}\, d^3 \arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{3e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\, x} -}{c}$
default	$\frac{\frac{a(cex+cd)^3}{3c^2e} + \frac{bc \operatorname{arcsec}(cx) d^3}{3e} + b \operatorname{arcsec}(cx) d^2 cx + bce \operatorname{arcsec}(cx) d x^2 + \frac{bc\,e^2 \operatorname{arcsec}(cx) x^3}{3} + \frac{b\sqrt{c^2x^2-1}\, d^3 \arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{3e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\, x} -}{c}$

input
```
int((e*x+d)^2*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*(e*x+d)^3/e+1/3*b*e^2*arcsec(c*x)*x^3+b*e*arcsec(c*x)*x^2*d+b*arcsec
(c*x)*x*d^2+1/3*b/e*arcsec(c*x)*d^3+1/3*b/c/e*(c^2*x^2-1)^(1/2)/((c^2*x^2-
1)/c^2/x^2)^(1/2)/x*d^3*arctan(1/(c^2*x^2-1)^(1/2))-1/6*b/c^3*e^2*(c^2*x^2
-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)-b/c^2*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x
^2)^(1/2)/x*d^2*ln(c*x+(c^2*x^2-1)^(1/2))-b/c^3*e*(c^2*x^2-1)/((c^2*x^2-1)
/c^2/x^2)^(1/2)/x*d-1/6*b/c^4*e^2*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^
(1/2)/x*ln(c*x+(c^2*x^2-1)^(1/2))
```

3.57.5 Fricas [A] (verification not implemented)

Time $= 0.32$ (sec) , antiderivative size $= 209$, normalized size of antiderivative $= 1.69$

$$\int (d+ex)^2 \left(a + b \sec^{-1}(cx)\right)\, dx$$

$$= \frac{2\,ac^3e^2x^3 + 6\,ac^3dex^2 + 6\,ac^3d^2x + 2\left(bc^3e^2x^3 + 3\,bc^3dex^2 + 3\,bc^3d^2x - 3\,bc^3d^2 - 3\,bc^3de - bc^3e^2\right)\operatorname{arcsec}(}{}$$

input
```
integrate((e*x+d)^2*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/6*(2*a*c^3*e^2*x^3 + 6*a*c^3*d*e*x^2 + 6*a*c^3*d^2*x + 2*(b*c^3*e^2*x^3
+ 3*b*c^3*d*e*x^2 + 3*b*c^3*d^2*x - 3*b*c^3*d^2 - 3*b*c^3*d*e - b*c^3*e^2)
*arcsec(c*x) + 4*(3*b*c^3*d^2 + 3*b*c^3*d*e + b*c^3*e^2)*arctan(-c*x + sqr
t(c^2*x^2 - 1)) + (6*b*c^2*d^2 + b*e^2)*log(-c*x + sqrt(c^2*x^2 - 1)) - (b
*c*e^2*x + 6*b*c*d*e)*sqrt(c^2*x^2 - 1))/c^3
```

3.57.6 Sympy [A] (verification not implemented)

Time = 3.53 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.84

$$\int (d+ex)^2\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$= ad^2x + adex^2 + \frac{ae^2x^3}{3} + bd^2x\,\mathrm{asec}\,(cx) + bdex^2\,\mathrm{asec}\,(cx) + \frac{be^2x^3\,\mathrm{asec}\,(cx)}{3}$$
$$- \frac{bd^2\left(\begin{cases}\mathrm{acosh}\,(cx) & \text{for } |c^2x^2|>1\\ -i\,\mathrm{asin}\,(cx) & \text{otherwise}\end{cases}\right)}{c} - \frac{bde\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{c}$$
$$- \frac{be^2\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\mathrm{acosh}\,(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\,\mathrm{asin}\,(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}$$

input
```
integrate((e*x+d)**2*(a+b*asec(c*x)),x)
```

output
```
a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 + b*d**2*x*asec(c*x) + b*d*e*x**2*as
ec(c*x) + b*e**2*x**3*asec(c*x)/3 - b*d**2*Piecewise((acosh(c*x), Abs(c**2
*x**2) > 1), (-I*asin(c*x), True))/c - b*d*e*Piecewise((sqrt(c**2*x**2 - 1
)/c, Abs(c**2*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, True))/c - b*e**2*Pie
cewise((x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2)
> 1), (-I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1))
 - I*asin(c*x)/(2*c**2), True))/(3*c)
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.61

$$\begin{aligned}
&\int (d+ex)^2\left(a+b\sec^{-1}(cx)\right)\,dx \\
&= \frac{1}{3}ae^2x^3 + adex^2 + \left(x^2\operatorname{arcsec}(cx) - \frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)bde \\
&\quad + \frac{1}{12}\left(4x^3\operatorname{arcsec}(cx) - \frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)be^2 \\
&\quad + ad^2x + \frac{\left(2cx\operatorname{arcsec}(cx) - \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) + \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bd^2}{2c}
\end{aligned}$$

input
```
integrate((e*x+d)^2*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*e^2*x^3 + a*d*e*x^2 + (x^2*arcsec(c*x) - x*sqrt(-1/(c^2*x^2) + 1)/c)
*b*d*e + 1/12*(4*x^3*arcsec(c*x) - (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c^2*
x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2*
x^2) + 1) - 1)/c^2)/c)*b*e^2 + a*d^2*x + 1/2*(2*c*x*arcsec(c*x) - log(sqrt
(-1/(c^2*x^2) + 1) + 1) + log(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*d^2/c
```

3.57.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6416 vs. 2(110) = 220.

Time = 2.89 (sec) , antiderivative size = 6416, normalized size of antiderivative = 51.74

$$\int (d+ex)^2\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output

```
-1/6*(6*b*c^3*d*e*x^2*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/(c^4 + 3*c^4*(1/(c
^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +
 c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6) + 6*a*c^3*d*e*x^2*(1/(c^2*x^2) -
 1)/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) -
1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6) + 18*b*c^3
*d*e*x^2*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^4 + 3*c^4*(1/(c^2*x^2) -
1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c
^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) + 18*a*c^3*d*e*x^2*(1/(c^
2*x^2) - 1)^2/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(
c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)
*(1/(c*x) + 1)^2) + 18*b*c^3*d*e*x^2*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/(
(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2
/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^
4) + 18*a*c^3*d*e*x^2*(1/(c^2*x^2) - 1)^3/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/
(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*
x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^4) + 6*b*c^3*d*e*x^2*(1/(c^2*x^
2) - 1)^4*arccos(1/(c*x))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
+ 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(
c*x) + 1)^6)*(1/(c*x) + 1)^6) + 6*b*c^2*d*e*x*sqrt(-1/(c^2*x^2) + 1)/(c^4
+ 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(...
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2 \left(a+b\sec^{-1}(cx)\right)\,dx = \int \left(a+b\,\text{acos}\left(\frac{1}{c\,x}\right)\right)\,(d+e\,x)^2\,dx$$

input

```
int((a + b*acos(1/(c*x)))*(d + e*x)^2,x)
```

output

```
int((a + b*acos(1/(c*x)))*(d + e*x)^2, x)
```

3.58 $\int (d+ex)\left(a+b\sec^{-1}(cx)\right)\,dx$

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3.58.1 Optimal result

Integrand size = 14, antiderivative size = 84

$$\int (d+ex)\left(a+b\sec^{-1}(cx)\right)\,dx = -\frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} + \frac{bd^2\csc^{-1}(cx)}{2e} + \frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e} - \frac{bd\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$$

output
```
1/2*b*d^2*arccsc(c*x)/e+1/2*(e*x+d)^2*(a+b*arcsec(c*x))/e-b*d*arctanh((1-1
/c^2/x^2)^(1/2))/c-1/2*b*e*x*(1-1/c^2/x^2)^(1/2)/c
```

3.58.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.36

$$\int (d+ex)\left(a+b\sec^{-1}(cx)\right)\,dx = adx + \frac{1}{2}aex^2 - \frac{bex\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{2c} + bdx\sec^{-1}(cx) + \frac{1}{2}bex^2\sec^{-1}(cx) - \frac{bd\sqrt{1-\frac{1}{c^2x^2}}\,x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{-1+c^2x^2}}$$

input
```
Integrate[(d + e*x)*(a + b*ArcSec[c*x]),x]
```

output
```
a*d*x + (a*e*x^2)/2 - (b*e*x*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(2*c) + b*d*x
*ArcSec[c*x] + (b*e*x^2*ArcSec[c*x])/2 - (b*d*Sqrt[1 - 1/(c^2*x^2)]*x*ArcT
anh[(c*x)/Sqrt[-1 + c^2*x^2]])/Sqrt[-1 + c^2*x^2]
```

3.58.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {5749, 1892, 1730, 540, 25, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\downarrow 5749$$

$$\frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e} - \frac{b\int \frac{(d+ex)^2}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{2ce}$$

$$\downarrow 1892$$

$$\frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e} - \frac{b\int \frac{\left(\frac{d}{x}+e\right)^2}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{2ce}$$

$$\downarrow 1730$$

$$\frac{b\int \frac{\left(\frac{d}{x}+e\right)^2 x^2}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{2ce} + \frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e}$$

$$\downarrow 540$$

$$\frac{b\left(e^2x\left(-\sqrt{1-\frac{1}{c^2x^2}}\right) - \int -\frac{d\left(\frac{d}{x}+2e\right)x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}\right)}{2ce} + \frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e}$$

$$\downarrow 25$$

$$\frac{b\left(\int \frac{d\left(\frac{d}{x}+2e\right)x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x} - e^2x\sqrt{1-\frac{1}{c^2x^2}}\right)}{2ce} + \frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e}$$

$$\downarrow 27$$

$$\frac{b\left(d\int\frac{\left(\frac{d}{x}+2e\right)x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-e^2x\sqrt{1-\frac{1}{c^2x^2}}\right)}{2ce}+\frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e}$$

$$\downarrow 538$$

$$\frac{b\left(d\left(d\int\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}+2e\int\frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}\right)-e^2x\sqrt{1-\frac{1}{c^2x^2}}\right)}{2ce}+\frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e}$$

$$\downarrow 223$$

$$\frac{b\left(d\left(2e\int\frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}+cd\arcsin\left(\frac{1}{cx}\right)\right)-e^2x\sqrt{1-\frac{1}{c^2x^2}}\right)}{2ce}+\frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e}$$

$$\downarrow 243$$

$$\frac{b\left(d\left(e\int\frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}+cd\arcsin\left(\frac{1}{cx}\right)\right)-e^2x\sqrt{1-\frac{1}{c^2x^2}}\right)}{2ce}+\frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e}$$

$$\downarrow 73$$

$$\frac{b\left(d\left(cd\arcsin\left(\frac{1}{cx}\right)-2c^2e\int\frac{1}{c^2-c^2\sqrt{1-\frac{1}{c^2x^2}}}d\sqrt{1-\frac{1}{c^2x^2}}\right)-e^2x\sqrt{1-\frac{1}{c^2x^2}}\right)}{2ce}+\frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e}$$

$$\downarrow 221$$

$$\frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e}+\frac{b\left(d\left(cd\arcsin\left(\frac{1}{cx}\right)-2e\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)\right)-e^2x\sqrt{1-\frac{1}{c^2x^2}}\right)}{2ce}$$

input
```
Int[(d + e*x)*(a + b*ArcSec[c*x]),x]
```

output
```
((d + e*x)^2*(a + b*ArcSec[c*x]))/(2*e) + (b*(-(e^2*Sqrt[1 - 1/(c^2*x^2)]*
x) + d*(c*d*ArcSin[1/(c*x)] - 2*e*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])))/(2*c*e
)
```

3.58.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 1730
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol
] :> -Subst[Int[(d + e/x^n)^q*((a + c/x^(2*n))^p/x^2), x], x, 1/x] /; FreeQ
[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && ILtQ[n, 0]
```

rule 1892
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

rule 5749
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.58.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.31

method	result	size
parts	$a\left(\frac{1}{2}e\,x^2+dx\right)+\frac{b\left(\frac{c\,\mathrm{arcsec}(cx)x^2e}{2}+\mathrm{arcsec}(cx)xcd-\frac{\sqrt{c^2x^2-1}\left(2dc\ln\left(cx+\sqrt{c^2x^2-1}\right)+e\sqrt{c^2x^2-1}\right)}{2c^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}\right)}{c}$	110
derivativedivides	$\frac{\frac{a\left(dx\,c^2+\frac{1}{2}e\,c^2x^2\right)}{c}+\frac{b\left(\mathrm{arcsec}(cx)d\,c^2x+\frac{\mathrm{arcsec}(cx)e\,c^2x^2}{2}-\frac{\sqrt{c^2x^2-1}\left(2dc\ln\left(cx+\sqrt{c^2x^2-1}\right)+e\sqrt{c^2x^2-1}\right)}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c}}{c}$	127
default	$\frac{\frac{a\left(dx\,c^2+\frac{1}{2}e\,c^2x^2\right)}{c}+\frac{b\left(\mathrm{arcsec}(cx)d\,c^2x+\frac{\mathrm{arcsec}(cx)e\,c^2x^2}{2}-\frac{\sqrt{c^2x^2-1}\left(2dc\ln\left(cx+\sqrt{c^2x^2-1}\right)+e\sqrt{c^2x^2-1}\right)}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c}}{c}$	127

input
```
int((e*x+d)*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/2*e*x^2+d*x)+b/c*(1/2*c*arcsec(c*x)*x^2*e+arcsec(c*x)*x*c*d-1/2/c^2/(
(c^2*x^2-1)/c^2/x^2)^(1/2)/x*(c^2*x^2-1)^(1/2)*(2*d*c*ln(c*x+(c^2*x^2-1)^(
1/2))+e*(c^2*x^2-1)^(1/2)))
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.55

$$\int (d+ex)\left(a+b\operatorname{sec}^{-1}(cx)\right)\,dx$$
$$=\frac{ac^2ex^2+2\,ac^2dx+2\,bcd\log\left(-cx+\sqrt{c^2x^2-1}\right)-\sqrt{c^2x^2-1}be+\left(bc^2ex^2+2\,bc^2dx-2\,bc^2d-bc^2e\right)\arcs}{2\,c^2}$$

input
```
integrate((e*x+d)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/2*(a*c^2*e*x^2 + 2*a*c^2*d*x + 2*b*c*d*log(-c*x + sqrt(c^2*x^2 - 1)) - s
qrt(c^2*x^2 - 1)*b*e + (b*c^2*e*x^2 + 2*b*c^2*d*x - 2*b*c^2*d - b*c^2*e)*a
rcsec(c*x) + 2*(2*b*c^2*d + b*c^2*e)*arctan(-c*x + sqrt(c^2*x^2 - 1)))/c^2
```

3.58.6 Sympy [A] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.24

$$\int (d+ex)\left(a+b\operatorname{sec}^{-1}(cx)\right)\,dx = adx+\frac{aex^2}{2}+bdx\operatorname{asec}(cx)+\frac{bex^2\operatorname{asec}(cx)}{2}$$
$$-\frac{bd\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c}$$
$$-\frac{be\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c}$$

input
```
integrate((e*x+d)*(a+b*asec(c*x)),x)
```

output
```
a*d*x + a*e*x**2/2 + b*d*x*asec(c*x) + b*e*x**2*asec(c*x)/2 - b*d*Piecewis
e((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x), True))/c - b*e*Piecewis
e((sqrt(c**2*x**2 - 1)/c, Abs(c**2*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c,
True))/(2*c)
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.11

$$\begin{aligned}&\int (d+ex)\left(a+b\sec^{-1}(cx)\right)\,dx\\&=\frac{1}{2}aex^2+\frac{1}{2}\left(x^2\operatorname{arcsec}(cx)-\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)be+adx\\&\quad+\frac{\left(2\,cx\operatorname{arcsec}(cx)-\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)+\log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bd}{2\,c}\end{aligned}$$

input
```
integrate((e*x+d)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/2*a*e*x^2 + 1/2*(x^2*arcsec(c*x) - x*sqrt(-1/(c^2*x^2) + 1)/c)*b*e + a*d
*x + 1/2*(2*c*x*arcsec(c*x) - log(sqrt(-1/(c^2*x^2) + 1) + 1) + log(-sqrt(
-1/(c^2*x^2) + 1) + 1))*b*d/c
```

3.58.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1547 vs. 2(74) = 148.

Time = 0.53 (sec) , antiderivative size = 1547, normalized size of antiderivative = 18.42

$$\int (d+ex)\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
1/2*(2*b*c*d*arccos(1/(c*x))/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^
2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) - 2*b*c*d*log(abs(sqrt(-1/(c^
2*x^2) + 1) + 1/(c*x) + 1))/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) + 2*b*c*d*log(abs(sqrt(-1/(c^2
*x^2) + 1) - 1/(c*x) - 1))/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
+ c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) + 2*a*c*d/(c^3 + 2*c^3*(1/(c^2*
x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) + b*e
*arccos(1/(c*x))/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(
c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) - 4*b*c*d*(1/(c^2*x^2) - 1)*log(abs(sqrt(
-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x)
 + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) + 4*b*
c*d*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^3
 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c
*x) + 1)^4)*(1/(c*x) + 1)^2) + a*e/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x)
 + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) - 2*b*e*(1/(c^2*x^2) -
1)*arccos(1/(c*x))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(
1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) - 2*b*c*d*(1/(c^2*x^2
) - 1)^2*arccos(1/(c*x))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 +
 c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4) - 2*b*c*d*(1/(c
^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^3 + 2...
```

3.58.9 Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

$$\int (d+ex)\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{a\,x\,(2\,d+e\,x)}{2} - \frac{b\,d\,\operatorname{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{c^2\,x^2}}}\right)}{c} + b\,d\,x\,\operatorname{acos}\left(\frac{1}{c\,x}\right) - \frac{b\,e\,x\left(\sqrt{1-\frac{1}{c^2\,x^2}} - c\,x\,\operatorname{acos}\left(\frac{1}{c\,x}\right)\right)}{2\,c}$$

input
```
int((a + b*acos(1/(c*x)))*(d + e*x),x)
```

output
```
(a*x*(2*d + e*x))/2 - (b*d*atanh(1/(1 - 1/(c^2*x^2))^(1/2)))/c + b*d*x*aco
s(1/(c*x)) - (b*e*x*((1 - 1/(c^2*x^2))^(1/2) - c*x*acos(1/(c*x))))/(2*c)
```

3.59 $\int (a + b \sec^{-1}(cx))\, dx$

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3.59.1 Optimal result

Integrand size = 8, antiderivative size = 32

$$\int (a + b \sec^{-1}(cx))\, dx = ax + bx \sec^{-1}(cx) - \frac{b \operatorname{arctanh}\left(\sqrt{1 - \frac{1}{c^2 x^2}}\right)}{c}$$

output
```
a*x+b*x*arcsec(c*x)-b*arctanh((1-1/c^2/x^2)^(1/2))/c
```

3.59.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.84

$$\int (a + b \sec^{-1}(cx))\, dx = ax + bx \sec^{-1}(cx) - \frac{b\sqrt{1 - \frac{1}{c^2 x^2}}\, x \operatorname{arctanh}\left(\frac{cx}{\sqrt{-1 + c^2 x^2}}\right)}{\sqrt{-1 + c^2 x^2}}$$

input
```
Integrate[a + b*ArcSec[c*x],x]
```

output
```
a*x + b*x*ArcSec[c*x] - (b*Sqrt[1 - 1/(c^2*x^2)]*x*ArcTanh[(c*x)/Sqrt[-1 +
 c^2*x^2]])/Sqrt[-1 + c^2*x^2]
```

3.59.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a + b \sec^{-1}(cx)\right)\, dx$$

$$\downarrow \text{2009}$$

$$ax - \frac{b \operatorname{arctanh}\left(\sqrt{1 - \frac{1}{c^2 x^2}}\right)}{c} + bx \sec^{-1}(cx)$$

input
```
Int[a + b*ArcSec[c*x],x]
```

output
```
a*x + b*x*ArcSec[c*x] - (b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/c
```

3.59.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.59.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

method	result	size
default	$ax + bx \operatorname{arcsec}(cx) - \frac{b \ln\left(cx + cx\sqrt{1 - \frac{1}{c^2 x^2}}\right)}{c}$	38
parts	$ax + bx \operatorname{arcsec}(cx) - \frac{b \ln\left(cx + cx\sqrt{1 - \frac{1}{c^2 x^2}}\right)}{c}$	38
derivativedivides	$\frac{acx + b\left(cx \operatorname{arcsec}(cx) - \ln\left(cx + cx\sqrt{1 - \frac{1}{c^2 x^2}}\right)\right)}{c}$	42

input
```
int(a+b*arcsec(c*x),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*x*arcsec(c*x)-b/c*ln(c*x+c*x*(1-1/c^2/x^2)^(1/2))
```

3.59.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(30) = 60$.

Time $= 0.28$ (sec) , antiderivative size $= 63$, normalized size of antiderivative $= 1.97$

$$\int \left(a + b\sec^{-1}(cx)\right)\, dx$$
$$= \frac{acx + 2\,bc\arctan\left(-cx + \sqrt{c^2x^2 - 1}\right) + (bcx - bc)\operatorname{arcsec}(cx) + b\log\left(-cx + \sqrt{c^2x^2 - 1}\right)}{c}$$

input
```
integrate(a+b*arcsec(c*x),x, algorithm="fricas")
```

output
```
(a*c*x + 2*b*c*arctan(-c*x + sqrt(c^2*x^2 - 1)) + (b*c*x - b*c)*arcsec(c*x
) + b*log(-c*x + sqrt(c^2*x^2 - 1)))/c
```

3.59.6 Sympy [A] (verification not implemented)

Time $= 1.19$ (sec) , antiderivative size $= 32$, normalized size of antiderivative $= 1.00$

$$\int \left(a + b\sec^{-1}(cx)\right)\, dx = ax + b\left(x\operatorname{asec}(cx) - \frac{\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2| > 1 \\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}}{c}\right)$$

input
```
integrate(a+b*asec(c*x),x)
```

output
```
a*x + b*(x*asec(c*x) - Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asi
n(c*x), True))/c)
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.66

$$\int \left(a + b \sec^{-1}(cx)\right)\, dx$$
$$= ax + \frac{\left(2\, cx \operatorname{arcsec}(cx) - \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) + \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)b}{2\, c}$$

input
```
integrate(a+b*arcsec(c*x),x, algorithm="maxima")
```

output
```
a*x + 1/2*(2*c*x*arcsec(c*x) - log(sqrt(-1/(c^2*x^2) + 1) + 1) + log(-sqrt
(-1/(c^2*x^2) + 1) + 1))*b/c
```

3.59.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(30) = 60.

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.97

$$\int \left(a + b \sec^{-1}(cx)\right)\, dx$$
$$= \frac{1}{2}\, bc \left(\frac{2\, x \arccos\left(\frac{1}{cx}\right)}{c} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) - \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} \right) + ax$$

input
```
integrate(a+b*arcsec(c*x),x, algorithm="giac")
```

output
```
1/2*b*c*(2*x*arccos(1/(c*x))/c - (log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-s
qrt(-1/(c^2*x^2) + 1) + 1))/c^2) + a*x
```

3.59.9 Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \left(a + b \sec^{-1}(cx)\right)\, dx = a\, x + b\, x \operatorname{acos}\left(\frac{1}{c\, x}\right) - \frac{b \operatorname{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{c^2\, x^2}}}\right)}{c}$$

input
```
int(a + b*acos(1/(c*x)),x)
```

output
```
a*x + b*x*acos(1/(c*x)) - (b*atanh(1/(1 - 1/(c^2*x^2))^(1/2)))/c
```

3.60 $\int \frac{a+b\sec^{-1}(cx)}{d+ex}\,dx$

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3.60.1 Optimal result

Integrand size = 16, antiderivative size = 247

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex}\,dx = \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{\left(e-\sqrt{-c^2d^2+e^2}\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e} + \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{\left(e+\sqrt{-c^2d^2+e^2}\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e} - \frac{(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{\left(e-\sqrt{-c^2d^2+e^2}\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{\left(e+\sqrt{-c^2d^2+e^2}\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e} + \frac{ib\,\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2e}$$

output

```
-(a+b*arcsec(c*x))*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)/e+(a+b*arcsec(c*x
))*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(e-(-c^2*d^2+e^2)^(1/2))/c/d)/e+(a+b
*arcsec(c*x))*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(e+(-c^2*d^2+e^2)^(1/2))/
c/d)/e+1/2*I*b*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)/e-I*b*polylog(2
,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(e-(-c^2*d^2+e^2)^(1/2))/c/d)/e-I*b*polylo
g(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(e+(-c^2*d^2+e^2)^(1/2))/c/d)/e
```

3.60.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.35

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex}\,dx = \frac{a\log(d+ex)}{e} + \frac{b\left(4i\arcsin\left(\frac{\sqrt{1+\frac{e}{cd}}}{\sqrt{2}}\right)\arctan\left(\frac{(-cd+e)\tan\left(\frac{1}{2}\sec^{-1}(cx)\right)}{\sqrt{-c^2d^2+e^2}}\right)+\left(\sec^{-1}(cx)+2\arcsin\left(\frac{\sqrt{1+\frac{e}{cd}}}{\sqrt{2}}\right)\right)\log\left(1+\frac{\left(e-\sqrt{}\right.\right.\right.}{}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(d + e*x),x]
```

output
```
(a*Log[d + e*x])/e + (b*((4*I)*ArcSin[Sqrt[1 + e/(c*d)]/Sqrt[2]]*ArcTan[((
-(c*d) + e)*Tan[ArcSec[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] + (ArcSec[c*x] + 2
*ArcSin[Sqrt[1 + e/(c*d)]/Sqrt[2]])*Log[1 + ((e - Sqrt[-(c^2*d^2) + e^2])*
E^(I*ArcSec[c*x]))/(c*d)] + (ArcSec[c*x] - 2*ArcSin[Sqrt[1 + e/(c*d)]/Sqrt
[2]])*Log[1 + ((e + Sqrt[-(c^2*d^2) + e^2])*E^(I*ArcSec[c*x]))/(c*d)] - Ar
cSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] - I*(PolyLog[2, ((-e + Sqrt[-(c^2
*d^2) + e^2])*E^(I*ArcSec[c*x]))/(c*d)] + PolyLog[2, -(((e + Sqrt[-(c^2*d^
2) + e^2])*E^(I*ArcSec[c*x]))/(c*d))]) + (I/2)*PolyLog[2, -E^((2*I)*ArcSec
[c*x])]))/e
```

3.60.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5747, 2998}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex}\,dx$$
$$\downarrow 5747$$

$$-\frac{b\int\frac{\log\left(\frac{e^{i\sec^{-1}(cx)}\left(e-\sqrt{e^2-c^2d^2}\right)}{cd}+1\right)}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{ce}-\frac{b\int\frac{\log\left(\frac{e^{i\sec^{-1}(cx)}\left(e+\sqrt{e^2-c^2d^2}\right)}{cd}+1\right)}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{ce}+$$
$$\frac{b\int\frac{\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{ce}+\frac{\left(a+b\sec^{-1}(cx)\right)\log\left(1+\frac{\left(e-\sqrt{e^2-c^2d^2}\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e}+$$
$$\frac{\left(a+b\sec^{-1}(cx)\right)\log\left(1+\frac{\left(\sqrt{e^2-c^2d^2}+e\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e}-\frac{\log\left(1+e^{2i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)}{e}$$

$$\downarrow 2998$$

$$\frac{\left(a+b\sec^{-1}(cx)\right)\log\left(1+\frac{\left(e-\sqrt{e^2-c^2d^2}\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e}+$$
$$\frac{\left(a+b\sec^{-1}(cx)\right)\log\left(1+\frac{\left(\sqrt{e^2-c^2d^2}+e\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e}-\frac{\log\left(1+e^{2i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)}{e}-$$
$$\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{\left(e-\sqrt{e^2-c^2d^2}\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e}-\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{\left(e+\sqrt{e^2-c^2d^2}\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e}+$$
$$\frac{ib\,\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2e}$$

input
```
Int[(a + b*ArcSec[c*x])/(d + e*x),x]
```

output
```
((a + b*ArcSec[c*x])*Log[1 + ((e - Sqrt[-(c^2*d^2) + e^2])*E^(I*ArcSec[c*x
]))/(c*d)])/e + ((a + b*ArcSec[c*x])*Log[1 + ((e + Sqrt[-(c^2*d^2) + e^2])
*E^(I*ArcSec[c*x]))/(c*d)])/e - ((a + b*ArcSec[c*x])*Log[1 + E^((2*I)*ArcS
ec[c*x])])/e - (I*b*PolyLog[2, -(((e - Sqrt[-(c^2*d^2) + e^2])*E^(I*ArcSec
[c*x]))/(c*d))])/e - (I*b*PolyLog[2, -(((e + Sqrt[-(c^2*d^2) + e^2])*E^(I*
ArcSec[c*x]))/(c*d))])/e + ((I/2)*b*PolyLog[2, -E^((2*I)*ArcSec[c*x])])/e
```

3.60.3.1 Defintions of rubi rules used

rule 2998
```
Int[Log[v_]*(u_), x_Symbol] :> With[{w = DerivativeDivides[v, u*(1 - v), x]
}, Simp[w*PolyLog[2, 1 - v], x] /;  !FalseQ[w]]
```

rule 5747
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))/((d_.) + (e_.)*(x_)), x_Symbol] :> S
imp[(a + b*ArcSec[c*x])*(Log[1 + (e - Sqrt[(-c^2)*d^2 + e^2])*(E^(I*ArcSec[
c*x])/(c*d))]/e), x] + (Simp[(a + b*ArcSec[c*x])*(Log[1 + (e + Sqrt[(-c^2)*
d^2 + e^2])*(E^(I*ArcSec[c*x])/(c*d))]/e), x] - Simp[(a + b*ArcSec[c*x])*(L
og[1 + E^(2*I*ArcSec[c*x])]/e), x] - Simp[b/(c*e)   Int[Log[1 + (e - Sqrt[(
-c^2)*d^2 + e^2])*(E^(I*ArcSec[c*x])/(c*d))]/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x
], x] - Simp[b/(c*e)   Int[Log[1 + (e + Sqrt[(-c^2)*d^2 + e^2])*(E^(I*ArcSe
c[c*x])/(c*d))]/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] + Simp[b/(c*e)   Int[Lo
g[1 + E^(2*I*ArcSec[c*x])]/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x]) /; FreeQ[{a
, b, c, d, e}, x]
```

3.60.4 Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.83

method	result
parts	$\frac{a\ln(ex+d)}{e}+\frac{b\operatorname{arcsec}(cx)\ln\left(\frac{-cd\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)+\sqrt{-c^2d^2+e^2}-e}{-e+\sqrt{-c^2d^2+e^2}}\right)}{e}+\frac{b\operatorname{arcsec}(cx)\ln\left(\frac{cd\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)+\sqrt{-c^2}}{e+\sqrt{-c^2d^2+e^2}}\right.}{e}$
derivativedivides	$\frac{ac\ln(cex+cd)}{e}+bc\left(\frac{\operatorname{arcsec}(cx)\ln\left(\frac{-cd\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)+\sqrt{-c^2d^2+e^2}-e}{-e+\sqrt{-c^2d^2+e^2}}\right)}{e}+\frac{\operatorname{arcsec}(cx)\ln\left(\frac{cd\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)+\sqrt{-c^2d^2+e^2}}{e+\sqrt{-c^2d^2+e^2}}\right.}{e}\right.$
default	$\frac{ac\ln(cex+cd)}{e}+bc\left(\frac{\operatorname{arcsec}(cx)\ln\left(\frac{-cd\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)+\sqrt{-c^2d^2+e^2}-e}{-e+\sqrt{-c^2d^2+e^2}}\right)}{e}+\frac{\operatorname{arcsec}(cx)\ln\left(\frac{cd\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)+\sqrt{-c^2d^2+e^2}}{e+\sqrt{-c^2d^2+e^2}}\right.}{e}\right.$

input
```
int((a+b*arcsec(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
a*ln(e*x+d)/e+b/e*arcsec(c*x)*ln((-c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))+(-c^2
*d^2+e^2)^(1/2)-e)/(-e+(-c^2*d^2+e^2)^(1/2)))+b/e*arcsec(c*x)*ln((c*d*(1/c
/x+I*(1-1/c^2/x^2)^(1/2))+(-c^2*d^2+e^2)^(1/2)+e)/(e+(-c^2*d^2+e^2)^(1/2))
)-I*b/e*dilog((c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))+(-c^2*d^2+e^2)^(1/2)+e)/(
e+(-c^2*d^2+e^2)^(1/2)))-I*b/e*dilog((-c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))+(
-c^2*d^2+e^2)^(1/2)-e)/(-e+(-c^2*d^2+e^2)^(1/2)))-b/e*arcsec(c*x)*ln(1+I*(
1/c/x+I*(1-1/c^2/x^2)^(1/2)))-b/e*arcsec(c*x)*ln(1-I*(1/c/x+I*(1-1/c^2/x^2
)^(1/2)))+I*b/e*dilog(1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))+I*b/e*dilog(1-I*(
1/c/x+I*(1-1/c^2/x^2)^(1/2)))
```

3.60.5 Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex}\,dx = \int \frac{b\operatorname{arcsec}(cx)+a}{ex+d}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*arcsec(c*x) + a)/(e*x + d), x)
```

3.60.6 Sympy [F]

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex}\,dx = \int \frac{a+b\operatorname{asec}(cx)}{d+ex}\,dx$$

input
```
integrate((a+b*asec(c*x))/(e*x+d),x)
```

output
```
Integral((a + b*asec(c*x))/(d + e*x), x)
```

3.60.7 Maxima [F]

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex}\,dx = \int \frac{b\,\text{arcsec}\,(cx)+a}{ex+d}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d),x, algorithm="maxima")
```

output
```
b*integrate(arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x + d), x) + a*log(e*x
+ d)/e
```

3.60.8 Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex}\,dx = \int \frac{a+b\,\text{acos}\left(\frac{1}{c\,x}\right)}{d+e\,x}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(d + e*x),x)
```

output
```
int((a + b*acos(1/(c*x)))/(d + e*x), x)
```

3.61 $\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^2}\,dx$

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3.61.1 Optimal result

Integrand size = 16, antiderivative size = 104

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^2}\,dx = -\frac{b\csc^{-1}(cx)}{de} - \frac{a+b\sec^{-1}(cx)}{e(d+ex)} - \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{c^2d^2-e^2}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{d\sqrt{c^2d^2-e^2}}$$

output
```
-b*arccsc(c*x)/d/e+(-a-b*arcsec(c*x))/e/(e*x+d)-b*arctanh((c^2*d+e/x)/c/(c
^2*d^2-e^2)^(1/2)/(1-1/c^2/x^2)^(1/2))/d/(c^2*d^2-e^2)^(1/2)
```

3.61.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.37

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^2}\,dx = -\frac{a}{e(d+ex)} - \frac{b\sec^{-1}(cx)}{e(d+ex)} - \frac{b\arcsin\left(\frac{1}{cx}\right)}{de} - \frac{b\log(d+ex)}{d\sqrt{c^2d^2-e^2}} + \frac{b\log\left(e+c\left(cd-\sqrt{c^2d^2-e^2}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{d\sqrt{c^2d^2-e^2}}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(d + e*x)^2,x]
```

output
```
-(a/(e*(d + e*x))) - (b*ArcSec[c*x])/(e*(d + e*x)) - (b*ArcSin[1/(c*x)])/(
d*e) - (b*Log[d + e*x])/(d*Sqrt[c^2*d^2 - e^2]) + (b*Log[e + c*(c*d - Sqrt
[c^2*d^2 - e^2]*Sqrt[1 - 1/(c^2*x^2)])*x])/(d*Sqrt[c^2*d^2 - e^2])
```

3.61.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5749, 1892, 1803, 605, 223, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^2}\,dx$$

$$\downarrow 5749$$

$$\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2(d+ex)}dx}{ce} - \frac{a+b\sec^{-1}(cx)}{e(d+ex)}$$

$$\downarrow 1892$$

$$\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)x^3}dx}{ce} - \frac{a+b\sec^{-1}(cx)}{e(d+ex)}$$

$$\downarrow 1803$$

$$-\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)x}d\frac{1}{x}}{ce} - \frac{a+b\sec^{-1}(cx)}{e(d+ex)}$$

$$\downarrow 605$$

$$-\frac{b\left(\frac{\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{d} - \frac{e\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}}{d}\right)}{ce} - \frac{a+b\sec^{-1}(cx)}{e(d+ex)}$$

$$\downarrow 223$$

$$-\frac{b\left(\frac{c\arcsin\left(\frac{1}{cx}\right)}{d} - \frac{e\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}}{d}\right)}{ce} - \frac{a+b\sec^{-1}(cx)}{e(d+ex)}$$

$$\downarrow 488$$

$$-\frac{b\left(\frac{e\int \frac{1}{d^2-\frac{e^2}{c^2}-\frac{1}{x^2}}d\frac{d+\frac{e}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}}{d} + \frac{c\arcsin\left(\frac{1}{cx}\right)}{d}\right)}{ce} - \frac{a+b\sec^{-1}(cx)}{e(d+ex)}$$

$$\downarrow 219$$

$$-\frac{a+b\sec^{-1}(cx)}{e(d+ex)}-\frac{b\left(\frac{c\arcsin\left(\frac{1}{cx}\right)}{d}+\frac{ce\operatorname{arctanh}\left(\frac{c\left(\frac{e}{c^2x}+d\right)}{\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d^2-e^2}}\right)}{d\sqrt{c^2d^2-e^2}}\right)}{ce}$$

input
```
Int[(a + b*ArcSec[c*x])/(d + e*x)^2,x]
```

output
```
-((a + b*ArcSec[c*x])/(e*(d + e*x))) - (b*((c*ArcSin[1/(c*x)])/d + (c*e*Ar
cTanh[(c*(d + e/(c^2*x)))/(Sqrt[c^2*d^2 - e^2]*Sqrt[1 - 1/(c^2*x^2)])])/(d
*Sqrt[c^2*d^2 - e^2])))/(c*e)
```

3.61.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 605
```
Int[((x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol]
:> Simp[1/d   Int[x^(m - 1)*(a + b*x^2)^p, x], x] - Simp[c/d   Int[x^(m - 1
)*((a + b*x^2)^p/(c + d*x)), x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m,
 0] && LtQ[-1, p, 0]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1892
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

rule 5749
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.61.4 Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.86

method	result
parts	$-\frac{a}{(ex+d)e}+\frac{b\left(-\frac{c^2\operatorname{arcsec}(cx)}{(cex+cd)e}-\frac{\sqrt{c^2x^2-1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}-\ln\left(\frac{2\sqrt{\frac{c^2d^2-e^2}{e^2}}\sqrt{c^2x^2-1}\,e-2dx\,c^2-2e}{cex+cd}\right)\right)}{e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,xd\sqrt{\frac{c^2d^2-e^2}{e^2}}}\right)}{c}$
derivativedivides	$-\frac{a\,c^2}{(cex+cd)e}+b\,c^2\left(-\frac{\operatorname{arcsec}(cx)}{(cex+cd)e}-\frac{\sqrt{c^2x^2-1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}-\ln\left(\frac{2\sqrt{\frac{c^2d^2-e^2}{e^2}}\sqrt{c^2x^2-1}\,e-2dx\,c^2-2e}{cex+cd}\right)\right)}{e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2xd\sqrt{\frac{c^2d^2-e^2}{e^2}}}\right)$
default	$-\frac{a\,c^2}{(cex+cd)e}+b\,c^2\left(-\frac{\operatorname{arcsec}(cx)}{(cex+cd)e}-\frac{\sqrt{c^2x^2-1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}-\ln\left(\frac{2\sqrt{\frac{c^2d^2-e^2}{e^2}}\sqrt{c^2x^2-1}\,e-2dx\,c^2-2e}{cex+cd}\right)\right)}{e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2xd\sqrt{\frac{c^2d^2-e^2}{e^2}}}\right)$

input
```
int((a+b*arcsec(c*x))/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-a/(e*x+d)/e+b/c*(-c^2/(c*e*x+c*d)/e*arcsec(c*x)-1/e*(c^2*x^2-1)^(1/2)*(ar
ctan(1/(c^2*x^2-1)^(1/2))*((c^2*d^2-e^2)/e^2)^(1/2)-ln(2*(((c^2*d^2-e^2)/e
^2)^(1/2)*(c^2*x^2-1)^(1/2)*e-d*x*c^2-e)/(c*e*x+c*d)))/((c^2*x^2-1)/c^2/x^
2)^(1/2)/x/d/((c^2*d^2-e^2)/e^2)^(1/2))
```

3.61.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(98) = 196.

Time = 0.31 (sec) , antiderivative size = 477, normalized size of antiderivative = 4.59

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^2}\,dx$$

$$= \left[-\frac{ac^2d^3-ade^2-\sqrt{c^2d^2-e^2}(be^2x+bde)\log\left(\frac{c^3d^2x+cde-\sqrt{c^2d^2-e^2}(c^2dx+e)+\left(c^2d^2-\sqrt{c^2d^2-e^2}cd-e^2\right)\sqrt{c^2x^2-1}}{ex+d}\right)+}{c^2d^4e-d^2e^3+(c^2d^3e} \right.$$

$$\left. -\frac{ac^2d^3-ade^2-2\sqrt{-c^2d^2+e^2}(be^2x+bde)\arctan\left(-\frac{\sqrt{-c^2d^2+e^2}\sqrt{c^2x^2-1}e-\sqrt{-c^2d^2+e^2}(cex+cd)}{c^2d^2-e^2}\right)+(bc^2d^3-b}{c^2d^4e-d^2e^3+(c^2d^3e^2-d} \right]$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^2,x, algorithm="fricas")
```

output
```
[-(a*c^2*d^3 - a*d*e^2 - sqrt(c^2*d^2 - e^2)*(b*e^2*x + b*d*e)*log((c^3*d^
2*x + c*d*e - sqrt(c^2*d^2 - e^2)*(c^2*d*x + e) + (c^2*d^2 - sqrt(c^2*d^2
 - e^2)*c*d - e^2)*sqrt(c^2*x^2 - 1))/(e*x + d)) + (b*c^2*d^3 - b*d*e^2)*ar
csec(c*x) - 2*(b*c^2*d^3 - b*d*e^2 + (b*c^2*d^2*e - b*e^3)*x)*arctan(-c*x
 + sqrt(c^2*x^2 - 1)))/(c^2*d^4*e - d^2*e^3 + (c^2*d^3*e^2 - d*e^4)*x), -(a
*c^2*d^3 - a*d*e^2 - 2*sqrt(-c^2*d^2 + e^2)*(b*e^2*x + b*d*e)*arctan(-(sqr
t(-c^2*d^2 + e^2)*sqrt(c^2*x^2 - 1)*e - sqrt(-c^2*d^2 + e^2)*(c*e*x + c*d)
)/(c^2*d^2 - e^2)) + (b*c^2*d^3 - b*d*e^2)*arcsec(c*x) - 2*(b*c^2*d^3 - b*
d*e^2 + (b*c^2*d^2*e - b*e^3)*x)*arctan(-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^
4*e - d^2*e^3 + (c^2*d^3*e^2 - d*e^4)*x)]
```

3.61.6 Sympy [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^2}\,dx = \int \frac{a+b\operatorname{asec}(cx)}{(d+ex)^2}\,dx$$

input
```
integrate((a+b*asec(c*x))/(e*x+d)**2,x)
```

output
```
Integral((a + b*asec(c*x))/(d + e*x)**2, x)
```

3.61.7 Maxima [F]

$$\int \frac{a + b \sec^{-1}(cx)}{(d+ex)^2}\,dx = \int \frac{b \operatorname{arcsec}(cx) + a}{(ex+d)^2}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^2,x, algorithm="maxima")
```

output
```
((c^2*e^2*x + c^2*d*e)*integrate(x*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1))
/(c^2*e^2*x^3 + c^2*d*e*x^2 - e^2*x - d*e + (c^2*e^2*x^3 + c^2*d*e*x^2 - e
^2*x - d*e)*e^(log(c*x + 1) + log(c*x - 1))), x) - arctan(sqrt(c*x + 1)*sq
rt(c*x - 1)))*b/(e^2*x + d*e) - a/(e^2*x + d*e)
```

3.61.8 Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \sec^{-1}(cx)}{(d+ex)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.61.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \sec^{-1}(cx)}{(d+ex)^2}\,dx = \int \frac{a + b \operatorname{acos}\left(\frac{1}{c\,x}\right)}{(d+e\,x)^2}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(d + e*x)^2,x)
```

output
```
int((a + b*acos(1/(c*x)))/(d + e*x)^2, x)
```

3.62 $\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^3}\,dx$

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3.62.1 Optimal result

Integrand size = 16, antiderivative size = 172

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^3}\,dx = \frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{2d\left(c^2d^2-e^2\right)\left(e+\frac{d}{x}\right)} - \frac{b\csc^{-1}(cx)}{2d^2e} - \frac{a+b\sec^{-1}(cx)}{2e(d+ex)^2} - \frac{b(2c^2d^2-e^2)\operatorname{arctanh}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{c^2d^2-e^2}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2d^2\left(c^2d^2-e^2\right)^{3/2}}$$

output

```
-1/2*b*arccsc(c*x)/d^2/e+1/2*(-a-b*arcsec(c*x))/e/(e*x+d)^2-1/2*b*(2*c^2*d
^2-e^2)*arctanh((c^2*d+e/x)/c/(c^2*d^2-e^2)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^2
/(c^2*d^2-e^2)^(3/2)+1/2*b*c*e*(1-1/c^2/x^2)^(1/2)/d/(c^2*d^2-e^2)/(e+d/x)
```

3.62.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.44

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^3}\,dx = \frac{1}{2}\left(-\frac{a}{e(d+ex)^2} + \frac{bce\sqrt{1-\frac{1}{c^2x^2}}x}{d\left(c^2d^2-e^2\right)(d+ex)} - \frac{b\sec^{-1}(cx)}{e(d+ex)^2} - \frac{b\arcsin\left(\frac{1}{cx}\right)}{d^2e} + \frac{b(-2c^2d^2+e^2)\log(d+ex)}{d^2(cd-e)(cd+e)\sqrt{c^2d^2-e^2}} + \frac{b(2c^2d^2-e^2)\log\left(e+c\left(cd-\sqrt{c^2d^2-e^2}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{d^2(cd-e)(cd+e)\sqrt{c^2d^2-e^2}}\right)$$

input
```
Integrate[(a + b*ArcSec[c*x])/(d + e*x)^3,x]
```

output
```
(-(a/(e*(d + e*x)^2)) + (b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x)/(d*(c^2*d^2 - e^2)
*(d + e*x)) - (b*ArcSec[c*x])/(e*(d + e*x)^2) - (b*ArcSin[1/(c*x)])/(d^2*e
) + (b*(-2*c^2*d^2 + e^2)*Log[d + e*x])/(d^2*(c*d - e)*(c*d + e)*Sqrt[c^2*
d^2 - e^2]) + (b*(2*c^2*d^2 - e^2)*Log[e + c*(c*d - Sqrt[c^2*d^2 - e^2]*Sq
rt[1 - 1/(c^2*x^2)])*x])/(d^2*(c*d - e)*(c*d + e)*Sqrt[c^2*d^2 - e^2]))/2
```

3.62.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5749, 1892, 1803, 603, 719, 223, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^3}\,dx$$

$$\downarrow 5749$$

$$\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2(d+ex)^2}dx}{2ce} - \frac{a+b\sec^{-1}(cx)}{2e(d+ex)^2}$$

$$\downarrow 1892$$

$$\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)^2x^4}dx}{2ce} - \frac{a+b\sec^{-1}(cx)}{2e(d+ex)^2}$$

$$\downarrow 1803$$

$$-\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)^2x^2}d\frac{1}{x}}{2ce} - \frac{a+b\sec^{-1}(cx)}{2e(d+ex)^2}$$

$$\downarrow 603$$

$$-\frac{b\left(-\frac{\int \frac{e-\frac{d-\frac{e^2}{c^2d}}{x}}{\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}}{d^2-\frac{e^2}{c^2}} - \frac{e^2\sqrt{1-\frac{1}{c^2x^2}}}{d\left(d^2-\frac{e^2}{c^2}\right)\left(\frac{d}{x}+e\right)}\right)}{2ce} - \frac{a+b\sec^{-1}(cx)}{2e(d+ex)^2}$$

$$\downarrow 719$$

$$-\frac{b\left(-\frac{e\left(2-\frac{e^2}{c^2d^2}\right)\int\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}-\left(1-\frac{e^2}{c^2d^2}\right)\int\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{d^2-\frac{e^2}{c^2}}-\frac{e^2\sqrt{1-\frac{1}{c^2x^2}}}{d\left(d^2-\frac{e^2}{c^2}\right)\left(\frac{d}{x}+e\right)}\right)}{2ce}-\frac{a+b\sec^{-1}(cx)}{2e(d+ex)^2}$$

$$\downarrow 223$$

$$-\frac{b\left(-\frac{e\left(2-\frac{e^2}{c^2d^2}\right)\int\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}-c\arcsin\left(\frac{1}{cx}\right)\left(1-\frac{e^2}{c^2d^2}\right)}{d^2-\frac{e^2}{c^2}}-\frac{e^2\sqrt{1-\frac{1}{c^2x^2}}}{d\left(d^2-\frac{e^2}{c^2}\right)\left(\frac{d}{x}+e\right)}\right)}{2ce}-\frac{a+b\sec^{-1}(cx)}{2e(d+ex)^2}$$

$$\downarrow 488$$

$$-\frac{b\left(-\frac{-e\left(2-\frac{e^2}{c^2d^2}\right)\int\frac{1}{d^2-\frac{e^2}{c^2}-\frac{1}{x^2}}d\frac{d+\frac{e}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}-c\arcsin\left(\frac{1}{cx}\right)\left(1-\frac{e^2}{c^2d^2}\right)}{d^2-\frac{e^2}{c^2}}-\frac{e^2\sqrt{1-\frac{1}{c^2x^2}}}{d\left(d^2-\frac{e^2}{c^2}\right)\left(\frac{d}{x}+e\right)}\right)}{2ce}-\frac{a+b\sec^{-1}(cx)}{2e(d+ex)^2}$$

$$\downarrow 219$$

$$-\frac{a+b\sec^{-1}(cx)}{2e(d+ex)^2}-\frac{b\left(-\frac{-c\arcsin\left(\frac{1}{cx}\right)\left(1-\frac{e^2}{c^2d^2}\right)-\frac{ce\left(2-\frac{e^2}{c^2d^2}\right)\operatorname{arctanh}\left(\frac{c\left(\frac{e}{c^2x}+d\right)}{\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d^2-e^2}}\right)}{\sqrt{c^2d^2-e^2}}}{d^2-\frac{e^2}{c^2}}-\frac{e^2\sqrt{1-\frac{1}{c^2x^2}}}{d\left(d^2-\frac{e^2}{c^2}\right)\left(\frac{d}{x}+e\right)}\right)}{2ce}$$

input

```
Int[(a + b*ArcSec[c*x])/(d + e*x)^3,x]
```

output

```
-1/2*(a + b*ArcSec[c*x])/(e*(d + e*x)^2) - (b*(-((e^2*Sqrt[1 - 1/(c^2*x^2)
])/(d*(d^2 - e^2/c^2)*(e + d/x))) - (-(c*(1 - e^2/(c^2*d^2))*ArcSin[1/(c*x
)]) - (c*e*(2 - e^2/(c^2*d^2))*ArcTanh[(c*(d + e/(c^2*x)))/(Sqrt[c^2*d^2 -
 e^2]*Sqrt[1 - 1/(c^2*x^2)])])/Sqrt[c^2*d^2 - e^2])/(d^2 - e^2/c^2)))/(2*c
*e)
```

3.62.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 603
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[x^m, c + d*x, x], R = PolynomialRemainde
r[x^m, c + d*x, x]}, Simp[d*R*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n +
1)*(b*c^2 + a*d^2))), x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)
^(n + 1)*(a + b*x^2)^p*ExpandToSum[(n + 1)*(b*c^2 + a*d^2)*Qx + b*c*R*(n +
1) - b*d*R*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IGt
Q[m, 1] && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1892
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

rule 5749

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.62.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(159) = 318$.

Time = 2.07 (sec) , antiderivative size = 574, normalized size of antiderivative = 3.34

method	result
parts	$-\frac{a}{2(ex+d)^2e}+b\left(-\frac{c^3\operatorname{arcsec}(cx)}{2(cex+cd)^2e}+\frac{\sqrt{c^2x^2-1}\left(-\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}\,c^3d^2ex-\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}\,c^3d\right.}{}\right.$
derivativedivides	$-\frac{a\,c^3}{2(cex+cd)^2e}+b\,c^3\left(-\frac{\operatorname{arcsec}(cx)}{2(cex+cd)^2e}-\frac{\sqrt{c^2x^2-1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}\,c^3d^3+\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}\,c^3d^2ex\right.}{}\right.$
default	$-\frac{a\,c^3}{2(cex+cd)^2e}+b\,c^3\left(-\frac{\operatorname{arcsec}(cx)}{2(cex+cd)^2e}-\frac{\sqrt{c^2x^2-1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}\,c^3d^3+\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}\,c^3d^2ex\right.}{}\right.$

input

```
int((a+b*arcsec(c*x))/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output

```
-1/2*a/(e*x+d)^2/e+b/c*(-1/2*c^3/(c*e*x+c*d)^2/e*arcsec(c*x)+1/2/e*(c^2*x^
2-1)^(1/2)*(-arctan(1/(c^2*x^2-1)^(1/2))*((c^2*d^2-e^2)/e^2)^(1/2)*c^3*d^2
*e*x-arctan(1/(c^2*x^2-1)^(1/2))*((c^2*d^2-e^2)/e^2)^(1/2)*c^3*d^3+2*ln(2*
(((c^2*d^2-e^2)/e^2)^(1/2)*(c^2*x^2-1)^(1/2)*e-d*x*c^2-e)/(c*e*x+c*d))*c^3
*d^2*e*x+2*ln(2*(((c^2*d^2-e^2)/e^2)^(1/2)*(c^2*x^2-1)^(1/2)*e-d*x*c^2-e)/
(c*e*x+c*d))*c^3*d^3+(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*c*d*e^2+a
rctan(1/(c^2*x^2-1)^(1/2))*((c^2*d^2-e^2)/e^2)^(1/2)*e^3*c*x+arctan(1/(c^2
*x^2-1)^(1/2))*((c^2*d^2-e^2)/e^2)^(1/2)*c*d*e^2-ln(2*(((c^2*d^2-e^2)/e^2)
^(1/2)*(c^2*x^2-1)^(1/2)*e-d*x*c^2-e)/(c*e*x+c*d))*e^3*c*x-ln(2*(((c^2*d^2
-e^2)/e^2)^(1/2)*(c^2*x^2-1)^(1/2)*e-d*x*c^2-e)/(c*e*x+c*d))*c*d*e^2)/((c^
2*x^2-1)/c^2/x^2)^(1/2)/x/d^2/(c^2*d^2-e^2)/(c*e*x+c*d)/((c^2*d^2-e^2)/e^2
)^(1/2))
```

3.62.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(156) = 312$.

Time = 0.55 (sec) , antiderivative size = 1117, normalized size of antiderivative = 6.49

$$\int \frac{a + b\sec^{-1}(cx)}{(d+ex)^3}\,dx$$

$$= \left[-\frac{ac^4d^6 - bc^3d^5e - 2\,ac^2d^4e^2 + bcd^3e^3 + ad^2e^4 - (bc^3d^3e^3 - bcde^5)x^2 - (2\,bc^2d^4e - bd^2e^3 + (2\,bc^2d^2e^3 -}{} \right.$$

$$\left. -\frac{ac^4d^6 - bc^3d^5e - 2\,ac^2d^4e^2 + bcd^3e^3 + ad^2e^4 - (bc^3d^3e^3 - bcde^5)x^2 - 2\,(2\,bc^2d^4e - bd^2e^3 + (2\,bc^2d^2e^3 -}{} \right.$$

input

```
integrate((a+b*arcsec(c*x))/(e*x+d)^3,x, algorithm="fricas")
```

output

```
[-1/2*(a*c^4*d^6 - b*c^3*d^5*e - 2*a*c^2*d^4*e^2 + b*c*d^3*e^3 + a*d^2*e^4
 - (b*c^3*d^3*e^3 - b*c*d*e^5)*x^2 - (2*b*c^2*d^4*e - b*d^2*e^3 + (2*b*c^2
*d^2*e^3 - b*e^5)*x^2 + 2*(2*b*c^2*d^3*e^2 - b*d*e^4)*x)*sqrt(c^2*d^2 - e^
2)*log((c^3*d^2*x + c*d*e - sqrt(c^2*d^2 - e^2)*(c^2*d*x + e) + (c^2*d^2 -
 sqrt(c^2*d^2 - e^2)*c*d - e^2)*sqrt(c^2*x^2 - 1))/(e*x + d)) - 2*(b*c^3*d
^4*e^2 - b*c*d^2*e^4)*x + (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4)*arcsec
(c*x) - 2*(b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2 - 2*b*
c^2*d^2*e^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e - 2*b*c^2*d^3*e^3 + b*d*e^5)*x)*
arctan(-c*x + sqrt(c^2*x^2 - 1)) - (b*c^2*d^4*e^2 - b*d^2*e^4 + (b*c^2*d^3
*e^3 - b*d*e^5)*x)*sqrt(c^2*x^2 - 1))/(c^4*d^8*e - 2*c^2*d^6*e^3 + d^4*e^5
 + (c^4*d^6*e^3 - 2*c^2*d^4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 - 2*c^2*d^
5*e^4 + d^3*e^6)*x), -1/2*(a*c^4*d^6 - b*c^3*d^5*e - 2*a*c^2*d^4*e^2 + b*c
*d^3*e^3 + a*d^2*e^4 - (b*c^3*d^3*e^3 - b*c*d*e^5)*x^2 - 2*(2*b*c^2*d^4*e
- b*d^2*e^3 + (2*b*c^2*d^2*e^3 - b*e^5)*x^2 + 2*(2*b*c^2*d^3*e^2 - b*d*e^4
)*x)*sqrt(-c^2*d^2 + e^2)*arctan(-(sqrt(-c^2*d^2 + e^2)*sqrt(c^2*x^2 - 1)*
e - sqrt(-c^2*d^2 + e^2)*(c*e*x + c*d))/(c^2*d^2 - e^2)) - 2*(b*c^3*d^4*e^
2 - b*c*d^2*e^4)*x + (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4)*arcsec(c*x)
 - 2*(b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2 - 2*b*c^2*d
^2*e^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e - 2*b*c^2*d^3*e^3 + b*d*e^5)*x)*arcta
n(-c*x + sqrt(c^2*x^2 - 1)) - (b*c^2*d^4*e^2 - b*d^2*e^4 + (b*c^2*d^3*e...
```

3.62.6 Sympy [F]

$$\int \frac{a + b \sec^{-1}(cx)}{(d+ex)^3}\,dx = \int \frac{a + b \operatorname{asec}(cx)}{(d+ex)^3}\,dx$$

input

```
integrate((a+b*asec(c*x))/(e*x+d)**3,x)
```

output

```
Integral((a + b*asec(c*x))/(d + e*x)**3, x)
```

3.62.7 Maxima [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^3}\,dx = \int \frac{b\operatorname{arcsec}(cx)+a}{(ex+d)^3}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(c^2*e^3*x^2 + 2*c^2*d*e^2*x + c^2*d^2*e)*integrate(1/2*x*e^(1/2*lo
g(c*x + 1) + 1/2*log(c*x - 1))/(c^2*e^3*x^4 + 2*c^2*d*e^2*x^3 - 2*d*e^2*x
- d^2*e + (c^2*d^2*e - e^3)*x^2 + (c^2*e^3*x^4 + 2*c^2*d*e^2*x^3 - 2*d*e^2
*x - d^2*e + (c^2*d^2*e - e^3)*x^2)*e^(log(c*x + 1) + log(c*x - 1))), x) -
 arctan(sqrt(c*x + 1)*sqrt(c*x - 1)))*b/(e^3*x^2 + 2*d*e^2*x + d^2*e) - 1/
2*a/(e^3*x^2 + 2*d*e^2*x + d^2*e)
```

3.62.8 Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^3}\,dx = \int \frac{a+b\operatorname{acos}\left(\frac{1}{cx}\right)}{(d+e\,x)^3}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(d + e*x)^3,x)
```

output
```
int((a + b*acos(1/(c*x)))/(d + e*x)^3, x)
```

3.63 $\int (d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx$

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3.63.1 Optimal result

Integrand size = 18, antiderivative size = 372

$$\int (d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{4be\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e} + \frac{28bd\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} + \frac{4b(2c^2d^2+e^2)\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^4\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{4bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5ce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

output

```
2/5*(e*x+d)^(5/2)*(a+b*arcsec(c*x))/e+4/15*b*e*(-c^2*x^2+1)*(e*x+d)^(1/2)/
c^3/x/(1-1/c^2/x^2)^(1/2)+28/15*b*d*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2
^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/x/(1-1/c^2/
x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)+4/15*b*(2*c^2*d^2+e^2)*EllipticF(1/2*
(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2
)*(-c^2*x^2+1)^(1/2)/c^4/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/5*b*d^3*Ell
ipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)
/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

3.63.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.35 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.90

$$\int (d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{1}{15}\left(-\frac{4be\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}{c}\right.$$
$$+\frac{6a(d+ex)^{5/2}}{e}+\frac{6b(d+ex)^{5/2}\sec^{-1}(cx)}{e}$$
$$+\frac{4ib\sqrt{\frac{e(1+cx)}{-cd+e}}\sqrt{\frac{e-cex}{cd+e}}\left(-7cd(cd-e)E\left(i\text{arcsinh}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)|\frac{cd+e}{cd-e}\right)+(9c^2d^2-7cde+e^2)\,\text{EllipticF}\right.}{c^3e\sqrt{-\frac{c}{cd+e}}\sqrt{1}}$$

input
```
Integrate[(d + e*x)^(3/2)*(a + b*ArcSec[c*x]),x]
```

output
```
((-4*b*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])/c + (6*a*(d + e*x)^(5/2))/
e + (6*b*(d + e*x)^(5/2)*ArcSec[c*x])/e + ((4*I)*b*Sqrt[(e*(1 + c*x))/(-(c
*d) + e)]*Sqrt[(e - c*e*x)/(c*d + e)]*(-7*c*d*(c*d - e)*EllipticE[I*ArcSin
h[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] + (9*c^2*d^2 -
 7*c*d*e + e^2)*EllipticF[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (
c*d + e)/(c*d - e)] - 3*c^2*d^2*EllipticPi[1 + e/(c*d), I*ArcSinh[Sqrt[-(c
/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)]))/(c^3*e*Sqrt[-(c/(c*d +
 e))]*Sqrt[1 - 1/(c^2*x^2)]*x))/15
```

3.63.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.06, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.944$, Rules used = {5749, 1898, 634, 633, 632, 186, 413, 412, 2185, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$\downarrow 5749$$

$$\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}-\frac{2b\int\frac{(d+ex)^{5/2}}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{5ce}$$

$$\downarrow 1898$$

$$\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\int\frac{(d+ex)^{5/2}}{x\sqrt{x^2-\frac{1}{c^2}}}dx}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\downarrow 634$$

$$\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(d^3\int\frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx-\int\frac{-x^2e^3-3dxe^2-3d^2e}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\downarrow 633$$

$$\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{d^3\sqrt{1-c^2x^2}\int\frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{\sqrt{x^2-\frac{1}{c^2}}}-\int\frac{-x^2e^3-3dxe^2-3d^2e}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\downarrow 632$$

$$\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{d^3\sqrt{1-c^2x^2}\int\frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\sqrt{d+ex}}dx}{\sqrt{x^2-\frac{1}{c^2}}}-\int\frac{-x^2e^3-3dxe^2-3d^2e}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\downarrow 186$$

$$\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2d^3\sqrt{1-c^2x^2}\int\frac{1}{cx\sqrt{cx+1}\sqrt{d+\frac{e}{c}-\frac{e(1-cx)}{c}}}d\sqrt{1-cx}}{\sqrt{x^2-\frac{1}{c^2}}}-\int\frac{-x^2e^3-3dxe^2-3d^2e}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\downarrow 413$$

$$\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e} -$$
$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2d^3\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\int\frac{1}{cx\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}}d\sqrt{1-cx}}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\int\frac{-x^2e^3-3dxe^2-3d^2e}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\downarrow 412$$

$$\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e} -$$
$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\int\frac{-x^2e^3-3dxe^2-3d^2e}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{2d^3\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\downarrow 2185$$

$$\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e} -$$
$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2\int-\frac{e^3\left(9d^2c^2+7dexc^2+e^2\right)}{2c^2\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{3e^2}-\frac{2d^3\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}+\frac{2}{3}e^2\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+\right.}$$
$$5cex\sqrt{1-\frac{1}{c^2x^2}}$$

$$\downarrow 27$$

$$\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e} -$$
$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{e\int\frac{9d^2c^2+7dexc^2+e^2}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{3c^2}-\frac{2d^3\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}+\frac{2}{3}e^2\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\downarrow 600$$

$$\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e} -$$
$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{e\left(\left(2c^2d^2+e^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx+7c^2d\int\frac{\sqrt{d+ex}}{\sqrt{x^2-\frac{1}{c^2}}}dx\right)}{3c^2}-\frac{2d^3\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}+\right.$$
$$5cex\sqrt{1-\frac{1}{c^2x^2}}$$

$$\downarrow 509$$

$$\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e} -$$

$$2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{e\left(\left(2c^2d^2+e^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx+\frac{7c^2d\sqrt{1-c^2x^2}\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx}{\sqrt{x^2-\frac{1}{c^2}}}\right)}{3c^2}-\frac{2d^3\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\right.}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right)$$

$$5cex\sqrt{1-\frac{1}{c^2x^2}}$$

↓ 508

$$\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e} -$$

$$2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{e\left(\left(2c^2d^2+e^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{14cd\sqrt{1-c^2x^2}\sqrt{d+ex}\int\frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3c^2}-\frac{2d^3\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,a\right.}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+}}\right)$$

$$5cex\sqrt{1-\frac{1}{c^2x^2}}$$

↓ 327

$$\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e} -$$

$$2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{e\left(\left(2c^2d^2+e^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{14cd\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3c^2}-\frac{2d^3\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(\right.}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}}}\right)$$

$$5cex\sqrt{1-\frac{1}{c^2x^2}}$$

↓ 512

$$\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e} -$$

$$2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{e\left(\frac{\sqrt{1-c^2x^2}\left(2c^2d^2+e^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{\sqrt{x^2-\frac{1}{c^2}}}-\frac{14cd\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3c^2}-\frac{2d^3\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{Ell}}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-}}\right)$$

$$5cex\sqrt{1-\frac{1}{c^2x^2}}$$

↓ 511

$$\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e} -$$

$$2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{e\left(-\frac{2\sqrt{1-c^2x^2}\left(2c^2d^2+e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}\int\frac{1}{\sqrt{1-\frac{e(1-cx)}{cd+e}}\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}}-\frac{14cd\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3c^2}-\frac{2c}{}\right)$$

$$5cex\sqrt{1-\frac{1}{c^2x^2}}$$

$$\downarrow 321$$

$$\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e} -$$

$$2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2d^3\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}+\frac{e\left(-\frac{2\sqrt{1-c^2x^2}\left(2c^2d^2+e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-c}}{\sqrt{2}}\right)\right.\right.}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}}\right.$$

$$5cex\sqrt{1-\frac{1}{c^2x^2}}$$

input
```
Int[(d + e*x)^(3/2)*(a + b*ArcSec[c*x]),x]
```

output
```
(2*(d + e*x)^(5/2)*(a + b*ArcSec[c*x]))/(5*e) - (2*b*Sqrt[-c^(-2) + x^2]*(
(2*e^2*Sqrt[d + e*x]*Sqrt[-c^(-2) + x^2])/3 + (e*((-14*c*d*Sqrt[d + e*x]*S
qrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]
)/(Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[-c^(-2) + x^2]) - (2*(2*c^2*d^2 + e^
2)*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1
 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*Sqrt[d + e*x]*Sqrt[-c^(-2) + x^2]))
)/(3*c^2) - (2*d^3*Sqrt[1 - c^2*x^2]*Sqrt[1 - (e*(1 - c*x))/(c*d + e)]*Ell
ipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[-c^(-2)
+ x^2]*Sqrt[d + e/c - (e*(1 - c*x))/c])))/(5*c*e*Sqrt[1 - 1/(c^2*x^2)]*x)
```

3.63.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 634
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] - Int[(
1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]))*ExpandToSum[(c^(n + 1/2) - (c + d*x)^(n
+ 1/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n - 1/2, 0]
```

rule 1898

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p])   Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n
))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !I
ntegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

rule 2185

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

rule 5749

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.63.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 797 vs. $2(335) = 670$.

Time = 9.08 (sec) , antiderivative size = 798, normalized size of antiderivative = 2.15

method	result
derivativedivides	$\frac{2a(ex+d)^{\frac{5}{2}}}{5}+2b\left(\frac{(ex+d)^{\frac{5}{2}}\operatorname{arcsec}(cx)}{5}-\frac{2\left(\sqrt{\frac{c}{cd-e}}\,c^2(ex+d)^{\frac{5}{2}}+9d^2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\,\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd}}\right.\right.\right.$
default	$\frac{2a(ex+d)^{\frac{5}{2}}}{5}+2b\left(\frac{(ex+d)^{\frac{5}{2}}\operatorname{arcsec}(cx)}{5}-\frac{2\left(\sqrt{\frac{c}{cd-e}}\,c^2(ex+d)^{\frac{5}{2}}+9d^2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\,\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd}}\right.\right.\right.$
parts	$\frac{2a(ex+d)^{\frac{5}{2}}}{5e}+2b\left(\frac{(ex+d)^{\frac{5}{2}}\operatorname{arcsec}(cx)}{5}-\frac{2\left(\sqrt{\frac{c}{cd-e}}\,c^2(ex+d)^{\frac{5}{2}}-2\sqrt{\frac{c}{cd-e}}\,c^2d(ex+d)^{\frac{3}{2}}+9d^2\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\sqrt{-\frac{c(ex+d)-c}{cd+e}}\right.\right.\right.$

input
```
int((e*x+d)^(3/2)*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
2/e*(1/5*a*(e*x+d)^(5/2)+b*(1/5*(e*x+d)^(5/2)*arcsec(c*x)-2/15/c^3*((c/(c*
d-e))^(1/2)*c^2*(e*x+d)^(5/2)+9*d^2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-
c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),
((c*d-e)/(c*d+e))^(1/2))*c^2-7*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*
x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d
-e)/(c*d+e))^(1/2))*c^2*d^2-3*d^2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*
(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1
/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^2-2*(c/(c*d-e))^(1/2)*
c^2*d*(e*x+d)^(3/2)+7*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+
e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+
e))^(1/2))*c*d*e-7*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/
(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))
^(1/2))*c*d*e+(c/(c*d-e))^(1/2)*c^2*d^2*(e*x+d)^(1/2)+((-c*(e*x+d)+c*d-e)/
(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*
(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e^2-(c/(c*d-e))^(1/2)*e^2*(e*x+
d)^(1/2))/(c/(c*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)
/c^2/e^2/x^2)^(1/2)))
```

3.63.5 Fricas [F(-1)]

Timed out.

$$\int (d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
Timed out
```

3.63.6 Sympy [F]

$$\int (d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int (a+b\,\text{asec}\,(cx))\,(d+ex)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x+d)**(3/2)*(a+b*asec(c*x)),x)
```

output
```
Integral((a + b*asec(c*x))*(d + e*x)**(3/2), x)
```

3.63.7 Maxima [F(-2)]

Exception generated.

$$\int (d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

3.63.8 Giac [F]

$$\int (d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int (ex+d)^{\frac{3}{2}}(b\operatorname{arcsec}(cx)+a)\,dx$$

input
```
integrate((e*x+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x + d)^(3/2)*(b*arcsec(c*x) + a), x)
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int \left(a+b\operatorname{acos}\left(\frac{1}{c\,x}\right)\right)\,(d+e\,x)^{3/2}\,dx$$

input
```
int((a + b*acos(1/(c*x)))*(d + e*x)^(3/2),x)
```

output
```
int((a + b*acos(1/(c*x)))*(d + e*x)^(3/2), x)
```

3.64 $\int \sqrt{d+ex}(a+b\sec^{-1}(cx))\,dx$

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3.64.1 Optimal result

Integrand size = 18, antiderivative size = 315

$$\int \sqrt{d+ex}(a+b\sec^{-1}(cx))\,dx$$
$$= \frac{2(d+ex)^{3/2}(a+b\sec^{-1}(cx))}{3e} + \frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$+ \frac{4bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$+ \frac{4bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3ce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

output

```
2/3*(e*x+d)^(3/2)*(a+b*arcsec(c*x))/e+4/3*b*EllipticE(1/2*(-c*x+1)^(1/2)*2
^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/x/(
1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)+4/3*b*d*EllipticF(1/2*(-c*x+1
)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2
*x^2+1)^(1/2)/c^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/3*b*d^2*EllipticPi
(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e
))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

3.64.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.34 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.88

$$\int \sqrt{d+ex}\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$= \frac{2\left(a(d+ex)^{3/2}+b(d+ex)^{3/2}\sec^{-1}(cx)+\frac{2ib\sqrt{\frac{e(1+cx)}{-cd+e}}\sqrt{\frac{e-cex}{cd+e}}\left((-cd+e)E\left(i\text{arcsinh}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)|\frac{cd+e}{cd-e}\right)+(2cd-e)\text{E}\right.\right.}{3e}$$

input
```
Integrate[Sqrt[d + e*x]*(a + b*ArcSec[c*x]),x]
```

output
```
(2*(a*(d + e*x)^(3/2) + b*(d + e*x)^(3/2)*ArcSec[c*x] + ((2*I)*b*Sqrt[(e*(
1 + c*x))/(-(c*d) + e)]*Sqrt[(e - c*e*x)/(c*d + e)]*((-(c*d) + e)*Elliptic
E[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] + (2
*c*d - e)*EllipticF[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d +
e)/(c*d - e)] - c*d*EllipticPi[1 + e/(c*d), I*ArcSinh[Sqrt[-(c/(c*d + e))]
*Sqrt[d + e*x]], (c*d + e)/(c*d - e)]))/(c^2*Sqrt[-(c/(c*d + e))]*Sqrt[1 -
 1/(c^2*x^2)]*x)))/(3*e)
```

3.64.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.11, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5749, 1898, 634, 600, 509, 508, 327, 512, 511, 321, 633, 632, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex}\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\downarrow 5749$$

$$\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{2b\int\frac{(d+ex)^{3/2}}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{3ce}$$

$$\downarrow 1898$$

$$\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\int\frac{(d+ex)^{3/2}}{x\sqrt{x^2-\frac{1}{c^2}}}dx}{3cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\downarrow 634$$

$$\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(d^2\int\frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx-\int\frac{-xe^2-2de}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\downarrow 600$$

$$\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(d^2\int\frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx+de\int\frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx+e\int\frac{\sqrt{d+ex}}{\sqrt{x^2-\frac{1}{c^2}}}dx\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\downarrow 509$$

$$\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(d^2\int\frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx+de\int\frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx+\frac{e\sqrt{1-c^2x^2}\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx}{\sqrt{x^2-\frac{1}{c^2}}}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\downarrow 508$$

$$\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(d^2\int\frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx+de\int\frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{2e\sqrt{1-c^2x^2}\sqrt{d+ex}\int\frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\downarrow 327$$

$$\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(d^2\int\frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx+de\int\frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{2e\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\downarrow 512$$

$$\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e} - 2b\sqrt{x^2-\frac{1}{c^2}}\left(d^2\int\frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx + \frac{de\sqrt{1-c^2x^2}\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{\sqrt{x^2-\frac{1}{c^2}}} - \frac{2e\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}\right) \Big/ \left(3cex\sqrt{1-\frac{1}{c^2x^2}}\right)$$

$$\downarrow 511$$

$$\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e} - 2b\sqrt{x^2-\frac{1}{c^2}}\left(d^2\int\frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx - \frac{2de\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\int\frac{1}{\sqrt{1-\frac{e(1-cx)}{cd+e}}\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}} - \frac{2e\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}\right) \Big/ \left(3cex\sqrt{1-\frac{1}{c^2x^2}}\right)$$

$$\downarrow 321$$

$$\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e} - 2b\sqrt{x^2-\frac{1}{c^2}}\left(d^2\int\frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx - \frac{2de\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}} - \frac{2e\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}\right) \Big/ \left(3cex\sqrt{1-\frac{1}{c^2x^2}}\right)$$

$$\downarrow 633$$

$$\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e} - 2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{d^2\sqrt{1-c^2x^2}\int\frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{\sqrt{x^2-\frac{1}{c^2}}} - \frac{2de\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}} - \frac{2e\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\right.}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(}{c}}}\right) \Big/ \left(3cex\sqrt{1-\frac{1}{c^2x^2}}\right)$$

$$\downarrow 632$$

$$\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e} - 2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{d^2\sqrt{1-c^2x^2}\int\frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\sqrt{d+ex}}dx}{\sqrt{x^2-\frac{1}{c^2}}} - \frac{2de\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}} - \frac{2e\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\right.}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{}}\right) \Big/ \left(3cex\sqrt{1-\frac{1}{c^2x^2}}\right)$$

$$\downarrow 186$$

$$\frac{\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e} - 2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2d^2\sqrt{1-c^2x^2}\int\frac{1}{cx\sqrt{cx+1}\sqrt{d+\frac{e}{c}-\frac{e(1-cx)}{c}}}d\sqrt{1-cx}}{\sqrt{x^2-\frac{1}{c^2}}} - \frac{2de\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}} - \frac{2e\sqrt{1-c^2x}}{}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\downarrow 413$$

$$\frac{\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e} - 2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2d^2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\int\frac{1}{cx\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}}d\sqrt{1-cx}}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}} - \frac{2de\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}} - \frac{2}{}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\downarrow 412$$

$$\frac{\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e} - 2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2d^2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}} - \frac{2de\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}$$

input
```
Int[Sqrt[d + e*x]*(a + b*ArcSec[c*x]),x]
```

output
```
(2*(d + e*x)^(3/2)*(a + b*ArcSec[c*x]))/(3*e) - (2*b*Sqrt[-c^(-2) + x^2]*(
(-2*e*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[
2]], (2*e)/(c*d + e)])/(c*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[-c^(-2) + x^2
]) - (2*d*e*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcS
in[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*Sqrt[d + e*x]*Sqrt[-c^(-2)
 + x^2]) - (2*d^2*Sqrt[1 - c^2*x^2]*Sqrt[1 - (e*(1 - c*x))/(c*d + e)]*Elli
pticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[-c^(-2) +
 x^2]*Sqrt[d + e/c - (e*(1 - c*x))/c]))/(3*c*e*Sqrt[1 - 1/(c^2*x^2)]*x)
```

3.64.3.1 Defintions of rubi rules used

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 634
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] - Int[(
1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]))*ExpandToSum[(c^(n + 1/2) - (c + d*x)^(n
+ 1/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n - 1/2, 0]
```

rule 1898
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p])   Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n
))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !I
ntegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

rule 5749

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.64.4 Maple [A] (verified)

Time = 7.74 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{2(ex+d)^{\frac{3}{2}}a}{3}+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arcsec}(cx)}{3}-\frac{2\left(2d\,\text{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c-\text{EllipticE}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)cd-d\,\text{Ellip}\right.}{}\right.$
default	$\frac{2(ex+d)^{\frac{3}{2}}a}{3}+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arcsec}(cx)}{3}-\frac{2\left(2d\,\text{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c-\text{EllipticE}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)cd-d\,\text{Ellip}\right.}{}\right.$
parts	$\frac{2a(ex+d)^{\frac{3}{2}}}{3e}+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arcsec}(cx)}{3}-\frac{2\left(2d\,\text{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c-\text{EllipticE}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)cd-d\,\text{E}\right.}{}\right.$

input

```
int((e*x+d)^(1/2)*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output

```
2/e*(1/3*(e*x+d)^(3/2)*a+b*(1/3*(e*x+d)^(3/2)*arcsec(c*x)-2/3/c^2*(2*d*Ell
ipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c-Elliptic
E((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d-d*EllipticP
i((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-
e))^(1/2))*c+EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(
1/2))*e-EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))
*e)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)/
(c/(c*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x
^2)^(1/2)))
```

3.64.5 Fricas [F]

$$\int \sqrt{d+ex}\left(a+b\sec^{-1}(cx)\right)\,dx = \int \sqrt{ex+d}(b\,\text{arcsec}\,(cx)+a)\,dx$$

input
```
integrate((e*x+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(b*arcsec(c*x) + a), x)
```

3.64.6 Sympy [F]

$$\int \sqrt{d+ex}\left(a+b\sec^{-1}(cx)\right)\,dx = \int (a+b\,\text{asec}\,(cx))\sqrt{d+ex}\,dx$$

input
```
integrate((e*x+d)**(1/2)*(a+b*asec(c*x)),x)
```

output
```
Integral((a + b*asec(c*x))*sqrt(d + e*x), x)
```

3.64.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex}\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

3.64.8 Giac [F]

$$\int \sqrt{d+ex}\left(a+b\sec^{-1}(cx)\right)\,dx = \int \sqrt{ex+d}(b\,\text{arcsec}\,(cx)+a)\,dx$$

input
```
integrate((e*x+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)*(b*arcsec(c*x) + a), x)
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex}\left(a+b\sec^{-1}(cx)\right)\,dx = \int \left(a+b\,\text{acos}\left(\frac{1}{c\,x}\right)\right)\,\sqrt{d+e\,x}\,dx$$

input
```
int((a + b*acos(1/(c*x)))*(d + e*x)^(1/2),x)
```

output
```
int((a + b*acos(1/(c*x)))*(d + e*x)^(1/2), x)
```

3.65 $\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex}}\,dx$

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3.65.1 Optimal result

Integrand size = 18, antiderivative size = 212

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{d+ex}(a+b\sec^{-1}(cx))}{e} + \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{4bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{ce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

output

```
2*(a+b*arcsec(c*x))*(e*x+d)^(1/2)/e+4*b*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/
2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)
/c^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4*b*d*EllipticPi(1/2*(-c*x+1)^(1/
2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^
2+1)^(1/2)/c/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

3.65.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 16.11 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex}}\,dx$$
$$= \frac{2\left(a\sqrt{d+ex}+b\sqrt{d+ex}\,\sec^{-1}(cx)+\frac{2ib\sqrt{\frac{e(1+cx)}{-cd+e}}\sqrt{\frac{e-cex}{cd+e}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right),\frac{cd+e}{cd-e}\right)-\text{EllipticPi}\left(1+\frac{e}{cd},i\right.\right.\right.}{c\sqrt{-\frac{c}{cd+e}}\sqrt{1-\frac{1}{c^2x^2}}x}$$
$$e$$

input
```
Integrate[(a + b*ArcSec[c*x])/Sqrt[d + e*x],x]
```

output
```
(2*(a*Sqrt[d + e*x] + b*Sqrt[d + e*x]*ArcSec[c*x] + ((2*I)*b*Sqrt[(e*(1 +
c*x))/(-(c*d) + e)]*Sqrt[(e - c*e*x)/(c*d + e)]*(EllipticF[I*ArcSinh[Sqrt[
-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] - EllipticPi[1 + e/(c
*d), I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)]))
/(c*Sqrt[-(c/(c*d + e))]*Sqrt[1 - 1/(c^2*x^2)]*x)))/e
```

3.65.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5749, 1898, 637, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex}}\,dx$$
$$\downarrow 5749$$
$$\frac{2\sqrt{d+ex}\left(a+b\sec^{-1}(cx)\right)}{e}-\frac{2b\int\frac{\sqrt{d+ex}}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{ce}$$
$$\downarrow 1898$$
$$\frac{2\sqrt{d+ex}\left(a+b\sec^{-1}(cx)\right)}{e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\int\frac{\sqrt{d+ex}}{x\sqrt{x^2-\frac{1}{c^2}}}dx}{cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\downarrow 637$$

$$\frac{2\sqrt{d+ex}\left(a+b\sec^{-1}(cx)\right)}{e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\int\left(\frac{d}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}+\frac{e}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}\right)dx}{cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\downarrow 2009$$

$$\frac{2\sqrt{d+ex}\left(a+b\sec^{-1}(cx)\right)}{e}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2e\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}}-\frac{2d\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}}\right)}{cex\sqrt{1-\frac{1}{c^2x^2}}}$$

input
```
Int[(a + b*ArcSec[c*x])/Sqrt[d + e*x],x]
```

output
```
(2*Sqrt[d + e*x]*(a + b*ArcSec[c*x]))/e - (2*b*Sqrt[-c^(-2) + x^2]*((-2*e*
Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 -
c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*Sqrt[d + e*x]*Sqrt[-c^(-2) + x^2]) - (
2*d*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[S
qrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[d + e*x]*Sqrt[-c^(-2) + x^2
]))/(c*e*Sqrt[1 - 1/(c^2*x^2)]*x)
```

3.65.3.1 Defintions of rubi rules used

rule 637
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p/Sqrt[c + d*x], x^m*(c + d*x)^(n + 1
/2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[p + 1/2] && IntegerQ[n
 + 1/2] && IntegerQ[m]
```

rule 1898
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p])   Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n
))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !I
ntegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5749
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.65.4 Maple [A] (verified)

Time = 5.21 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{2\sqrt{ex+d}\,a+2b\left(\sqrt{ex+d}\,\operatorname{arcsec}(cx)-\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\left(\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)-\operatorname{EllipticPi}\right.\right.}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,x\sqrt{\frac{c}{cd-e}}}\right)}{e}$
default	$\frac{2\sqrt{ex+d}\,a+2b\left(\sqrt{ex+d}\,\operatorname{arcsec}(cx)-\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\left(\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)-\operatorname{EllipticPi}\right.\right.}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,x\sqrt{\frac{c}{cd-e}}}\right)}{e}$
parts	$\frac{2a\sqrt{ex+d}}{e}+\frac{2b\left(\sqrt{ex+d}\,\operatorname{arcsec}(cx)-\frac{2\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\left(\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)-\operatorname{EllipticI}\right.\right.}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,x\sqrt{\frac{c}{cd-e}}}\right)}{e}$

input
```
int((a+b*arcsec(c*x))/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/e*((e*x+d)^(1/2)*a+b*((e*x+d)^(1/2)*arcsec(c*x)-2/c*((-c*(e*x+d)+c*d-e)/
(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*(EllipticF((e*x+d)^(1/2)
*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))-EllipticPi((e*x+d)^(1/2)*(c/(c
*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2)))/((c^2*(e*
x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)/x/(c/(c*d-e))^(1/2)
))
```

3.65.5 Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex}}\,dx = \int \frac{b\,\text{arcsec}\,(cx)+a}{\sqrt{ex+d}}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*arcsec(c*x) + a)/sqrt(e*x + d), x)
```

3.65.6 Sympy [F]

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex}}\,dx = \int \frac{a+b\,\text{asec}\,(cx)}{\sqrt{d+ex}}\,dx$$

input
```
integrate((a+b*asec(c*x))/(e*x+d)**(1/2),x)
```

output
```
Integral((a + b*asec(c*x))/sqrt(d + e*x), x)
```

3.65.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

3.65.8 Giac [F]

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex}}\,dx = \int \frac{b\,\text{arcsec}\,(cx)+a}{\sqrt{ex+d}}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/sqrt(e*x + d), x)
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex}}\,dx = \int \frac{a+b\,\text{acos}\left(\frac{1}{cx}\right)}{\sqrt{d+e\,x}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(d + e*x)^(1/2),x)
```

output
```
int((a + b*acos(1/(c*x)))/(d + e*x)^(1/2), x)
```

3.66 $\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{3/2}}\,dx$

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3.66.1 Optimal result

Integrand size = 18, antiderivative size = 119

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{3/2}}\,dx = -\frac{2(a+b\sec^{-1}(cx))}{e\sqrt{d+ex}} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{ce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

output
```
-2*(a+b*arcsec(c*x))/e/(e*x+d)^(1/2)-4*b*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(
1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(
1/2)/c/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

3.66.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.04

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{3/2}}\,dx = -\frac{2\left(\left(-1+c^2x^2\right)\left(a+b\sec^{-1}(cx)\right)+2bc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)\right.}{e\sqrt{d+ex}\left(-1+c^2x^2\right)}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(d + e*x)^(3/2),x]
```

output
```
(-2*((-1 + c^2*x^2)*(a + b*ArcSec[c*x]) + 2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sq
rt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1
- c*x]/Sqrt[2]], (2*e)/(c*d + e)]))/(e*Sqrt[d + e*x]*(-1 + c^2*x^2))
```

3.66.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5749, 1898, 633, 632, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{3/2}}\,dx$$

$$\downarrow 5749$$

$$\frac{2b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}dx}{ce} - \frac{2\left(a+b\sec^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

$$\downarrow 1898$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{cex\sqrt{1-\frac{1}{c^2x^2}}} - \frac{2\left(a+b\sec^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

$$\downarrow 633$$

$$\frac{2b\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{cex\sqrt{1-\frac{1}{c^2x^2}}} - \frac{2\left(a+b\sec^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

$$\downarrow 632$$

$$\frac{2b\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\sqrt{d+ex}}dx}{cex\sqrt{1-\frac{1}{c^2x^2}}} - \frac{2\left(a+b\sec^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

$$\downarrow 186$$

$$-\frac{4b\sqrt{1-c^2x^2}\int \frac{1}{cx\sqrt{cx+1}\sqrt{d+\frac{e}{c}-\frac{e(1-cx)}{c}}}d\sqrt{1-cx}}{cex\sqrt{1-\frac{1}{c^2x^2}}} - \frac{2\left(a+b\sec^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

$$\downarrow 413$$

$$-\frac{4b\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\int\frac{1}{cx\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}}d\sqrt{1-cx}}{cex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

$$\downarrow 412$$

$$-\frac{2\left(a+b\sec^{-1}(cx)\right)}{e\sqrt{d+ex}}-\frac{4b\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{cex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}$$

input
```
Int[(a + b*ArcSec[c*x])/(d + e*x)^(3/2),x]
```

output
```
(-2*(a + b*ArcSec[c*x]))/(e*Sqrt[d + e*x]) - (4*b*Sqrt[1 - c^2*x^2]*Sqrt[1
 - (e*(1 - c*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (
2*e)/(c*d + e)])/(c*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e/c - (e*(1 - c*x))
/c])
```

3.66.3.1 Defintions of rubi rules used

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
 - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 1898
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p])   Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n
))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !I
ntegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

rule 5749
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.66.4 Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.81

method	result	s
derivativedivides	$\frac{-\frac{2a}{\sqrt{ex+d}}+2b\left(-\frac{\operatorname{arcsec}(cx)}{\sqrt{ex+d}}-\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\ \operatorname{EllipticPi}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\frac{cd-e}{cd},\frac{\sqrt{\frac{c}{cd+e}}}{\sqrt{\frac{c}{cd-e}}}\right)}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,xd\sqrt{\frac{c}{cd-e}}}\right)}{e}$	2
default	$\frac{-\frac{2a}{\sqrt{ex+d}}+2b\left(-\frac{\operatorname{arcsec}(cx)}{\sqrt{ex+d}}-\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\ \operatorname{EllipticPi}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\frac{cd-e}{cd},\frac{\sqrt{\frac{c}{cd+e}}}{\sqrt{\frac{c}{cd-e}}}\right)}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,xd\sqrt{\frac{c}{cd-e}}}\right)}{e}$	2
parts	$-\frac{2a}{\sqrt{ex+d}\,e}+\frac{2b\left(-\frac{\operatorname{arcsec}(cx)}{\sqrt{ex+d}}-\frac{2\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\ \operatorname{EllipticPi}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\frac{cd-e}{cd},\frac{\sqrt{\frac{c}{cd+e}}}{\sqrt{\frac{c}{cd-e}}}\right)}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,xd\sqrt{\frac{c}{cd-e}}}\right)}{e}$	2

input
```
int((a+b*arcsec(c*x))/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/e*(-a/(e*x+d)^(1/2)+b*(-1/(e*x+d)^(1/2)*arcsec(c*x)-2/c/((c^2*(e*x+d)^2-
2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)/x/d/(c/(c*d-e))^(1/2)*((-c
*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*Elliptic
Pi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d
-e))^(1/2))))
```

3.66.5 Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{3/2}}\,dx = \int \frac{b\operatorname{arcsec}(cx)+a}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(b*arcsec(c*x) + a)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

3.66.6 Sympy [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{3/2}}\,dx = \int \frac{a+b\operatorname{asec}(cx)}{(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asec(c*x))/(e*x+d)**(3/2),x)
```

output
```
Integral((a + b*asec(c*x))/(d + e*x)**(3/2), x)
```

3.66.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

3.66.8 Giac [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{3/2}}\,dx = \int \frac{b\operatorname{arcsec}(cx)+a}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/(e*x + d)^(3/2), x)
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{3/2}}\,dx = \int \frac{a+b\operatorname{acos}\left(\frac{1}{cx}\right)}{(d+e\,x)^{3/2}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(d + e*x)^(3/2),x)
```

output
```
int((a + b*acos(1/(c*x)))/(d + e*x)^(3/2), x)
```

3.67 $\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{5/2}}\,dx$

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3.67.1 Optimal result

Integrand size = 18, antiderivative size = 298

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{5/2}}\,dx = -\frac{4be(1-c^2x^2)}{3cd\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$-\frac{2(a+b\sec^{-1}(cx))}{3e(d+ex)^{3/2}}+\frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3d\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$-\frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3cde\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

output

```
-2/3*(a+b*arcsec(c*x))/e/(e*x+d)^(3/2)-4/3*b*e*(-c^2*x^2+1)/c/d/(c^2*d^2-e
^2)/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/3*b*EllipticE(1/2*(-c*x+1)^(1/2)
*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^
2*d^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)-4/3*b*EllipticP
i(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+
e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/d/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

3.67.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.38 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.09

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{5/2}}\,dx = \frac{2\left(-\frac{a}{(d+ex)^{3/2}}+\frac{2bce^2\sqrt{1-\frac{1}{c^2x^2}}x}{(c^2d^3-de^2)\sqrt{d+ex}}-\frac{b\sec^{-1}(cx)}{(d+ex)^{3/2}}-\frac{2ib\sqrt{\frac{e(1+cx)}{-cd+e}}\sqrt{\frac{e-cex}{cd+e}}\left(-cdE\left(i\text{arcsinh}\left(\sqrt{-\frac{c}{cd+e}}\right.\right.\right.\right.}{}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(d + e*x)^(5/2),x]
```

output
```
(2*(-(a/(d + e*x)^(3/2)) + (2*b*c*e^2*Sqrt[1 - 1/(c^2*x^2)]*x)/((c^2*d^3 -
 d*e^2)*Sqrt[d + e*x]) - (b*ArcSec[c*x])/(d + e*x)^(3/2) - ((2*I)*b*Sqrt[(
e*(1 + c*x))/(-(c*d) + e)]*Sqrt[(e - c*e*x)/(c*d + e)]*(-(c*d*EllipticE[I*
ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)]) + c*d*E
llipticF[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e
)] + (c*d + e)*EllipticPi[1 + e/(c*d), I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt
[d + e*x]], (c*d + e)/(c*d - e)]))/(d^2*(-(c/(c*d + e)))^(3/2)*(c*d + e)^2
*Sqrt[1 - 1/(c^2*x^2)]*x)))/(3*e)
```

3.67.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.07, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5749, 1898, 635, 25, 27, 498, 27, 509, 508, 327, 633, 632, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{5/2}}\,dx$$

$$\downarrow 5749$$

$$\frac{2b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}}dx}{3ce}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$$\downarrow 1898$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\int\frac{1}{x(d+ex)^{3/2}\sqrt{x^2-\frac{1}{c^2}}}dx}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$$\downarrow 635$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\int-\frac{e}{d(d+ex)^{3/2}\sqrt{x^2-\frac{1}{c^2}}}dx+\frac{\int\frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$$\downarrow 25$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{\int\frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d}-\int\frac{e}{d(d+ex)^{3/2}\sqrt{x^2-\frac{1}{c^2}}}dx\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$$\downarrow 27$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{\int\frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d}-\frac{e\int\frac{1}{(d+ex)^{3/2}\sqrt{x^2-\frac{1}{c^2}}}dx}{d}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$$\downarrow 498$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{\int\frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d}-\frac{e\left(\frac{2\int-\frac{\sqrt{d+ex}}{2\sqrt{x^2-\frac{1}{c^2}}}dx}{d^2-\frac{e^2}{c^2}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}}{\left(d^2-\frac{e^2}{c^2}\right)\sqrt{d+ex}}\right)}{d}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$$\downarrow 27$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{\int\frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d}-\frac{e\left(\frac{\int\frac{\sqrt{d+ex}}{\sqrt{x^2-\frac{1}{c^2}}}dx}{d^2-\frac{e^2}{c^2}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}}{\left(d^2-\frac{e^2}{c^2}\right)\sqrt{d+ex}}\right)}{d}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$$\downarrow 509$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{\int\frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d}-\frac{e\left(\frac{\sqrt{1-c^2x^2}\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx}{\sqrt{x^2-\frac{1}{c^2}}\left(d^2-\frac{e^2}{c^2}\right)}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}}{\left(d^2-\frac{e^2}{c^2}\right)\sqrt{d+ex}}\right)}{d}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$$\downarrow 508$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{\int\frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d}-\frac{e\left(-\frac{2\sqrt{1-c^2x^2}\sqrt{d+ex}\int\frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{x^2-\frac{1}{c^2}}\left(d^2-\frac{e^2}{c^2}\right)\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}}{\left(d^2-\frac{e^2}{c^2}\right)\sqrt{d+ex}}\right)}{d}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$$\downarrow 327$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{\int\frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d}-\frac{e\left(-\frac{2\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\left(d^2-\frac{e^2}{c^2}\right)\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}}{\left(d^2-\frac{e^2}{c^2}\right)\sqrt{d+ex}}\right)}{d}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$$\downarrow 633$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{\sqrt{1-c^2x^2}\int\frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d\sqrt{x^2-\frac{1}{c^2}}}-\frac{e\left(-\frac{2\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\left(d^2-\frac{e^2}{c^2}\right)\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}}{\left(d^2-\frac{e^2}{c^2}\right)\sqrt{d+ex}}\right)}{d}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$$\downarrow 632$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{\sqrt{1-c^2x^2}\int\frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\sqrt{d+ex}}dx}{d\sqrt{x^2-\frac{1}{c^2}}}-\frac{e\left(-\frac{2\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\left(d^2-\frac{e^2}{c^2}\right)\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}}{\left(d^2-\frac{e^2}{c^2}\right)\sqrt{d+ex}}\right)}{d}\right)}{\frac{3cex\sqrt{1-\frac{1}{c^2x^2}}}{2\left(a+b\sec^{-1}(cx)\right)}}}{3e(d+ex)^{3/2}}-$$

$$\downarrow 186$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2\sqrt{1-c^2x^2}\int\frac{1}{cx\sqrt{cx+1}\sqrt{d+\frac{e}{c}-\frac{e(1-cx)}{c}}}d\sqrt{1-cx}}{d\sqrt{x^2-\frac{1}{c^2}}}-\frac{e\left(-\frac{2\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\left(d^2-\frac{e^2}{c^2}\right)\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}}{\left(d^2-\frac{e^2}{c^2}\right)\sqrt{d+ex}}\right)}{d}\right)}{\frac{3cex\sqrt{1-\frac{1}{c^2x^2}}}{2\left(a+b\sec^{-1}(cx)\right)}}}{3e(d+ex)^{3/2}}-$$

$$\downarrow 413$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\int\frac{1}{cx\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}}d\sqrt{1-cx}}{d\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{e\left(-\frac{2\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\left(d^2-\frac{e^2}{c^2}\right)\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}}{\left(d^2-\frac{e^2}{c^2}\right)\sqrt{d+e}}\right.}{d}}{\frac{3cex\sqrt{1-\frac{1}{c^2x^2}}}{2\left(a+b\sec^{-1}(cx)\right)}}}{3e(d+ex)^{3/2}}$$

$$\downarrow 412$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{e\left(-\frac{2\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\left(d^2-\frac{e^2}{c^2}\right)\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}}{\left(d^2-\frac{e^2}{c^2}\right)\sqrt{d+ex}}\right)}{d}-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\right.}{d\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right)}{\frac{3cex\sqrt{1-\frac{1}{c^2x^2}}}{2\left(a+b\sec^{-1}(cx)\right)}}}{3e(d+ex)^{3/2}}$$

input
```
Int[(a + b*ArcSec[c*x])/(d + e*x)^(5/2),x]
```

output
```
(-2*(a + b*ArcSec[c*x]))/(3*e*(d + e*x)^(3/2)) + (2*b*Sqrt[-c^(-2) + x^2]*
(-((e*((-2*e*Sqrt[-c^(-2) + x^2])/((d^2 - e^2/c^2)*Sqrt[d + e*x]) - (2*Sqr
t[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e
)/(c*d + e)])/(c*(d^2 - e^2/c^2)*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[-c^(-2
) + x^2]))))/d) - (2*Sqrt[1 - c^2*x^2]*Sqrt[1 - (e*(1 - c*x))/(c*d + e)]*El
lipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(d*Sqrt[-c^(-
2) + x^2]*Sqrt[d + e/c - (e*(1 - c*x))/c]))/(3*c*e*Sqrt[1 - 1/(c^2*x^2)]*
x)
```

3.67.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 635
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] + Int[(
(c + d*x)^n/Sqrt[a + b*x^2])*ExpandToSum[(1 - c^(n + 1/2)*(c + d*x)^(-n - 1
/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[n + 1/2, 0]
```

rule 1898
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p])   Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n
))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !I
ntegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

rule 5749
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.67.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 874 vs. $2(270) = 540$.

Time = 7.33 (sec) , antiderivative size = 875, normalized size of antiderivative = 2.94

method	result
derivativedivides	$-\frac{2a}{3(ex+d)^{\frac{3}{2}}}+2b\left(-\frac{\operatorname{arcsec}(cx)}{3(ex+d)^{\frac{3}{2}}}-\frac{2\left(\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c^2d^2\sqrt{ex+d}-\sqrt{=}\right.$
default	$-\frac{2a}{3(ex+d)^{\frac{3}{2}}}+2b\left(-\frac{\operatorname{arcsec}(cx)}{3(ex+d)^{\frac{3}{2}}}-\frac{2\left(\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c^2d^2\sqrt{ex+d}-\sqrt{=}\right.$
parts	$-\frac{2a}{3(ex+d)^{\frac{3}{2}}e}+2b\left(-\frac{\operatorname{arcsec}(cx)}{3(ex+d)^{\frac{3}{2}}}+\frac{2\sqrt{\frac{c}{cd-e}}c^2d(ex+d)^2}{3}-\frac{2\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}}\right.\right.}{3}$

input
```
int((a+b*arcsec(c*x))/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/e*(-1/3*a/(e*x+d)^(3/2)+b*(-1/3/(e*x+d)^(3/2)*arcsec(c*x)-2/3/c*(((-c*(e
*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((
e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2*(e*x+d)^(1
/2)-((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*
EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2
*(e*x+d)^(1/2)+((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d
+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*
d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^2*d^2*(e*x+d)^(1/2)-(c/(c*d-e))^(1/2)*c^2
*d*(e*x+d)^2+((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e
))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2)
)*c*d*e*(e*x+d)^(1/2)-((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+
e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+
e))^(1/2))*c*d*e*(e*x+d)^(1/2)+2*(c/(c*d-e))^(1/2)*c^2*d^2*(e*x+d)-((-c*(e
*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi(
(e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e)
)^(1/2))*e^2*(e*x+d)^(1/2)-(c/(c*d-e))^(1/2)*c^2*d^3+(c/(c*d-e))^(1/2)*d*e
^2)/(c*d-e)/(c/(c*d-e))^(1/2)/(e*x+d)^(1/2)/(c*d+e)/d^2/x/((c^2*(e*x+d)^2-
2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

3.67.5 Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{5/2}}\,dx = \int \frac{b\operatorname{arcsec}(cx)+a}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(b*arcsec(c*x) + a)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*
e*x + d^3), x)
```

3.67.6 Sympy [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{5/2}}\,dx = \int \frac{a+b\operatorname{asec}(cx)}{(d+ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*asec(c*x))/(e*x+d)**(5/2),x)
```

output
```
Integral((a + b*asec(c*x))/(d + e*x)**(5/2), x)
```

3.67.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

3.67.8 Giac [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{5/2}}\,dx = \int \frac{b\operatorname{arcsec}(cx)+a}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/(e*x + d)^(5/2), x)
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{5/2}}\,dx = \int \frac{a+b\operatorname{acos}\left(\frac{1}{cx}\right)}{(d+e\,x)^{5/2}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(d + e*x)^(5/2),x)
```

output
```
int((a + b*acos(1/(c*x)))/(d + e*x)^(5/2), x)
```

3.68 $\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

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3.68.1 Optimal result

Integrand size = 18, antiderivative size = 540

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{7/2}}\,dx = -\frac{4be(1-c^2x^2)}{15cd\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x(d+ex)^{3/2}}$$
$$-\frac{16bce(1-c^2x^2)}{15\left(c^2d^2-e^2\right)^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}-\frac{4be(1-c^2x^2)}{5cd^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$-\frac{2(a+b\sec^{-1}(cx))}{5e(d+ex)^{5/2}}+\frac{4b(7c^2d^2-3e^2)\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15\left(c^2d^3-de^2\right)^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$-\frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15d\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$-\frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5cd^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

output
```
-2/5*(a+b*arcsec(c*x))/e/(e*x+d)^(5/2)-4/15*b*e*(-c^2*x^2+1)/c/d/(c^2*d^2-
e^2)/x/(e*x+d)^(3/2)/(1-1/c^2/x^2)^(1/2)-16/15*b*c*e*(-c^2*x^2+1)/(c^2*d^2
-e^2)^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/5*b*e*(-c^2*x^2+1)/c/d^2/(c^
2*d^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/15*b*(7*c^2*d^2-3*e^2)*El
lipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2
)*(-c^2*x^2+1)^(1/2)/(c^2*d^3-d*e^2)^2/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c
*d+e))^(1/2)-4/15*b*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e
))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*d^2-e^2)/x/(
1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/5*b*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/
2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/
2)/c/d^2/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

3.68.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.62 (sec) , antiderivative size = 407, normalized size of antiderivative = 0.75

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{7/2}}\,dx = \frac{2\left(-\frac{3a}{(d+ex)^{5/2}}+\frac{2bce^2\sqrt{1-\frac{1}{c^2x^2}}x\left(-e^2(4d+3ex)+c^2d^2(8d+7ex)\right)}{\left(c^2d^3-de^2\right)^2(d+ex)^{3/2}}-\frac{3b\sec^{-1}(cx)}{(d+ex)^{5/2}}+\frac{2ib\sqrt{\frac{e(1+cx)}{-cd+e}}\sqrt{\frac{e-cex}{cd+e}}}{}\right.$$

input
```
Integrate[(a + b*ArcSec[c*x])/(d + e*x)^(7/2),x]
```

output
```
(2*((-3*a)/(d + e*x)^(5/2) + (2*b*c*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*(-(e^2*(4*
d + 3*e*x)) + c^2*d^2*(8*d + 7*e*x)))/((c^2*d^3 - d*e^2)^2*(d + e*x)^(3/2)
) - (3*b*ArcSec[c*x])/(d + e*x)^(5/2) + ((2*I)*b*Sqrt[(e*(1 + c*x))/(-(c*d
) + e)]*Sqrt[(e - c*e*x)/(c*d + e)]*(c*d*(7*c^2*d^2 - 3*e^2)*EllipticE[I*A
rcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] - c*d*(6*
c^2*d^2 - c*d*e - 3*e^2)*EllipticF[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d +
 e*x]], (c*d + e)/(c*d - e)] - 3*(c*d - e)*(c*d + e)^2*EllipticPi[1 + e/(c
*d), I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)]))
/(d^3*(c*d - e)*(-(c/(c*d + e)))^(3/2)*(c*d + e)^3*Sqrt[1 - 1/(c^2*x^2)]*x
)))/(15*e)
```

3.68.3 Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 516, normalized size of antiderivative = 0.96, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 1.056$, Rules used = {5749, 1898, 635, 633, 632, 186, 413, 412, 688, 27, 688, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{7/2}}\,dx$$

$$\downarrow 5749$$

$$\frac{2b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2(d+ex)^{5/2}}dx}{5ce} - \frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\downarrow 1898$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\int \frac{1}{x(d+ex)^{5/2}\sqrt{x^2-\frac{1}{c^2}}}dx}{5cex\sqrt{1-\frac{1}{c^2x^2}}} - \frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\downarrow 635$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\int \frac{-\frac{xe^2}{d^2}-\frac{2e}{d}}{(d+ex)^{5/2}\sqrt{x^2-\frac{1}{c^2}}}dx + \frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d^2}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}} - \frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\downarrow 633$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\int \frac{-\frac{xe^2}{d^2}-\frac{2e}{d}}{(d+ex)^{5/2}\sqrt{x^2-\frac{1}{c^2}}}dx + \frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d^2\sqrt{x^2-\frac{1}{c^2}}}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}} - \frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\downarrow 632$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\int \frac{-\frac{xe^2}{d^2}-\frac{2e}{d}}{(d+ex)^{5/2}\sqrt{x^2-\frac{1}{c^2}}}dx + \frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\sqrt{d+ex}}dx}{d^2\sqrt{x^2-\frac{1}{c^2}}}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}} - \frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\downarrow 186$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\int\frac{-\frac{xe^2}{d^2}-\frac{2e}{d}}{(d+ex)^{5/2}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{2\sqrt{1-c^2x^2}\int\frac{1}{cx\sqrt{cx+1}\sqrt{d+\frac{e}{c}-\frac{e(1-cx)}{c}}}d\sqrt{1-cx}}{d^2\sqrt{x^2-\frac{1}{c^2}}}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\downarrow 413$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\int\frac{-\frac{xe^2}{d^2}-\frac{2e}{d}}{(d+ex)^{5/2}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\int\frac{1}{cx\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}}d\sqrt{1-cx}}{d^2\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\downarrow 412$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\int\frac{-\frac{xe^2}{d^2}-\frac{2e}{d}}{(d+ex)^{5/2}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{d^2\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\downarrow 688$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2\int\frac{e\left(3d\left(2-\frac{e^2}{c^2d^2}\right)-ex\right)}{2d(d+ex)^{3/2}\sqrt{x^2-\frac{1}{c^2}}}dx}{3\left(d^2-\frac{e^2}{c^2}\right)}-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{d^2\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}+\frac{2e^2\sqrt{x^2-\frac{1}{c^2}}}{3d\left(d^2-\frac{e^2}{c^2}\right)(d+ex)^{3/2}}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\downarrow 27$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{e\int\frac{3\left(2d-\frac{e^2}{c^2d}\right)-ex}{(d+ex)^{3/2}\sqrt{x^2-\frac{1}{c^2}}}dx}{3d\left(d^2-\frac{e^2}{c^2}\right)}-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{d^2\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}+\frac{2e^2\sqrt{x^2-\frac{1}{c^2}}}{3d\left(d^2-\frac{e^2}{c^2}\right)(d+ex)^{3/2}}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\downarrow 688$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{e\left(-\frac{2\int -\frac{6d^2-\frac{2e^2}{c^2}+e\left(7d-\frac{3e^2}{c^2d}\right)x}{2\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d^2-\frac{e^2}{c^2}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}\left(7c^2d^2-3e^2\right)}{d\left(c^2d^2-e^2\right)\sqrt{d+ex}}\right)}{3d\left(d^2-\frac{e^2}{c^2}\right)}-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right.}{d^2\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\downarrow 27$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{e\left(\frac{2\left(3d^2-\frac{e^2}{c^2}\right)+e\left(7d-\frac{3e^2}{c^2d}\right)x}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{2e\sqrt{x^2-\frac{1}{c^2}}\left(7c^2d^2-3e^2\right)}{d\left(c^2d^2-e^2\right)\sqrt{d+ex}}\right)}{3d\left(d^2-\frac{e^2}{c^2}\right)}-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right.}{d^2\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\downarrow 600$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{e\left(\frac{\left(7d-\frac{3e^2}{c^2d}\right)\int\frac{\sqrt{d+ex}}{\sqrt{x^2-\frac{1}{c^2}}}dx-\left(d^2-\frac{e^2}{c^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d^2-\frac{e^2}{c^2}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}\left(7c^2d^2-3e^2\right)}{d\left(c^2d^2-e^2\right)\sqrt{d+ex}}\right)}{3d\left(d^2-\frac{e^2}{c^2}\right)}-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(\right.}{d^2\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-}}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\downarrow 509$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{e\left(\frac{\frac{\sqrt{1-c^2x^2}\left(7d-\frac{3e^2}{c^2d}\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx}{\sqrt{x^2-\frac{1}{c^2}}}-\left(d^2-\frac{e^2}{c^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d^2-\frac{e^2}{c^2}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}\left(7c^2d^2-3e^2\right)}{d\left(c^2d^2-e^2\right)\sqrt{d+ex}}\right)}{3d\left(d^2-\frac{e^2}{c^2}\right)}-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{E}}{d^2\sqrt{x^2-\frac{}{c}}}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\downarrow 508$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{e\left(\frac{-\left(\left(d^2-\frac{e^2}{c^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx\right)-\frac{2\sqrt{1-c^2x^2}\left(7d-\frac{3e^2}{c^2d}\right)\sqrt{d+ex}\int\frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}}{d^2-\frac{e^2}{c^2}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}\left(7c^2d^2-3e^2\right)}{d\left(c^2d^2-e^2\right)\sqrt{d+ex}}\right)}{3d\left(d^2-\frac{e^2}{c^2}\right)}-\frac{2\sqrt{}}{}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\downarrow 327$$

$$2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{e\left(\frac{-\left(d^2-\frac{e^2}{c^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{2\sqrt{1-c^2x^2}\left(7d-\frac{3e^2}{c^2d}\right)\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}}{d^2-\frac{e^2}{c^2}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}\left(7c^2d^2-3e^2\right)}{d\left(c^2d^2-e^2\right)\sqrt{d+ex}}\right)}{3d\left(d^2-\frac{e^2}{c^2}\right)}-\frac{2\sqrt{1-}}{5cex\sqrt{1-\frac{1}{c^2x^2}}}\right.$$

$$\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\downarrow 512$$

$$2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{e\left(\frac{-\frac{\sqrt{1-c^2x^2}\left(d^2-\frac{e^2}{c^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{\sqrt{x^2-\frac{1}{c^2}}}-\frac{2\sqrt{1-c^2x^2}\left(7d-\frac{3e^2}{c^2d}\right)\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}}{d^2-\frac{e^2}{c^2}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}\left(7c^2d^2-3e^2\right)}{d\left(c^2d^2-e^2\right)\sqrt{d+ex}}\right)}{3d\left(d^2-\frac{e^2}{c^2}\right)}\right.$$

$$5cex\sqrt{1-\frac{1}{c^2x^2}}$$

$$\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\downarrow 511$$

$$\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow 321$$

$$\frac{2\left(a+b\sec^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

input `Int[(a + b*ArcSec[c*x])/(d + e*x)^(7/2),x]`

output
```
(-2*(a + b*ArcSec[c*x]))/(5*e*(d + e*x)^(5/2)) + (2*b*Sqrt[-c^(-2) + x^2]*
((2*e^2*Sqrt[-c^(-2) + x^2])/(3*d*(d^2 - e^2/c^2)*(d + e*x)^(3/2)) - (e*((
-2*e*(7*c^2*d^2 - 3*e^2)*Sqrt[-c^(-2) + x^2])/(d*(c^2*d^2 - e^2)*Sqrt[d +
e*x]) + ((-2*(7*d - (3*e^2)/(c^2*d))*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*Ellip
ticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*Sqrt[(c*(d + e*x)
)/(c*d + e)]*Sqrt[-c^(-2) + x^2]) + (2*(d^2 - e^2/c^2)*Sqrt[(c*(d + e*x))/
(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e
)/(c*d + e)])/(c*Sqrt[d + e*x]*Sqrt[-c^(-2) + x^2]))/(d^2 - e^2/c^2)))/(3*
d*(d^2 - e^2/c^2)) - (2*Sqrt[1 - c^2*x^2]*Sqrt[1 - (e*(1 - c*x))/(c*d + e)
]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(d^2*Sqrt
[-c^(-2) + x^2]*Sqrt[d + e/c - (e*(1 - c*x))/c])))/(5*c*e*Sqrt[1 - 1/(c^2*
x^2)]*x)
```

3.68.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 635
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] + Int[(
(c + d*x)^n/Sqrt[a + b*x^2])*ExpandToSum[(1 - c^(n + 1/2)*(c + d*x)^(-n - 1
/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[n + 1/2, 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1898
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p])   Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n
))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !I
ntegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

rule 5749
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.68.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1617 vs. 2(491) = 982.

Time = 8.78 (sec) , antiderivative size = 1618, normalized size of antiderivative = 3.00

method	result	size
derivativedivides	Expression too large to display	1618
default	Expression too large to display	1618
parts	Expression too large to display	1642

input
```
int((a+b*arcsec(c*x))/(e*x+d)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/e*(-1/5*a/(e*x+d)^(5/2)+b*(-1/5/(e*x+d)^(5/2)*arcsec(c*x)+2/15/c*(7*(c/(
c*d-e))^(1/2)*c^4*d^3*(e*x+d)^3-6*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*
(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((
c*d-e)/(c*d+e))^(1/2))*c^4*d^4*(e*x+d)^(3/2)+7*((-c*(e*x+d)+c*d-e)/(c*d-e)
)^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d
-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^4*d^4*(e*x+d)^(3/2)-3*((-c*(e*x+d)+c
*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)
^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2)
)*c^4*d^4*(e*x+d)^(3/2)-13*(c/(c*d-e))^(1/2)*c^4*d^4*(e*x+d)^2-7*((-c*(e*x
+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*
x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^3*d^3*e*(e*x+d)^(3
/2)+7*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2
)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^3*d
^3*e*(e*x+d)^(3/2)+5*(c/(c*d-e))^(1/2)*c^4*d^5*(e*x+d)-3*(c/(c*d-e))^(1/2)
*c^2*d*e^2*(e*x+d)^3+2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d
+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d
+e))^(1/2))*c^2*d^2*e^2*(e*x+d)^(3/2)-3*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)
*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1
/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2*e^2*(e*x+d)^(3/2)+6*((-c*(e*x+d)+c*d-
e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)...
```

3.68.5 Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{7/2}}\,dx = \int \frac{b\,\text{arcsec}\,(cx)+a}{(ex+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(b*arcsec(c*x) + a)/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*
e^2*x^2 + 4*d^3*e*x + d^4), x)
```

3.68.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*asec(c*x))/(e*x+d)**(7/2),x)
```

output
```
Timed out
```

3.68.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

3.68.8 Giac [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{7/2}}\,dx = \int \frac{b\,\text{arcsec}\,(cx)+a}{(ex+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x+d)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/(e*x + d)^(7/2), x)
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{7/2}}\,dx = \int \frac{a+b\,\text{acos}\left(\frac{1}{c\,x}\right)}{(d+e\,x)^{7/2}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(d + e*x)^(7/2),x)
```

output
```
int((a + b*acos(1/(c*x)))/(d + e*x)^(7/2), x)
```

3.69 $\int x^4(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$

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3.69.1 Optimal result

Integrand size = 19, antiderivative size = 206

$$\begin{aligned}\int x^4(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = &-\frac{b(42c^2d+25e)\,x^2\sqrt{-1+c^2x^2}}{560c^5\sqrt{c^2x^2}}\\ &-\frac{b(42c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}}-\frac{bex^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}}\\ &+\frac{1}{5}dx^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\sec^{-1}(cx)\right)\\ &-\frac{b(42c^2d+25e)\,x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{560c^6\sqrt{c^2x^2}}\end{aligned}$$

output

```
1/5*d*x^5*(a+b*arcsec(c*x))+1/7*e*x^7*(a+b*arcsec(c*x))-1/560*b*(42*c^2*d+
25*e)*x*arctanh(c*x/(c^2*x^2-1)^(1/2))/c^6/(c^2*x^2)^(1/2)-1/560*b*(42*c^2
*d+25*e)*x^2*(c^2*x^2-1)^(1/2)/c^5/(c^2*x^2)^(1/2)-1/840*b*(42*c^2*d+25*e)
*x^4*(c^2*x^2-1)^(1/2)/c^3/(c^2*x^2)^(1/2)-1/42*b*e*x^6*(c^2*x^2-1)^(1/2)/
c/(c^2*x^2)^(1/2)
```

3.69.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.68

$$\int x^4\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{48ac^7x^5(7d+5ex^2)-bc^2\sqrt{1-\frac{1}{c^2x^2}}x^2(75e+2c^2(63d+25ex^2)+c^4(84dx^2+40ex^4))+48bc^7x^5(7d+5ex$$
$$1680c^7$$

input
```
Integrate[x^4*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

output
```
(48*a*c^7*x^5*(7*d + 5*e*x^2) - b*c^2*Sqrt[1 - 1/(c^2*x^2)]*x^2*(75*e + 2*
c^2*(63*d + 25*e*x^2) + c^4*(84*d*x^2 + 40*e*x^4)) + 48*b*c^7*x^5*(7*d + 5
*e*x^2)*ArcSec[c*x] - 3*b*(42*c^2*d + 25*e)*Log[(1 + Sqrt[1 - 1/(c^2*x^2)]
)*x])/(1680*c^7)
```

3.69.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5761, 27, 363, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$\downarrow 5761$$
$$-\frac{bcx\int\frac{x^4\left(5ex^2+7d\right)}{35\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{1}{5}dx^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\sec^{-1}(cx)\right)$$
$$\downarrow 27$$
$$-\frac{bcx\int\frac{x^4\left(5ex^2+7d\right)}{\sqrt{c^2x^2-1}}dx}{35\sqrt{c^2x^2}}+\frac{1}{5}dx^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\sec^{-1}(cx)\right)$$
$$\downarrow 363$$
$$-\frac{bcx\left(\frac{1}{6}\left(\frac{25e}{c^2}+42d\right)\int\frac{x^4}{\sqrt{c^2x^2-1}}dx+\frac{5ex^5\sqrt{c^2x^2-1}}{6c^2}\right)}{35\sqrt{c^2x^2}}+\frac{1}{5}dx^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 262$$

$$-\frac{bcx\left(\frac{1}{6}\left(\frac{25e}{c^2}+42d\right)\left(\frac{3\int\frac{x^2}{\sqrt{c^2x^2-1}}dx}{4c^2}+\frac{x^3\sqrt{c^2x^2-1}}{4c^2}\right)+\frac{5ex^5\sqrt{c^2x^2-1}}{6c^2}\right)}{35\sqrt{c^2x^2}}+\frac{1}{5}dx^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 262$$

$$-\frac{bcx\left(\frac{1}{6}\left(\frac{25e}{c^2}+42d\right)\left(\frac{3\left(\frac{\int\frac{1}{\sqrt{c^2x^2-1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{c^2x^2-1}}{4c^2}\right)+\frac{5ex^5\sqrt{c^2x^2-1}}{6c^2}\right)}{35\sqrt{c^2x^2}}+\frac{1}{5}dx^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 224$$

$$-\frac{bcx\left(\frac{1}{6}\left(\frac{25e}{c^2}+42d\right)\left(\frac{3\left(\frac{\int\frac{1}{1-\frac{c^2x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}}{2c^2}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{c^2x^2-1}}{4c^2}\right)+\frac{5ex^5\sqrt{c^2x^2-1}}{6c^2}\right)}{35\sqrt{c^2x^2}}+\frac{1}{5}dx^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 219$$

$$\frac{1}{5}dx^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\sec^{-1}(cx)\right)-\frac{bcx\left(\frac{1}{6}\left(\frac{3\left(\frac{\operatorname{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{2c^3}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{c^2x^2-1}}{4c^2}\right)\left(\frac{25e}{c^2}+42d\right)+\frac{5ex^5\sqrt{c^2x^2-1}}{6c^2}\right)}{35\sqrt{c^2x^2}}$$

input
```
Int[x^4*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

output
```
(d*x^5*(a + b*ArcSec[c*x]))/5 + (e*x^7*(a + b*ArcSec[c*x]))/7 - (b*c*x*((5
*e*x^5*Sqrt[-1 + c^2*x^2])/(6*c^2) + ((42*d + (25*e)/c^2)*((x^3*Sqrt[-1 +
c^2*x^2])/(4*c^2) + (3*((x*Sqrt[-1 + c^2*x^2])/(2*c^2) + ArcTanh[(c*x)/Sqr
t[-1 + c^2*x^2]]/(2*c^3)))/(4*c^2)))/6))/(35*Sqrt[c^2*x^2])
```

3.69.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.69.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.59

method	result
parts	$a\left(\frac{1}{7}e\,x^7+\frac{1}{5}d\,x^5\right)+\frac{b\,\mathrm{arcsec}(cx)e\,x^7}{7}+\frac{b\,\mathrm{arcsec}(cx)x^5d}{5}-\frac{b\left(c^2x^2-1\right)x^4e}{42c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b\left(c^2x^2-1\right)x^2d}{20c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{5b\left(c^2x^2-1\right)}{168c^5\sqrt{\frac{c^2x^2-1}{c^2x^2}}}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{5}d\,c^7x^5+\frac{1}{7}e\,c^7x^7\right)}{c^2}+\frac{b\,\mathrm{arcsec}(cx)d\,c^5x^5}{5}+\frac{b\,c^5\,\mathrm{arcsec}(cx)e\,x^7}{7}-\frac{b\left(c^2x^2-1\right)c^2x^2d}{20\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b\,c^2\left(c^2x^2-1\right)x^4e}{42\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{3b\left(c^2x^2-1\right)d}{40\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{5b\left(c^2x^2}{168\sqrt{}}}{c^5}$
default	$\frac{\frac{a\left(\frac{1}{5}d\,c^7x^5+\frac{1}{7}e\,c^7x^7\right)}{c^2}+\frac{b\,\mathrm{arcsec}(cx)d\,c^5x^5}{5}+\frac{b\,c^5\,\mathrm{arcsec}(cx)e\,x^7}{7}-\frac{b\left(c^2x^2-1\right)c^2x^2d}{20\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b\,c^2\left(c^2x^2-1\right)x^4e}{42\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{3b\left(c^2x^2-1\right)d}{40\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{5b\left(c^2x^2}{168\sqrt{}}}{c^5}$

input
```
int(x^4*(e*x^2+d)*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/7*e*x^7+1/5*d*x^5)+1/7*b*arcsec(c*x)*e*x^7+1/5*b*arcsec(c*x)*x^5*d-1/
42*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^4*e-1/20*b/c^3*(c^2*x^2
-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^2*d-5/168*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)
/c^2/x^2)^(1/2)*x^2*e-3/40*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*d
-5/112*b/c^7*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*e-3/40*b/c^6*(c^2*x^2
-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d*ln(c*x+(c^2*x^2-1)^(1/2))-5/112*
b/c^8*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*e*ln(c*x+(c^2*x^2-1)
^(1/2))
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.93

$$\int x^4\left(d+ex^2\right)\left(a+b\,\mathrm{sec}^{-1}(cx)\right)\,dx$$

$$=\frac{240\,ac^7ex^7+336\,ac^7dx^5+48\left(5\,bc^7ex^7+7\,bc^7dx^5-7\,bc^7d-5\,bc^7e\right)\mathrm{arcsec}\left(cx\right)+96\left(7\,bc^7d+5\,bc^7e\right)\mathrm{arct}}{}$$

input
```
integrate(x^4*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/1680*(240*a*c^7*e*x^7 + 336*a*c^7*d*x^5 + 48*(5*b*c^7*e*x^7 + 7*b*c^7*d*
x^5 - 7*b*c^7*d - 5*b*c^7*e)*arcsec(c*x) + 96*(7*b*c^7*d + 5*b*c^7*e)*arct
an(-c*x + sqrt(c^2*x^2 - 1)) + 3*(42*b*c^2*d + 25*b*e)*log(-c*x + sqrt(c^2
*x^2 - 1)) - (40*b*c^5*e*x^5 + 2*(42*b*c^5*d + 25*b*c^3*e)*x^3 + 3*(42*b*c
^3*d + 25*b*c*e)*x)*sqrt(c^2*x^2 - 1))/c^7
```

3.69.6 Sympy [A] (verification not implemented)

Time = 10.86 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.98

$$\int x^4\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{adx^5}{5}+\frac{aex^7}{7}+\frac{bdx^5\,\text{asec}\,(cx)}{5}+\frac{bex^7\,\text{asec}\,(cx)}{7}$$
$$-\frac{bd\left(\begin{cases}\frac{cx^5}{4\sqrt{c^2x^2-1}}+\frac{x^3}{8c\sqrt{c^2x^2-1}}-\frac{3x}{8c^3\sqrt{c^2x^2-1}}+\frac{3\,\text{acosh}\,(cx)}{8c^4} & \text{for } |c^2x^2|>1\\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}}-\frac{ix^3}{8c\sqrt{-c^2x^2+1}}+\frac{3ix}{8c^3\sqrt{-c^2x^2+1}}-\frac{3i\,\text{asin}\,(cx)}{8c^4} & \text{otherwise}\end{cases}\right)}{5c}$$
$$-\frac{be\left(\begin{cases}\frac{cx^7}{6\sqrt{c^2x^2-1}}+\frac{x^5}{24c\sqrt{c^2x^2-1}}+\frac{5x^3}{48c^3\sqrt{c^2x^2-1}}-\frac{5x}{16c^5\sqrt{c^2x^2-1}}+\frac{5\,\text{acosh}\,(cx)}{16c^6} & \text{for } |c^2x^2|>1\\ -\frac{icx^7}{6\sqrt{-c^2x^2+1}}-\frac{ix^5}{24c\sqrt{-c^2x^2+1}}-\frac{5ix^3}{48c^3\sqrt{-c^2x^2+1}}+\frac{5ix}{16c^5\sqrt{-c^2x^2+1}}-\frac{5i\,\text{asin}\,(cx)}{16c^6} & \text{otherwise}\end{cases}\right)}{7c}$$

input
```
integrate(x**4*(e*x**2+d)*(a+b*asec(c*x)),x)
```

output
```
a*d*x**5/5 + a*e*x**7/7 + b*d*x**5*asec(c*x)/5 + b*e*x**7*asec(c*x)/7 - b*
d*Piecewise((c*x**5/(4*sqrt(c**2*x**2 - 1)) + x**3/(8*c*sqrt(c**2*x**2 - 1
)) - 3*x/(8*c**3*sqrt(c**2*x**2 - 1)) + 3*acosh(c*x)/(8*c**4), Abs(c**2*x*
*2) > 1), (-I*c*x**5/(4*sqrt(-c**2*x**2 + 1)) - I*x**3/(8*c*sqrt(-c**2*x**
2 + 1)) + 3*I*x/(8*c**3*sqrt(-c**2*x**2 + 1)) - 3*I*asin(c*x)/(8*c**4), Tr
ue))/(5*c) - b*e*Piecewise((c*x**7/(6*sqrt(c**2*x**2 - 1)) + x**5/(24*c*sq
rt(c**2*x**2 - 1)) + 5*x**3/(48*c**3*sqrt(c**2*x**2 - 1)) - 5*x/(16*c**5*s
qrt(c**2*x**2 - 1)) + 5*acosh(c*x)/(16*c**6), Abs(c**2*x**2) > 1), (-I*c*x
**7/(6*sqrt(-c**2*x**2 + 1)) - I*x**5/(24*c*sqrt(-c**2*x**2 + 1)) - 5*I*x*
*3/(48*c**3*sqrt(-c**2*x**2 + 1)) + 5*I*x/(16*c**5*sqrt(-c**2*x**2 + 1)) -
 5*I*asin(c*x)/(16*c**6), True))/(7*c)
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.44

$$\int x^4\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{1}{7}aex^7+\frac{1}{5}adx^5$$
$$+\frac{1}{80}\left(16\,x^5\operatorname{arcsec}(cx)+\frac{\frac{2\left(3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2c^4\left(\frac{1}{c^2x^2}-1\right)+c^4}-\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)bd$$
$$+\frac{1}{672}\left(96\,x^7\operatorname{arcsec}(cx)-\frac{\frac{2\left(15\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-40\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+33\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^6\left(\frac{1}{c^2x^2}-1\right)^3+3c^6\left(\frac{1}{c^2x^2}-1\right)^2+3c^6\left(\frac{1}{c^2x^2}-1\right)+c^6}+\frac{15\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^6}-\frac{15\log\left(\sqrt{-\frac{1}{c^2x^2}+1}\right.\right.}{c}\right.$$

input
```
integrate(x^4*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*e*x^7 + 1/5*a*d*x^5 + 1/80*(16*x^5*arcsec(c*x) + (2*(3*(-1/(c^2*x^2)
 + 1)^(3/2) - 5*sqrt(-1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(
1/(c^2*x^2) - 1) + c^4) - 3*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sq
rt(-1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*d + 1/672*(96*x^7*arcsec(c*x) - (2*(15
*(-1/(c^2*x^2) + 1)^(5/2) - 40*(-1/(c^2*x^2) + 1)^(3/2) + 33*sqrt(-1/(c^2*
x^2) + 1))/(c^6*(1/(c^2*x^2) - 1)^3 + 3*c^6*(1/(c^2*x^2) - 1)^2 + 3*c^6*(1
/(c^2*x^2) - 1) + c^6) + 15*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 - 15*log(s
qrt(-1/(c^2*x^2) + 1) - 1)/c^6)/c)*b*e
```

3.69.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17474 vs. 2(178) = 356.

Time = 2.76 (sec) , antiderivative size = 17474, normalized size of antiderivative = 84.83

$$\int x^4\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
1/1680*(336*b*c^2*d*arccos(1/(c*x))/(c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x
) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2
) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21
*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(
c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14) - 126*b*c^2*d*log
(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(c^8 + 7*c^8*(1/(c^2*x^2) - 1)
/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/
(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1
)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1
)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14) + 126*b*c
^2*d*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(c^8 + 7*c^8*(1/(c^2*x
^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35
*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(
c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*
x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14) +
 336*a*c^2*d/(c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c
^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^
6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^
5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(
c^2*x^2) - 1)^7/(1/(c*x) + 1)^14) - 1008*b*c^2*d*(1/(c^2*x^2) - 1)*arcc...
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int x^4(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx=\int x^4\left(e\,x^2+d\right)\left(a+b\,\text{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^4*(d + e*x^2)*(a + b*acos(1/(c*x))),x)
```

output
```
int(x^4*(d + e*x^2)*(a + b*acos(1/(c*x))), x)
```

3.70 $\int x^2(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$

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3.70.1 Optimal result

Integrand size = 19, antiderivative size = 161

$$\begin{aligned}\int x^2(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx &= -\frac{b(20c^2d+9e)\,x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}} - \frac{bex^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}\\ &\quad + \frac{1}{3}dx^3\left(a+b\sec^{-1}(cx)\right) + \frac{1}{5}ex^5\left(a+b\sec^{-1}(cx)\right)\\ &\quad - \frac{b(20c^2d+9e)\,x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{120c^4\sqrt{c^2x^2}}\end{aligned}$$

output
```
1/3*d*x^3*(a+b*arcsec(c*x))+1/5*e*x^5*(a+b*arcsec(c*x))-1/120*b*(20*c^2*d+
9*e)*x*arctanh(c*x/(c^2*x^2-1)^(1/2))/c^4/(c^2*x^2)^(1/2)-1/120*b*(20*c^2*
d+9*e)*x^2*(c^2*x^2-1)^(1/2)/c^3/(c^2*x^2)^(1/2)-1/20*b*e*x^4*(c^2*x^2-1)^
(1/2)/c/(c^2*x^2)^(1/2)
```

3.70.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.76

$$\begin{aligned}&\int x^2(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx\\ &= \frac{c^2x^2\left(8ac^3x(5d+3ex^2) - b\sqrt{1-\frac{1}{c^2x^2}}(9e+c^2(20d+6ex^2))\right) + 8bc^5x^3(5d+3ex^2)\sec^{-1}(cx) - b(20c^2d+}{120c^5}\end{aligned}$$

input
```
Integrate[x^2*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

output
```
(c^2*x^2*(8*a*c^3*x*(5*d + 3*e*x^2) - b*Sqrt[1 - 1/(c^2*x^2)]*(9*e + c^2*(
20*d + 6*e*x^2))) + 8*b*c^5*x^3*(5*d + 3*e*x^2)*ArcSec[c*x] - b*(20*c^2*d
+ 9*e)*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/(120*c^5)
```

3.70.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5761, 27, 363, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\downarrow 5761$$

$$-\frac{bcx\int\frac{x^2(3ex^2+5d)}{15\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{1}{3}dx^3\left(a+b\sec^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 27$$

$$-\frac{bcx\int\frac{x^2(3ex^2+5d)}{\sqrt{c^2x^2-1}}dx}{15\sqrt{c^2x^2}}+\frac{1}{3}dx^3\left(a+b\sec^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 363$$

$$-\frac{bcx\left(\frac{1}{4}\left(\frac{9e}{c^2}+20d\right)\int\frac{x^2}{\sqrt{c^2x^2-1}}dx+\frac{3ex^3\sqrt{c^2x^2-1}}{4c^2}\right)}{15\sqrt{c^2x^2}}+\frac{1}{3}dx^3\left(a+b\sec^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 262$$

$$-\frac{bcx\left(\frac{1}{4}\left(\frac{9e}{c^2}+20d\right)\left(\frac{\int\frac{1}{\sqrt{c^2x^2-1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)+\frac{3ex^3\sqrt{c^2x^2-1}}{4c^2}\right)}{15\sqrt{c^2x^2}}+\frac{1}{3}dx^3\left(a+b\sec^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 224$$

$$-\frac{bcx\left(\frac{1}{4}\left(\frac{9e}{c^2}+20d\right)\left(\frac{\int \frac{1}{1-\frac{c^2x^2}{c^2x^2-1}} d\frac{x}{\sqrt{c^2x^2-1}}}{2c^2}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)+\frac{3ex^3\sqrt{c^2x^2-1}}{4c^2}\right)}{15\sqrt{c^2x^2}}+\frac{1}{3}dx^3\left(a+b\sec^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 219$$

$$\frac{1}{3}dx^3\left(a+b\sec^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\sec^{-1}(cx)\right)-\frac{bcx\left(\frac{1}{4}\left(\frac{\operatorname{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{2c^3}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)\left(\frac{9e}{c^2}+20d\right)+\frac{3ex^3\sqrt{c^2x^2-1}}{4c^2}\right)}{15\sqrt{c^2x^2}}$$

input
```
Int[x^2*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

output
```
(d*x^3*(a + b*ArcSec[c*x]))/3 + (e*x^5*(a + b*ArcSec[c*x]))/5 - (b*c*x*((3
*e*x^3*Sqrt[-1 + c^2*x^2])/(4*c^2) + ((20*d + (9*e)/c^2)*((x*Sqrt[-1 + c^2
*x^2])/(2*c^2) + ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]]/(2*c^3)))/4))/(15*Sqrt[
c^2*x^2])
```

3.70.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))  Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))  Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])  u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])  Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.70.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.58

method	result
parts	$a\left(\frac{1}{5}e\,x^5+\frac{1}{3}d\,x^3\right)+\frac{b\,\mathrm{arcsec}(cx)e\,x^5}{5}+\frac{b\,\mathrm{arcsec}(cx)x^3d}{3}-\frac{b\left(c^2x^2-1\right)x^2e}{20c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b\left(c^2x^2-1\right)d}{6c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{3b\left(c^2x^2-1\right)}{40c^5\sqrt{\frac{c^2x^2-1}{c^2x^2}}}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{3}d\,c^5x^3+\frac{1}{5}e\,c^5x^5\right)}{c^2}+\frac{b\,\mathrm{arcsec}(cx)d\,c^3x^3}{3}+\frac{b\,c^3\,\mathrm{arcsec}(cx)e\,x^5}{5}-\frac{b\left(c^2x^2-1\right)d}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b\left(c^2x^2-1\right)x^2e}{20\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b\sqrt{c^2x^2-1}\,d\ln\left(cx+\sqrt{c^2x^2-1}\right)}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{c^3}$
default	$\frac{\frac{a\left(\frac{1}{3}d\,c^5x^3+\frac{1}{5}e\,c^5x^5\right)}{c^2}+\frac{b\,\mathrm{arcsec}(cx)d\,c^3x^3}{3}+\frac{b\,c^3\,\mathrm{arcsec}(cx)e\,x^5}{5}-\frac{b\left(c^2x^2-1\right)d}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b\left(c^2x^2-1\right)x^2e}{20\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b\sqrt{c^2x^2-1}\,d\ln\left(cx+\sqrt{c^2x^2-1}\right)}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{c^3}$

input
```
int(x^2*(e*x^2+d)*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*e*x^5+1/3*d*x^3)+1/5*b*arcsec(c*x)*e*x^5+1/3*b*arcsec(c*x)*x^3*d-1/
20*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^2*e-1/6*b/c^3*(c^2*x^2-
1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*d-3/40*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x
^2)^(1/2)*e-1/6*b/c^4*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d*ln
(c*x+(c^2*x^2-1)^(1/2))-3/40*b/c^6*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)
^(1/2)/x*e*ln(c*x+(c^2*x^2-1)^(1/2))
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.06

$$\int x^2\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{24\,ac^5ex^5+40\,ac^5dx^3+8\left(3\,bc^5ex^5+5\,bc^5dx^3-5\,bc^5d-3\,bc^5e\right)\operatorname{arcsec}\left(cx\right)+16\left(5\,bc^5d+3\,bc^5e\right)\arctan}{120}$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/120*(24*a*c^5*e*x^5 + 40*a*c^5*d*x^3 + 8*(3*b*c^5*e*x^5 + 5*b*c^5*d*x^3
- 5*b*c^5*d - 3*b*c^5*e)*arcsec(c*x) + 16*(5*b*c^5*d + 3*b*c^5*e)*arctan(-
c*x + sqrt(c^2*x^2 - 1)) + (20*b*c^2*d + 9*b*e)*log(-c*x + sqrt(c^2*x^2 -
1)) - (6*b*c^3*e*x^3 + (20*b*c^3*d + 9*b*c*e)*x)*sqrt(c^2*x^2 - 1))/c^5
```

3.70.6 Sympy [A] (verification not implemented)

Time = 4.38 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.83

$$\int x^2\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{adx^3}{3}+\frac{aex^5}{5}+\frac{bdx^3\operatorname{asec}\left(cx\right)}{3}+\frac{bex^5\operatorname{asec}\left(cx\right)}{5}$$
$$-\frac{bd\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\operatorname{acosh}\left(cx\right)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\operatorname{asin}\left(cx\right)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}$$
$$-\frac{be\left(\begin{cases}\frac{cx^5}{4\sqrt{c^2x^2-1}}+\frac{x^3}{8c\sqrt{c^2x^2-1}}-\frac{3x}{8c^3\sqrt{c^2x^2-1}}+\frac{3\operatorname{acosh}\left(cx\right)}{8c^4} & \text{for } |c^2x^2|>1\\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}}-\frac{ix^3}{8c\sqrt{-c^2x^2+1}}+\frac{3ix}{8c^3\sqrt{-c^2x^2+1}}-\frac{3i\operatorname{asin}\left(cx\right)}{8c^4} & \text{otherwise}\end{cases}\right)}{5c}$$

input
```
integrate(x**2*(e*x**2+d)*(a+b*asec(c*x)),x)
```

output
```
a*d*x**3/3 + a*e*x**5/5 + b*d*x**3*asec(c*x)/3 + b*e*x**5*asec(c*x)/5 - b*
d*Piecewise((x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x
**2) > 1), (-I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2
+ 1)) - I*asin(c*x)/(2*c**2), True))/(3*c) - b*e*Piecewise((c*x**5/(4*sqrt
(c**2*x**2 - 1)) + x**3/(8*c*sqrt(c**2*x**2 - 1)) - 3*x/(8*c**3*sqrt(c**2*
x**2 - 1)) + 3*acosh(c*x)/(8*c**4), Abs(c**2*x**2) > 1), (-I*c*x**5/(4*sqr
t(-c**2*x**2 + 1)) - I*x**3/(8*c*sqrt(-c**2*x**2 + 1)) + 3*I*x/(8*c**3*sqr
t(-c**2*x**2 + 1)) - 3*I*asin(c*x)/(8*c**4), True))/(5*c)
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.44

$$\int x^2\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{1}{5}aex^5+\frac{1}{3}adx^3$$
$$+\frac{1}{12}\left(4\,x^3\operatorname{arcsec}(cx)-\frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}-\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bd$$
$$+\frac{1}{80}\left(16\,x^5\operatorname{arcsec}(cx)+\frac{\frac{2\left(3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2\,c^4\left(\frac{1}{c^2x^2}-1\right)+c^4}-\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)be$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*e*x^5 + 1/3*a*d*x^3 + 1/12*(4*x^3*arcsec(c*x) - (2*sqrt(-1/(c^2*x^2)
 + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2
- log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d + 1/80*(16*x^5*arcsec(c*x) +
 (2*(3*(-1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(-1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x
^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^4) - 3*log(sqrt(-1/(c^2*x^2) + 1)
 + 1)/c^4 + 3*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*e
```

3.70.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9792 vs. 2(139) = 278.

Time = 1.93 (sec) , antiderivative size = 9792, normalized size of antiderivative = 60.82

$$\int x^2(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
1/120*(40*b*c^2*d*arccos(1/(c*x))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x)
+ 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2)
- 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(
1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10) - 20*b*c^2*d*log(abs(sqrt(-1/(c^2*x^2
) + 1) + 1/(c*x) + 1))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10
*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(
c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2)
 - 1)^5/(1/(c*x) + 1)^10) + 20*b*c^2*d*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/
(c*x) - 1))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^
2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6
 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/
(c*x) + 1)^10) + 40*a*c^2*d/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3
/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2
*x^2) - 1)^5/(1/(c*x) + 1)^10) - 40*b*c^2*d*(1/(c^2*x^2) - 1)*arccos(1/(c*
x))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2)
- 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^
6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) +
 1)^10)*(1/(c*x) + 1)^2) - 100*b*c^2*d*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(
c^2*x^2) + 1) + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x)) ...
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = \int x^2\left(e\,x^2+d\right)\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^2*(d + e*x^2)*(a + b*acos(1/(c*x))),x)
```

output
```
int(x^2*(d + e*x^2)*(a + b*acos(1/(c*x))), x)
```

3.71 $\int (d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$

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3.71.1 Optimal result

Integrand size = 16, antiderivative size = 109

$$\int (d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = -\frac{bex^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}} + dx\left(a+b\sec^{-1}(cx)\right) + \frac{1}{3}ex^3\left(a+b\sec^{-1}(cx)\right) - \frac{b(6c^2d+e)\,x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{6c^2\sqrt{c^2x^2}}$$

output
```
d*x*(a+b*arcsec(c*x))+1/3*e*x^3*(a+b*arcsec(c*x))-1/6*b*(6*c^2*d+e)*x*arct
anh(c*x/(c^2*x^2-1)^(1/2))/c^2/(c^2*x^2)^(1/2)-1/6*b*e*x^2*(c^2*x^2-1)^(1/
2)/c/(c^2*x^2)^(1/2)
```

3.71.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.38

$$\int (d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = adx + \frac{1}{3}aex^3 - \frac{bex^2\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{6c} + bdx\sec^{-1}(cx) + \frac{1}{3}bex^3\sec^{-1}(cx) - \frac{bd\sqrt{1-\frac{1}{c^2x^2}}\,x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{-1+c^2x^2}} - \frac{be\log\left(x\left(1+\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\right)\right)}{6c^3}$$

input
```
Integrate[(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

output
```
a*d*x + (a*e*x^3)/3 - (b*e*x^2*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(6*c) + b*d
*x*ArcSec[c*x] + (b*e*x^3*ArcSec[c*x])/3 - (b*d*Sqrt[1 - 1/(c^2*x^2)]*x*Ar
cTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/Sqrt[-1 + c^2*x^2] - (b*e*Log[x*(1 + Sqrt
[(-1 + c^2*x^2)/(c^2*x^2)])])/(6*c^3)
```

3.71.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5751, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\downarrow 5751$$

$$-\frac{bcx\int \frac{ex^2+3d}{3\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+dx\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 27$$

$$-\frac{bcx\int \frac{ex^2+3d}{\sqrt{c^2x^2-1}}dx}{3\sqrt{c^2x^2}}+dx\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 299$$

$$-\frac{bcx\left(\frac{\left(6c^2d+e\right)\int \frac{1}{\sqrt{c^2x^2-1}}dx}{2c^2}+\frac{ex\sqrt{c^2x^2-1}}{2c^2}\right)}{3\sqrt{c^2x^2}}+dx\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 224$$

$$-\frac{bcx\left(\frac{\left(6c^2d+e\right)\int \frac{1}{1-\frac{c^2x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}}{2c^2}+\frac{ex\sqrt{c^2x^2-1}}{2c^2}\right)}{3\sqrt{c^2x^2}}+dx\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 219$$

$$dx\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\sec^{-1}(cx)\right)-\frac{bcx\left(\frac{\operatorname{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)(6c^2d+e)}{2c^3}+\frac{ex\sqrt{c^2x^2-1}}{2c^2}\right)}{3\sqrt{c^2x^2}}$$

input
```
Int[(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

output
```
d*x*(a + b*ArcSec[c*x]) + (e*x^3*(a + b*ArcSec[c*x]))/3 - (b*c*x*((e*x*Sqr
t[-1 + c^2*x^2])/(2*c^2) + ((6*c^2*d + e)*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]
])/(2*c^3)))/(3*Sqrt[c^2*x^2])
```

3.71.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 5751
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSec[c*x])   u,
x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 -
 1]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1
/2, 0])
```

3.71.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.21

method	result
parts	$a\left(\frac{1}{3}x^3e+dx\right)+\frac{b\left(\frac{c\,\mathrm{arcsec}(cx)x^3e}{3}+\mathrm{arcsec}(cx)xcd-\frac{\sqrt{c^2x^2-1}\left(6dc^2\ln\left(cx+\sqrt{c^2x^2-1}\right)+ecx\sqrt{c^2x^2-1}+e\ln\left(cx+\sqrt{c^2x^2}\right.\right.\right.}{6c^3x\sqrt{\frac{c^2x^2-1}{c^2x^2}}}}{c}$
derivativedivides	$\frac{a\left(c^3dx+\frac{1}{3}ec^3x^3\right)}{c^2}+\frac{b\left(\mathrm{arcsec}(cx)dc^3x+\frac{\mathrm{arcsec}(cx)ec^3x^3}{3}-\frac{\sqrt{c^2x^2-1}\left(6dc^2\ln\left(cx+\sqrt{c^2x^2-1}\right)+ecx\sqrt{c^2x^2-1}+e\ln\left(cx+\sqrt{c^2x^2-1}\right.\right.\right.}{6cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}}}{c^2}}{c}$
default	$\frac{a\left(c^3dx+\frac{1}{3}ec^3x^3\right)}{c^2}+\frac{b\left(\mathrm{arcsec}(cx)dc^3x+\frac{\mathrm{arcsec}(cx)ec^3x^3}{3}-\frac{\sqrt{c^2x^2-1}\left(6dc^2\ln\left(cx+\sqrt{c^2x^2-1}\right)+ecx\sqrt{c^2x^2-1}+e\ln\left(cx+\sqrt{c^2x^2-1}\right.\right.\right.}{6cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}}}{c^2}}{c}$

input
```
int((e*x^2+d)*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*x^3*e+d*x)+b/c*(1/3*c*arcsec(c*x)*x^3*e+arcsec(c*x)*x*c*d-1/6/c^3*(
c^2*x^2-1)^(1/2)*(6*d*c^2*ln(c*x+(c^2*x^2-1)^(1/2))+e*c*x*(c^2*x^2-1)^(1/2
)+e*ln(c*x+(c^2*x^2-1)^(1/2)))/x/((c^2*x^2-1)/c^2/x^2)^(1/2))
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.29

$$\int \left(d+ex^2\right)\left(a+b\,\mathrm{sec}^{-1}(cx)\right)\,dx$$
$$=\frac{2\,ac^3ex^3+6\,ac^3dx-\sqrt{c^2x^2-1}bcex+2\left(bc^3ex^3+3\,bc^3dx-3\,bc^3d-bc^3e\right)\mathrm{arcsec}\left(cx\right)+4\left(3\,bc^3d+bc^3e\right)}{6\,c^3}$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/6*(2*a*c^3*e*x^3 + 6*a*c^3*d*x - sqrt(c^2*x^2 - 1)*b*c*e*x + 2*(b*c^3*e*
x^3 + 3*b*c^3*d*x - 3*b*c^3*d - b*c^3*e)*arcsec(c*x) + 4*(3*b*c^3*d + b*c^
3*e)*arctan(-c*x + sqrt(c^2*x^2 - 1)) + (6*b*c^2*d + b*e)*log(-c*x + sqrt(
c^2*x^2 - 1)))/c^3
```

3.71.6 Sympy [A] (verification not implemented)

Time = 3.22 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.40

$$\int \left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$= adx + \frac{aex^3}{3} + bdx\,\mathrm{asec}\left(cx\right) + \frac{bex^3\,\mathrm{asec}\left(cx\right)}{3} - \frac{bd\left(\begin{cases}\mathrm{acosh}\left(cx\right) & \text{for } \left|c^2x^2\right| > 1 \\ -i\,\mathrm{asin}\left(cx\right) & \text{otherwise}\end{cases}\right)}{c}$$
$$- \frac{be\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c} + \frac{\mathrm{acosh}\left(cx\right)}{2c^2} & \text{for } \left|c^2x^2\right| > 1 \\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}} + \frac{ix}{2c\sqrt{-c^2x^2+1}} - \frac{i\,\mathrm{asin}\left(cx\right)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}$$

input
```
integrate((e*x**2+d)*(a+b*asec(c*x)),x)
```

output
```
a*d*x + a*e*x**3/3 + b*d*x*asec(c*x) + b*e*x**3*asec(c*x)/3 - b*d*Piecewis
e((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x), True))/c - b*e*Piecewis
e((x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1),
 (-I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*
asin(c*x)/(2*c**2), True))/(3*c)
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.41

$$\int \left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$= \frac{1}{3}\,aex^3 + \frac{1}{12}\left(4\,x^3\,\mathrm{arcsec}\left(cx\right) - \frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)be$$
$$+ adx + \frac{\left(2\,cx\,\mathrm{arcsec}\left(cx\right) - \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) + \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bd}{2\,c}$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*e*x^3 + 1/12*(4*x^3*arcsec(c*x) - (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/
(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/
(c^2*x^2) + 1) - 1)/c^2)/c)*b*e + a*d*x + 1/2*(2*c*x*arcsec(c*x) - log(sqr
t(-1/(c^2*x^2) + 1) + 1) + log(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*d/c
```

3.71.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4051 vs. $2(95) = 190$.

Time $= 1.28$ (sec) , antiderivative size $= 4051$, normalized size of antiderivative $= 37.17$

$$\int \left(d + ex^2\right)\left(a + b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
1/6*(6*b*c^2*d*arccos(1/(c*x))/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1
)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/
(1/(c*x) + 1)^6) - 6*b*c^2*d*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1)
)/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)
^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6) + 6*b*c^2*d*
log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(c^4 + 3*c^4*(1/(c^2*x^2) -
 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(
c^2*x^2) - 1)^3/(1/(c*x) + 1)^6) + 6*a*c^2*d/(c^4 + 3*c^4*(1/(c^2*x^2) - 1
)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^
2*x^2) - 1)^3/(1/(c*x) + 1)^6) + 6*b*c^2*d*(1/(c^2*x^2) - 1)*arccos(1/(c*x
))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) -
1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) +
 1)^2) - 18*b*c^2*d*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(
c*x) + 1))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2
*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1
/(c*x) + 1)^2) + 18*b*c^2*d*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) +
1) - 1/(c*x) - 1))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4
*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) +
1)^6)*(1/(c*x) + 1)^2) + 6*a*c^2*d*(1/(c^2*x^2) - 1)/((c^4 + 3*c^4*(1/(c^2
*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 ...
```

3.71.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = \int (e\,x^2+d)\left(a+b\,\text{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int((d + e*x^2)*(a + b*acos(1/(c*x))),x)
```

output
```
int((d + e*x^2)*(a + b*acos(1/(c*x))), x)
```

3.72 $\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x^2}\,dx$

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3.72.1 Optimal result

Integrand size = 19, antiderivative size = 87

$$\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x^2}\,dx = \frac{bcd\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} - \frac{d(a+b\sec^{-1}(cx))}{x} + ex\left(a+b\sec^{-1}(cx)\right) - \frac{bex\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{c^2x^2}}$$

output
```
-d*(a+b*arcsec(c*x))/x+e*x*(a+b*arcsec(c*x))-b*e*x*arctanh(c*x/(c^2*x^2-1)
^(1/2))/(c^2*x^2)^(1/2)+b*c*d*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)
```

3.72.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x^2}\,dx = -\frac{ad}{x} + aex + bcd\sqrt{\frac{-1+c^2x^2}{c^2x^2}} - \frac{bd\sec^{-1}(cx)}{x} + bex\sec^{-1}(cx) - \frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{-1+c^2x^2}}$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^2,x]
```

output
```
-((a*d)/x) + a*e*x + b*c*d*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)] - (b*d*ArcSec[c*
x])/x + b*e*x*ArcSec[c*x] - (b*e*Sqrt[1 - 1/(c^2*x^2)]*x*ArcTanh[(c*x)/Sqr
t[-1 + c^2*x^2]])/Sqrt[-1 + c^2*x^2]
```

3.72.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5761, 25, 358, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)}{x^2}\,dx$$

$$\downarrow 5761$$

$$-\frac{bcx\int -\frac{d-ex^2}{x^2\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}} - \frac{d\left(a+b\sec^{-1}(cx)\right)}{x} + ex\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 25$$

$$\frac{bcx\int \frac{d-ex^2}{x^2\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}} - \frac{d\left(a+b\sec^{-1}(cx)\right)}{x} + ex\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 358$$

$$\frac{bcx\left(\frac{d\sqrt{c^2x^2-1}}{x} - e\int \frac{1}{\sqrt{c^2x^2-1}}dx\right)}{\sqrt{c^2x^2}} - \frac{d\left(a+b\sec^{-1}(cx)\right)}{x} + ex\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 224$$

$$\frac{bcx\left(\frac{d\sqrt{c^2x^2-1}}{x} - e\int \frac{1}{1-\frac{c^2x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}\right)}{\sqrt{c^2x^2}} - \frac{d\left(a+b\sec^{-1}(cx)\right)}{x} + ex\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 219$$

$$-\frac{d\left(a+b\sec^{-1}(cx)\right)}{x} + ex\left(a+b\sec^{-1}(cx)\right) + \frac{bcx\left(\frac{d\sqrt{c^2x^2-1}}{x} - \frac{e\operatorname{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{c}\right)}{\sqrt{c^2x^2}}$$

input
```
Int[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^2,x]
```

output
```
-((d*(a + b*ArcSec[c*x]))/x) + e*x*(a + b*ArcSec[c*x]) + (b*c*x*((d*Sqrt[-
1 + c^2*x^2])/x - (e*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/c))/Sqrt[c^2*x^2]
```

3.72.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.72.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.33

method	result	siz
parts	$a\left(ex-\frac{d}{x}\right)+bc\left(\frac{\operatorname{arcsec}(cx)ex}{c}-\frac{\operatorname{arcsec}(cx)d}{xc}-\frac{\sqrt{c^2x^2-1}\left(-dc^2\sqrt{c^2x^2-1}+e\ln\left(cx+\sqrt{c^2x^2-1}\right)cx\right)}{c^4x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right)$	11
derivativedivides	$c\left(\frac{a\left(cex-\frac{dc}{x}\right)}{c^2}+\frac{b\left(c\operatorname{arcsec}(cx)xe-\frac{\operatorname{arcsec}(cx)dc}{x}-\frac{\sqrt{c^2x^2-1}\left(-dc^2\sqrt{c^2x^2-1}+e\ln\left(cx+\sqrt{c^2x^2-1}\right)cx\right)}{c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right)}{c^2}\right)$	12
default	$c\left(\frac{a\left(cex-\frac{dc}{x}\right)}{c^2}+\frac{b\left(c\operatorname{arcsec}(cx)xe-\frac{\operatorname{arcsec}(cx)dc}{x}-\frac{\sqrt{c^2x^2-1}\left(-dc^2\sqrt{c^2x^2-1}+e\ln\left(cx+\sqrt{c^2x^2-1}\right)cx\right)}{c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right)}{c^2}\right)$	12

input
```
int((e*x^2+d)*(a+b*arcsec(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(e*x-d/x)+b*c*(1/c*arcsec(c*x)*e*x-arcsec(c*x)*d/x/c-1/c^4*(c^2*x^2-1)^(
1/2)*(-d*c^2*(c^2*x^2-1)^(1/2)+e*ln(c*x+(c^2*x^2-1)^(1/2))*c*x)/x^2/((c^2*
x^2-1)/c^2/x^2)^(1/2))
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.41

$$\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x^2}\,dx$$
$$=\frac{bc^2dx+acex^2+bex\log\left(-cx+\sqrt{c^2x^2-1}\right)+\sqrt{c^2x^2-1}bcd-acd-2\left(bcd-bce\right)x\arctan\left(-cx+\sqrt{c^2x}\right.}{cx}$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^2,x, algorithm="fricas")
```

output
```
(b*c^2*d*x + a*c*e*x^2 + b*e*x*log(-c*x + sqrt(c^2*x^2 - 1)) + sqrt(c^2*x^
2 - 1)*b*c*d - a*c*d - 2*(b*c*d - b*c*e)*x*arctan(-c*x + sqrt(c^2*x^2 - 1)
) + (b*c*e*x^2 - b*c*d + (b*c*d - b*c*e)*x)*arcsec(c*x))/(c*x)
```

3.72.6 Sympy [A] (verification not implemented)

Time = 2.72 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x^2}\,dx = -\frac{ad}{x} + aex + bcd\sqrt{1-\frac{1}{c^2x^2}} - \frac{bd\,\mathrm{asec}\,(cx)}{x} + bex\,\mathrm{asec}\,(cx) - \frac{be\left(\begin{cases}\mathrm{acosh}\,(cx) & \text{for } |c^2x^2|>1\\ -i\,\mathrm{asin}\,(cx) & \text{otherwise}\end{cases}\right)}{c}$$

input
```
integrate((e*x**2+d)*(a+b*asec(c*x))/x**2,x)
```

output
```
-a*d/x + a*e*x + b*c*d*sqrt(1 - 1/(c**2*x**2)) - b*d*asec(c*x)/x + b*e*x*a
sec(c*x) - b*e*Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x),
True))/c
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x^2}\,dx = \left(c\sqrt{-\frac{1}{c^2x^2}+1} - \frac{\mathrm{arcsec}\,(cx)}{x}\right)bd + aex + \frac{\left(2\,cx\,\mathrm{arcsec}\,(cx) - \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) + \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)be}{2\,c} - \frac{ad}{x}$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^2,x, algorithm="maxima")
```

output
```
(c*sqrt(-1/(c^2*x^2) + 1) - arcsec(c*x)/x)*b*d + a*e*x + 1/2*(2*c*x*arcsec
(c*x) - log(sqrt(-1/(c^2*x^2) + 1) + 1) + log(-sqrt(-1/(c^2*x^2) + 1) + 1)
)*b*e/c - a*d/x
```

3.72.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1088 vs. 2(79) = 158.

Time = 0.63 (sec) , antiderivative size = 1088, normalized size of antiderivative = 12.51

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^2,x, algorithm="giac")
```

output
```
-(b*c^2*d*arccos(1/(c*x))/(c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)
+ a*c^2*d/(c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) + 2*b*c^2*d*(1/(
c^2*x^2) - 1)*arccos(1/(c*x))/((c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1
)^4)*(1/(c*x) + 1)^2) - 2*b*c^2*d*sqrt(-1/(c^2*x^2) + 1)/((c^2 - c^2*(1/(c
^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)) + 2*a*c^2*d*(1/(c^2*x^2) -
1)/((c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) - b*e
*arccos(1/(c*x))/(c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) + b*c^2*d
*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c
*x) + 1)^4)*(1/(c*x) + 1)^4) + b*e*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x
) + 1))/(c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) - b*e*log(abs(sqrt
(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x)
 + 1)^4) + 2*b*c^2*d*(-1/(c^2*x^2) + 1)^(3/2)/((c^2 - c^2*(1/(c^2*x^2) - 1
)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^3) - a*e/(c^2 - c^2*(1/(c^2*x^2) - 1)^2
/(1/(c*x) + 1)^4) + a*c^2*d*(1/(c^2*x^2) - 1)^2/((c^2 - c^2*(1/(c^2*x^2) -
 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4) + 2*b*e*(1/(c^2*x^2) - 1)*arccos(1
/(c*x))/((c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2)
+ 2*a*e*(1/(c^2*x^2) - 1)/((c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)
*(1/(c*x) + 1)^2) - b*e*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^2 - c^2*(1
/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4) - b*e*(1/(c^2*x^2) - 1
)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^2 - c^2*(1/(c^2*...
```

3.72.9 Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^2}\,dx = a\,e\,x - \frac{d\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right) - b\,c\,x\,\sqrt{1-\frac{1}{c^2\,x^2}}\right)}{x} - \frac{b\,e\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{c^2\,x^2}}}\right)}{c} + b\,e\,x\,\mathrm{acos}\left(\frac{1}{c\,x}\right)$$

input
```
int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x^2,x)
```

output
```
a*e*x - (d*(a + b*acos(1/(c*x)) - b*c*x*(1 - 1/(c^2*x^2))^(1/2)))/x - (b*e
*atanh(1/(1 - 1/(c^2*x^2))^(1/2)))/c + b*e*x*acos(1/(c*x))
```

3.73 $\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x^4}\,dx$

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3.73.1 Optimal result

Integrand size = 19, antiderivative size = 105

$$\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x^4}\,dx = \frac{bc(2c^2d+9e)\sqrt{-1+c^2x^2}}{9\sqrt{c^2x^2}} + \frac{bcd\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}} - \frac{d(a+b\sec^{-1}(cx))}{3x^3} - \frac{e(a+b\sec^{-1}(cx))}{x}$$

output
```
-1/3*d*(a+b*arcsec(c*x))/x^3-e*(a+b*arcsec(c*x))/x+1/9*b*c*(2*c^2*d+9*e)*(
c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)+1/9*b*c*d*(c^2*x^2-1)^(1/2)/x^2/(c^2*x^2)
^(1/2)
```

3.73.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.66

$$\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x^4}\,dx = \frac{-3a(d+3ex^2)+bc\sqrt{1-\frac{1}{c^2x^2}}x(d+2c^2dx^2+9ex^2)-3b(d+3ex^2)\sec^{-1}(cx)}{9x^3}$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^4,x]
```

output
```
(-3*a*(d + 3*e*x^2) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + 2*c^2*d*x^2 + 9*e*x
^2) - 3*b*(d + 3*e*x^2)*ArcSec[c*x])/(9*x^3)
```

3.73.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {5761, 27, 359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x^4}\,dx$$

$$\downarrow 5761$$

$$-\frac{bcx\int -\frac{3ex^2+d}{3x^4\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}} - \frac{d\left(a+b\sec^{-1}(cx)\right)}{3x^3} - \frac{e\left(a+b\sec^{-1}(cx)\right)}{x}$$

$$\downarrow 27$$

$$\frac{bcx\int \frac{3ex^2+d}{x^4\sqrt{c^2x^2-1}}dx}{3\sqrt{c^2x^2}} - \frac{d\left(a+b\sec^{-1}(cx)\right)}{3x^3} - \frac{e\left(a+b\sec^{-1}(cx)\right)}{x}$$

$$\downarrow 359$$

$$\frac{bcx\left(\frac{1}{3}\left(2c^2d+9e\right)\int \frac{1}{x^2\sqrt{c^2x^2-1}}dx + \frac{d\sqrt{c^2x^2-1}}{3x^3}\right)}{3\sqrt{c^2x^2}} - \frac{d\left(a+b\sec^{-1}(cx)\right)}{3x^3} - \frac{e\left(a+b\sec^{-1}(cx)\right)}{x}$$

$$\downarrow 242$$

$$-\frac{d\left(a+b\sec^{-1}(cx)\right)}{3x^3} - \frac{e\left(a+b\sec^{-1}(cx)\right)}{x} + \frac{bcx\left(\frac{\sqrt{c^2x^2-1}\left(2c^2d+9e\right)}{3x} + \frac{d\sqrt{c^2x^2-1}}{3x^3}\right)}{3\sqrt{c^2x^2}}$$

input
```
Int[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^4,x]
```

output
```
(b*c*x*((d*Sqrt[-1 + c^2*x^2])/(3*x^3) + ((2*c^2*d + 9*e)*Sqrt[-1 + c^2*x^
2])/(3*x)))/(3*Sqrt[c^2*x^2]) - (d*(a + b*ArcSec[c*x]))/(3*x^3) - (e*(a +
b*ArcSec[c*x]))/x
```

3.73.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.73.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.03

method	result	size
parts	$a\left(-\frac{e}{x}-\frac{d}{3x^3}\right)+b\,c^3\left(-\frac{\operatorname{arcsec}(cx)e}{c^3x}-\frac{\operatorname{arcsec}(cx)d}{3x^3c^3}+\frac{\left(c^2x^2-1\right)\left(2c^4dx^2+9c^2ex^2+c^2d\right)}{9c^6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^4}\right)$	108
derivativedivides	$c^3\left(\frac{a\left(-\frac{d}{3cx^3}-\frac{e}{cx}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arcsec}(cx)d}{3cx^3}-\frac{\operatorname{arcsec}(cx)e}{cx}+\frac{\left(c^2x^2-1\right)\left(2c^4dx^2+9c^2ex^2+c^2d\right)}{9\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^4x^4}\right)}{c^2}\right)$	121
default	$c^3\left(\frac{a\left(-\frac{d}{3cx^3}-\frac{e}{cx}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arcsec}(cx)d}{3cx^3}-\frac{\operatorname{arcsec}(cx)e}{cx}+\frac{\left(c^2x^2-1\right)\left(2c^4dx^2+9c^2ex^2+c^2d\right)}{9\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^4x^4}\right)}{c^2}\right)$	121

input
```
int((e*x^2+d)*(a+b*arcsec(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
a*(-e/x-1/3*d/x^3)+b*c^3*(-1/c^3*arcsec(c*x)*e/x-1/3*arcsec(c*x)*d/x^3/c^3
+1/9/c^6*(c^2*x^2-1)*(2*c^4*d*x^2+9*c^2*e*x^2+c^2*d)/((c^2*x^2-1)/c^2/x^2)
^(1/2)/x^4)
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.64

$$\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x^4}\,dx$$
$$= -\frac{9\,aex^2+3\,ad+3\,(3\,bex^2+bd)\operatorname{arcsec}(cx)-\sqrt{c^2x^2-1}\left((2\,bc^2d+9\,be)x^2+bd\right)}{9\,x^3}$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^4,x, algorithm="fricas")
```

output
```
-1/9*(9*a*e*x^2 + 3*a*d + 3*(3*b*e*x^2 + b*d)*arcsec(c*x) - sqrt(c^2*x^2 -
 1)*((2*b*c^2*d + 9*b*e)*x^2 + b*d))/x^3
```

3.73.6 Sympy [A] (verification not implemented)

Time = 2.11 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.43

$$\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x^4}\,dx = -\frac{ad}{3x^3} - \frac{ae}{x} + bce\sqrt{1-\frac{1}{c^2x^2}} - \frac{bd\,\text{asec}\,(cx)}{3x^3} - \frac{be\,\text{asec}\,(cx)}{x} + \frac{bd\left(\begin{cases} \frac{2c^3\sqrt{c^2x^2-1}}{3x} + \frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2| > 1 \\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x} + \frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise} \end{cases}\right)}{3c}$$

input
```
integrate((e*x**2+d)*(a+b*asec(c*x))/x**4,x)
```

output
```
-a*d/(3*x**3) - a*e/x + b*c*e*sqrt(1 - 1/(c**2*x**2)) - b*d*asec(c*x)/(3*x
**3) - b*e*asec(c*x)/x + b*d*Piecewise((2*c**3*sqrt(c**2*x**2 - 1)/(3*x) +
 c*sqrt(c**2*x**2 - 1)/(3*x**3), Abs(c**2*x**2) > 1), (2*I*c**3*sqrt(-c**2
*x**2 + 1)/(3*x) + I*c*sqrt(-c**2*x**2 + 1)/(3*x**3), True))/(3*c)
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\begin{aligned}\int & \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x^4}\,dx \\ &= \left(c\sqrt{-\frac{1}{c^2x^2}+1} - \frac{\text{arcsec}\,(cx)}{x}\right)be \\ &\quad - \frac{1}{9}\,bd\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c} + \frac{3\,\text{arcsec}\,(cx)}{x^3}\right) - \frac{ae}{x} - \frac{ad}{3\,x^3}\end{aligned}$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^4,x, algorithm="maxima")
```

output
```
(c*sqrt(-1/(c^2*x^2) + 1) - arcsec(c*x)/x)*b*e - 1/9*b*d*((c^4*(-1/(c^2*x^
2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c + 3*arcsec(c*x)/x^3) - a*e
/x - 1/3*a*d/x^3
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^4}\,dx$$

$$= \frac{1}{9}\left(2\,bc^2d\sqrt{-\frac{1}{c^2x^2}+1}+9\,be\sqrt{-\frac{1}{c^2x^2}+1}-\frac{9\,be\arccos\left(\frac{1}{cx}\right)}{cx}+\frac{bd\sqrt{-\frac{1}{c^2x^2}+1}}{x^2}-\frac{9\,ae}{cx}-\frac{3\,bd\arccos\left(\frac{1}{cx}\right)}{cx^3}\right.$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^4,x, algorithm="giac")
```

output
```
1/9*(2*b*c^2*d*sqrt(-1/(c^2*x^2) + 1) + 9*b*e*sqrt(-1/(c^2*x^2) + 1) - 9*b
*e*arccos(1/(c*x))/(c*x) + b*d*sqrt(-1/(c^2*x^2) + 1)/x^2 - 9*a*e/(c*x) -
3*b*d*arccos(1/(c*x))/(c*x^3) - 3*a*d/(c*x^3))*c
```

3.73.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^4}\,dx = \int \frac{(e\,x^2+d)\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

input
```
int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x^4,x)
```

output
```
int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x^4, x)
```

3.74 $\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x^6}\,dx$

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3.74.1 Optimal result

Integrand size = 19, antiderivative size = 152

$$\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x^6}\,dx = \frac{2bc^3(12c^2d+25e)\sqrt{-1+c^2x^2}}{225\sqrt{c^2x^2}} + \frac{bcd\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}} + \frac{bc(12c^2d+25e)\sqrt{-1+c^2x^2}}{225x^2\sqrt{c^2x^2}} - \frac{d\left(a+b\sec^{-1}(cx)\right)}{5x^5} - \frac{e\left(a+b\sec^{-1}(cx)\right)}{3x^3}$$

output

```
-1/5*d*(a+b*arcsec(c*x))/x^5-1/3*e*(a+b*arcsec(c*x))/x^3+2/225*b*c^3*(12*c
^2*d+25*e)*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)+1/25*b*c*d*(c^2*x^2-1)^(1/2)/
x^4/(c^2*x^2)^(1/2)+1/225*b*c*(12*c^2*d+25*e)*(c^2*x^2-1)^(1/2)/x^2/(c^2*x
^2)^(1/2)
```

3.74.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.62

$$\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x^6}\,dx = \frac{-15a(3d+5ex^2)+bc\sqrt{1-\frac{1}{c^2x^2}}x(25ex^2(1+2c^2x^2)+3d(3+4c^2x^2+8c^4x^4))-15b(3d+5ex^2)\sec^{-1}(cx}{225x^5}$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^6,x]
```

output
```
(-15*a*(3*d + 5*e*x^2) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(25*e*x^2*(1 + 2*c^2*
x^2) + 3*d*(3 + 4*c^2*x^2 + 8*c^4*x^4)) - 15*b*(3*d + 5*e*x^2)*ArcSec[c*x]
)/(225*x^5)
```

3.74.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5761, 27, 359, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x^6}\,dx$$

$$\downarrow 5761$$

$$-\frac{bcx\int -\frac{5ex^2+3d}{15x^6\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}} - \frac{d\left(a+b\sec^{-1}(cx)\right)}{5x^5} - \frac{e\left(a+b\sec^{-1}(cx)\right)}{3x^3}$$

$$\downarrow 27$$

$$\frac{bcx\int \frac{5ex^2+3d}{x^6\sqrt{c^2x^2-1}}dx}{15\sqrt{c^2x^2}} - \frac{d\left(a+b\sec^{-1}(cx)\right)}{5x^5} - \frac{e\left(a+b\sec^{-1}(cx)\right)}{3x^3}$$

$$\downarrow 359$$

$$\frac{bcx\left(\frac{1}{5}\left(12c^2d+25e\right)\int \frac{1}{x^4\sqrt{c^2x^2-1}}dx + \frac{3d\sqrt{c^2x^2-1}}{5x^5}\right)}{15\sqrt{c^2x^2}} - \frac{d\left(a+b\sec^{-1}(cx)\right)}{5x^5} - \frac{e\left(a+b\sec^{-1}(cx)\right)}{3x^3}$$

$$\downarrow 245$$

$$\frac{bcx\left(\frac{1}{5}\left(12c^2d+25e\right)\left(\frac{2}{3}c^2\int \frac{1}{x^2\sqrt{c^2x^2-1}}dx + \frac{\sqrt{c^2x^2-1}}{3x^3}\right) + \frac{3d\sqrt{c^2x^2-1}}{5x^5}\right)}{15\sqrt{c^2x^2}} - \frac{d\left(a+b\sec^{-1}(cx)\right)}{5x^5} - \frac{e\left(a+b\sec^{-1}(cx)\right)}{3x^3}$$

$$\downarrow 242$$

$$\frac{-\frac{d\left(a+b\sec^{-1}(cx)\right)}{5x^5}-\frac{e\left(a+b\sec^{-1}(cx)\right)}{3x^3}+\frac{bcx\left(\frac{1}{5}\left(\frac{2c^2\sqrt{c^2x^2-1}}{3x}+\frac{\sqrt{c^2x^2-1}}{3x^3}\right)\left(12c^2d+25e\right)+\frac{3d\sqrt{c^2x^2-1}}{5x^5}\right)}{15\sqrt{c^2x^2}}$$

input
```
Int[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^6,x]
```

output
```
(b*c*x*((3*d*Sqrt[-1 + c^2*x^2])/(5*x^5) + ((12*c^2*d + 25*e)*(Sqrt[-1 + c
^2*x^2]/(3*x^3) + (2*c^2*Sqrt[-1 + c^2*x^2])/(3*x)))/5))/(15*Sqrt[c^2*x^2]
) - (d*(a + b*ArcSec[c*x]))/(5*x^5) - (e*(a + b*ArcSec[c*x]))/(3*x^3)
```

3.74.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 5761

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])  u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])  Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.74.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.84

method	result
parts	$a\left(-\frac{d}{5x^5}-\frac{e}{3x^3}\right)+bc^5\left(-\frac{\operatorname{arcsec}(cx)d}{5x^5c^5}-\frac{\operatorname{arcsec}(cx)e}{3c^5x^3}+\frac{\left(c^2x^2-1\right)\left(24c^6dx^4+50c^4ex^4+12c^4dx^2+25c^2ex^2+9\right.}{225c^8\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^6}\right.$
derivativedivides	$c^5\left(\frac{a\left(-\frac{d}{5c^3x^5}-\frac{e}{3c^3x^3}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arcsec}(cx)d}{5c^3x^5}-\frac{\operatorname{arcsec}(cx)e}{3c^3x^3}+\frac{\left(c^2x^2-1\right)\left(24c^6dx^4+50c^4ex^4+12c^4dx^2+25c^2ex^2+9c^2d\right)}{225\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right)}{c^2}\right.$
default	$c^5\left(\frac{a\left(-\frac{d}{5c^3x^5}-\frac{e}{3c^3x^3}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arcsec}(cx)d}{5c^3x^5}-\frac{\operatorname{arcsec}(cx)e}{3c^3x^3}+\frac{\left(c^2x^2-1\right)\left(24c^6dx^4+50c^4ex^4+12c^4dx^2+25c^2ex^2+9c^2d\right)}{225\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right)}{c^2}\right.$

input

```
int((e*x^2+d)*(a+b*arcsec(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output

```
a*(-1/5*d/x^5-1/3*e/x^3)+b*c^5*(-1/5*arcsec(c*x)*d/x^5/c^5-1/3/c^5*arcsec(
c*x)*e/x^3+1/225/c^8*(c^2*x^2-1)*(24*c^6*d*x^4+50*c^4*e*x^4+12*c^4*d*x^2+2
5*c^2*e*x^2+9*c^2*d)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^6)
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.59

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^6}\,dx =$$
$$-\frac{75\,aex^2+45\,ad+15\,(5\,bex^2+3\,bd)\operatorname{arcsec}(cx)-(2\,(12\,bc^4d+25\,bc^2e)x^4+(12\,bc^2d+25\,be)x^2+9\,bd)}{225\,x^5}$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^6,x, algorithm="fricas")
```

output
```
-1/225*(75*a*e*x^2 + 45*a*d + 15*(5*b*e*x^2 + 3*b*d)*arcsec(c*x) - (2*(12*
b*c^4*d + 25*b*c^2*e)*x^4 + (12*b*c^2*d + 25*b*e)*x^2 + 9*b*d)*sqrt(c^2*x^
2 - 1))/x^5
```

3.74.6 Sympy [A] (verification not implemented)

Time = 4.70 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.84

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^6}\,dx$$
$$= -\frac{ad}{5x^5}-\frac{ae}{3x^3}-\frac{bd\operatorname{asec}(cx)}{5x^5}-\frac{be\operatorname{asec}(cx)}{3x^3}$$
$$+\frac{bd\left(\begin{cases}\frac{8c^5\sqrt{c^2x^2-1}}{15x}+\frac{4c^3\sqrt{c^2x^2-1}}{15x^3}+\frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for } |c^2x^2|>1\\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x}+\frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3}+\frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise}\end{cases}\right)}{5c}$$
$$+\frac{be\left(\begin{cases}\frac{2c^3\sqrt{c^2x^2-1}}{3x}+\frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2|>1\\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x}+\frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)}{3c}$$

input
```
integrate((e*x**2+d)*(a+b*asec(c*x))/x**6,x)
```

output
```
-a*d/(5*x**5) - a*e/(3*x**3) - b*d*asec(c*x)/(5*x**5) - b*e*asec(c*x)/(3*x
**3) + b*d*Piecewise((8*c**5*sqrt(c**2*x**2 - 1)/(15*x) + 4*c**3*sqrt(c**2
*x**2 - 1)/(15*x**3) + c*sqrt(c**2*x**2 - 1)/(5*x**5), Abs(c**2*x**2) > 1)
, (8*I*c**5*sqrt(-c**2*x**2 + 1)/(15*x) + 4*I*c**3*sqrt(-c**2*x**2 + 1)/(1
5*x**3) + I*c*sqrt(-c**2*x**2 + 1)/(5*x**5), True))/(5*c) + b*e*Piecewise(
(2*c**3*sqrt(c**2*x**2 - 1)/(3*x) + c*sqrt(c**2*x**2 - 1)/(3*x**3), Abs(c*
*2*x**2) > 1), (2*I*c**3*sqrt(-c**2*x**2 + 1)/(3*x) + I*c*sqrt(-c**2*x**2
+ 1)/(3*x**3), True))/(3*c)
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^6}\,dx$$
$$= \frac{1}{75}\,bd\left(\frac{3\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} - 10\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 15\,c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c} - \frac{15\,\operatorname{arcsec}(cx)}{x^5}\right)$$
$$- \frac{1}{9}\,be\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c} + \frac{3\,\operatorname{arcsec}(cx)}{x^3}\right) - \frac{ae}{3\,x^3} - \frac{ad}{5\,x^5}$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^6,x, algorithm="maxima")
```

output
```
1/75*b*d*((3*c^6*(-1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(-1/(c^2*x^2) + 1)^(3/2
) + 15*c^6*sqrt(-1/(c^2*x^2) + 1))/c - 15*arcsec(c*x)/x^5) - 1/9*b*e*((c^4
*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c + 3*arcsec(c*x
)/x^3) - 1/3*a*e/x^3 - 1/5*a*d/x^5
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^6}\,dx$$
$$= \frac{1}{225}\left(24\,bc^4d\sqrt{-\frac{1}{c^2x^2}+1} + 50\,bc^2e\sqrt{-\frac{1}{c^2x^2}+1} + \frac{12\,bc^2d\sqrt{-\frac{1}{c^2x^2}+1}}{x^2} + \frac{25\,be\sqrt{-\frac{1}{c^2x^2}+1}}{x^2} - \frac{75\,be\,\operatorname{arcc}}{cx^3}\right.$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^6,x, algorithm="giac")
```

output
```
1/225*(24*b*c^4*d*sqrt(-1/(c^2*x^2) + 1) + 50*b*c^2*e*sqrt(-1/(c^2*x^2) +
1) + 12*b*c^2*d*sqrt(-1/(c^2*x^2) + 1)/x^2 + 25*b*e*sqrt(-1/(c^2*x^2) + 1)
/x^2 - 75*b*e*arccos(1/(c*x))/(c*x^3) + 9*b*d*sqrt(-1/(c^2*x^2) + 1)/x^4 -
 75*a*e/(c*x^3) - 45*b*d*arccos(1/(c*x))/(c*x^5) - 45*a*d/(c*x^5))*c
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^6}\,dx = \int \frac{(e\,x^2+d)\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

input
```
int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x^6,x)
```

output
```
int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x^6, x)
```

3.75 $\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x^8}\,dx$

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3.75.1 Optimal result

Integrand size = 19, antiderivative size = 197

$$\begin{aligned}\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x^8}\,dx &= \frac{8bc^5(30c^2d+49e)\sqrt{-1+c^2x^2}}{3675\sqrt{c^2x^2}} + \frac{bcd\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} \\ &+ \frac{bc(30c^2d+49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}} \\ &+ \frac{4bc^3(30c^2d+49e)\sqrt{-1+c^2x^2}}{3675x^2\sqrt{c^2x^2}} \\ &- \frac{d\left(a+b\sec^{-1}(cx)\right)}{7x^7} - \frac{e\left(a+b\sec^{-1}(cx)\right)}{5x^5}\end{aligned}$$

output

```
-1/7*d*(a+b*arcsec(c*x))/x^7-1/5*e*(a+b*arcsec(c*x))/x^5+8/3675*b*c^5*(30*
c^2*d+49*e)*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)+1/49*b*c*d*(c^2*x^2-1)^(1/2)
/x^6/(c^2*x^2)^(1/2)+1/1225*b*c*(30*c^2*d+49*e)*(c^2*x^2-1)^(1/2)/x^4/(c^2
*x^2)^(1/2)+4/3675*b*c^3*(30*c^2*d+49*e)*(c^2*x^2-1)^(1/2)/x^2/(c^2*x^2)^(
1/2)
```

3.75.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.56

$$\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x^8}\,dx$$
$$= \frac{-105a(5d+7ex^2)+bc\sqrt{1-\frac{1}{c^2x^2}}x(49ex^2(3+4c^2x^2+8c^4x^4)+15d(5+6c^2x^2+8c^4x^4+16c^6x^6))-105b}{3675x^7}$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^8,x]
```

output
```
(-105*a*(5*d + 7*e*x^2) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(49*e*x^2*(3 + 4*c^2
*x^2 + 8*c^4*x^4) + 15*d*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 16*c^6*x^6)) - 105*b
*(5*d + 7*e*x^2)*ArcSec[c*x])/(3675*x^7)
```

3.75.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5761, 27, 359, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x^8}\,dx$$
$$\downarrow 5761$$
$$-\frac{bcx\int -\frac{7ex^2+5d}{35x^8\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}} - \frac{d\left(a+b\sec^{-1}(cx)\right)}{7x^7} - \frac{e\left(a+b\sec^{-1}(cx)\right)}{5x^5}$$
$$\downarrow 27$$
$$\frac{bcx\int \frac{7ex^2+5d}{x^8\sqrt{c^2x^2-1}}dx}{35\sqrt{c^2x^2}} - \frac{d\left(a+b\sec^{-1}(cx)\right)}{7x^7} - \frac{e\left(a+b\sec^{-1}(cx)\right)}{5x^5}$$
$$\downarrow 359$$
$$\frac{bcx\left(\frac{1}{7}(30c^2d+49e)\int \frac{1}{x^6\sqrt{c^2x^2-1}}dx + \frac{5d\sqrt{c^2x^2-1}}{7x^7}\right)}{35\sqrt{c^2x^2}} - \frac{d\left(a+b\sec^{-1}(cx)\right)}{7x^7} - \frac{e\left(a+b\sec^{-1}(cx)\right)}{5x^5}$$
$$\downarrow 245$$

$$\frac{bcx\left(\frac{1}{7}\left(30c^2d+49e\right)\left(\frac{4}{5}c^2\int\frac{1}{x^4\sqrt{c^2x^2-1}}dx+\frac{\sqrt{c^2x^2-1}}{5x^5}\right)+\frac{5d\sqrt{c^2x^2-1}}{7x^7}\right)}{35\sqrt{c^2x^2}}-\frac{d\left(a+b\sec^{-1}(cx)\right)}{7x^7}-\frac{e\left(a+b\sec^{-1}(cx)\right)}{5x^5}$$

$$\downarrow 245$$

$$\frac{bcx\left(\frac{1}{7}\left(30c^2d+49e\right)\left(\frac{4}{5}c^2\left(\frac{2}{3}c^2\int\frac{1}{x^2\sqrt{c^2x^2-1}}dx+\frac{\sqrt{c^2x^2-1}}{3x^3}\right)+\frac{\sqrt{c^2x^2-1}}{5x^5}\right)+\frac{5d\sqrt{c^2x^2-1}}{7x^7}\right)}{35\sqrt{c^2x^2}}-\frac{d\left(a+b\sec^{-1}(cx)\right)}{7x^7}-\frac{e\left(a+b\sec^{-1}(cx)\right)}{5x^5}$$

$$\downarrow 242$$

$$-\frac{d\left(a+b\sec^{-1}(cx)\right)}{7x^7}-\frac{e\left(a+b\sec^{-1}(cx)\right)}{5x^5}+\frac{bcx\left(\frac{1}{7}\left(\frac{\sqrt{c^2x^2-1}}{5x^5}+\frac{4}{5}c^2\left(\frac{2c^2\sqrt{c^2x^2-1}}{3x}+\frac{\sqrt{c^2x^2-1}}{3x^3}\right)\right)\left(30c^2d+49e\right)+\frac{5d\sqrt{c^2x^2-1}}{7x^7}\right)}{35\sqrt{c^2x^2}}$$

input
```
Int[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^8,x]
```

output
```
(b*c*x*((5*d*Sqrt[-1 + c^2*x^2])/(7*x^7) + ((30*c^2*d + 49*e)*(Sqrt[-1 + c
^2*x^2]/(5*x^5) + (4*c^2*(Sqrt[-1 + c^2*x^2]/(3*x^3) + (2*c^2*Sqrt[-1 + c^
2*x^2])/(3*x)))/5))/7))/(35*Sqrt[c^2*x^2]) - (d*(a + b*ArcSec[c*x]))/(7*x^
7) - (e*(a + b*ArcSec[c*x]))/(5*x^5)
```

3.75.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 5761

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.75.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.74

method	result
parts	$a\left(-\frac{e}{5x^5}-\frac{d}{7x^7}\right)+b\,c^7\left(-\frac{\operatorname{arcsec}(cx)e}{5c^7x^5}-\frac{\operatorname{arcsec}(cx)d}{7x^7c^7}+\frac{(c^2x^2-1)\left(240c^8dx^6+392c^6ex^6+120c^6dx^4+196c^4e\right.}{3675c^{10}\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^8}\right.$
derivativedivides	$c^7\left(\frac{a\left(-\frac{d}{7c^5x^7}-\frac{e}{5c^5x^5}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arcsec}(cx)d}{7c^5x^7}-\frac{\operatorname{arcsec}(cx)e}{5c^5x^5}+\frac{\left(c^2x^2-1\right)\left(240c^8dx^6+392c^6ex^6+120c^6dx^4+196c^4ex^4+90c^4dx\right.}{3675\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^8x^8}\right.}{c^2}\right.$
default	$c^7\left(\frac{a\left(-\frac{d}{7c^5x^7}-\frac{e}{5c^5x^5}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arcsec}(cx)d}{7c^5x^7}-\frac{\operatorname{arcsec}(cx)e}{5c^5x^5}+\frac{\left(c^2x^2-1\right)\left(240c^8dx^6+392c^6ex^6+120c^6dx^4+196c^4ex^4+90c^4dx\right.}{3675\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^8x^8}\right.}{c^2}\right.$

input

```
int((e*x^2+d)*(a+b*arcsec(c*x))/x^8,x,method=_RETURNVERBOSE)
```

output

```
a*(-1/5*e/x^5-1/7*d/x^7)+b*c^7*(-1/5/c^7*arcsec(c*x)*e/x^5-1/7*arcsec(c*x)
*d/x^7/c^7+1/3675/c^10*(c^2*x^2-1)*(240*c^8*d*x^6+392*c^6*e*x^6+120*c^6*d*
x^4+196*c^4*e*x^4+90*c^4*d*x^2+147*c^2*e*x^2+75*c^2*d)/((c^2*x^2-1)/c^2/x^
2)^(1/2)/x^8)
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.56

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^8}\,dx =$$
$$-\frac{735\,aex^2+525\,ad+105\,(7\,bex^2+5\,bd)\operatorname{arcsec}(cx)-(8\,(30\,bc^6d+49\,bc^4e)x^6+4\,(30\,bc^4d+49\,bc^2e)x^4}{3675\,x^7}$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^8,x, algorithm="fricas")
```

output
```
-1/3675*(735*a*e*x^2 + 525*a*d + 105*(7*b*e*x^2 + 5*b*d)*arcsec(c*x) - (8*
(30*b*c^6*d + 49*b*c^4*e)*x^6 + 4*(30*b*c^4*d + 49*b*c^2*e)*x^4 + 3*(30*b*
c^2*d + 49*b*e)*x^2 + 75*b*d)*sqrt(c^2*x^2 - 1))/x^7
```

3.75.6 Sympy [A] (verification not implemented)

Time = 29.28 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.88

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^8}\,dx$$
$$= -\frac{ad}{7x^7}-\frac{ae}{5x^5}-\frac{bd\operatorname{asec}(cx)}{7x^7}-\frac{be\operatorname{asec}(cx)}{5x^5}$$
$$+\frac{bd\left(\begin{cases}\frac{16c^7\sqrt{c^2x^2-1}}{35x}+\frac{8c^5\sqrt{c^2x^2-1}}{35x^3}+\frac{6c^3\sqrt{c^2x^2-1}}{35x^5}+\frac{c\sqrt{c^2x^2-1}}{7x^7} & \text{for } |c^2x^2|>1\\ \frac{16ic^7\sqrt{-c^2x^2+1}}{35x}+\frac{8ic^5\sqrt{-c^2x^2+1}}{35x^3}+\frac{6ic^3\sqrt{-c^2x^2+1}}{35x^5}+\frac{ic\sqrt{-c^2x^2+1}}{7x^7} & \text{otherwise}\end{cases}\right)}{7c}$$
$$+\frac{be\left(\begin{cases}\frac{8c^5\sqrt{c^2x^2-1}}{15x}+\frac{4c^3\sqrt{c^2x^2-1}}{15x^3}+\frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for } |c^2x^2|>1\\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x}+\frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3}+\frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise}\end{cases}\right)}{5c}$$

input
```
integrate((e*x**2+d)*(a+b*asec(c*x))/x**8,x)
```

output
```
-a*d/(7*x**7) - a*e/(5*x**5) - b*d*asec(c*x)/(7*x**7) - b*e*asec(c*x)/(5*x
**5) + b*d*Piecewise((16*c**7*sqrt(c**2*x**2 - 1)/(35*x) + 8*c**5*sqrt(c**
2*x**2 - 1)/(35*x**3) + 6*c**3*sqrt(c**2*x**2 - 1)/(35*x**5) + c*sqrt(c**2
*x**2 - 1)/(7*x**7), Abs(c**2*x**2) > 1), (16*I*c**7*sqrt(-c**2*x**2 + 1)/
(35*x) + 8*I*c**5*sqrt(-c**2*x**2 + 1)/(35*x**3) + 6*I*c**3*sqrt(-c**2*x**
2 + 1)/(35*x**5) + I*c*sqrt(-c**2*x**2 + 1)/(7*x**7), True))/(7*c) + b*e*P
iecewise((8*c**5*sqrt(c**2*x**2 - 1)/(15*x) + 4*c**3*sqrt(c**2*x**2 - 1)/(
15*x**3) + c*sqrt(c**2*x**2 - 1)/(5*x**5), Abs(c**2*x**2) > 1), (8*I*c**5*
sqrt(-c**2*x**2 + 1)/(15*x) + 4*I*c**3*sqrt(-c**2*x**2 + 1)/(15*x**3) + I*
c*sqrt(-c**2*x**2 + 1)/(5*x**5), True))/(5*c)
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^8}\,dx =$$
$$-\frac{1}{245}\,bd\left(\frac{5\,c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}}-21\,c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+35\,c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-35\,c^8\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{35\ \mathrm{arcsec}\,(c}{x^7}\right.$$
$$+\frac{1}{75}\,be\left(\frac{3\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{15\ \mathrm{arcsec}\,(cx)}{x^5}\right)$$
$$-\frac{ae}{5\,x^5}-\frac{ad}{7\,x^7}$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^8,x, algorithm="maxima")
```

output
```
-1/245*b*d*((5*c^8*(-1/(c^2*x^2) + 1)^(7/2) - 21*c^8*(-1/(c^2*x^2) + 1)^(5
/2) + 35*c^8*(-1/(c^2*x^2) + 1)^(3/2) - 35*c^8*sqrt(-1/(c^2*x^2) + 1))/c +
 35*arcsec(c*x)/x^7) + 1/75*b*e*((3*c^6*(-1/(c^2*x^2) + 1)^(5/2) - 10*c^6*
(-1/(c^2*x^2) + 1)^(3/2) + 15*c^6*sqrt(-1/(c^2*x^2) + 1))/c - 15*arcsec(c*
x)/x^5) - 1/5*a*e/x^5 - 1/7*a*d/x^7
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^8}\,dx$$

$$= \frac{1}{3675}\left(240\,bc^6d\sqrt{-\frac{1}{c^2x^2}+1}+392\,bc^4e\sqrt{-\frac{1}{c^2x^2}+1}+\frac{120\,bc^4d\sqrt{-\frac{1}{c^2x^2}+1}}{x^2}+\frac{196\,bc^2e\sqrt{-\frac{1}{c^2x^2}+1}}{x^2}+\frac{90}{}\right.$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^8,x, algorithm="giac")
```

output
```
1/3675*(240*b*c^6*d*sqrt(-1/(c^2*x^2) + 1) + 392*b*c^4*e*sqrt(-1/(c^2*x^2)
 + 1) + 120*b*c^4*d*sqrt(-1/(c^2*x^2) + 1)/x^2 + 196*b*c^2*e*sqrt(-1/(c^2*
x^2) + 1)/x^2 + 90*b*c^2*d*sqrt(-1/(c^2*x^2) + 1)/x^4 + 147*b*e*sqrt(-1/(c
^2*x^2) + 1)/x^4 - 735*b*e*arccos(1/(c*x))/(c*x^5) + 75*b*d*sqrt(-1/(c^2*x
^2) + 1)/x^6 - 735*a*e/(c*x^5) - 525*b*d*arccos(1/(c*x))/(c*x^7) - 525*a*d
/(c*x^7))*c
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^8}\,dx = \int \frac{(e\,x^2+d)\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

input
```
int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x^8,x)
```

output
```
int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x^8, x)
```

3.76 $\int x^5(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$

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3.76.1 Optimal result

Integrand size = 19, antiderivative size = 196

$$\begin{aligned}\int x^5(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = &-\frac{b(4c^2d+3e)\,x\sqrt{-1+c^2x^2}}{24c^7\sqrt{c^2x^2}}\\ &-\frac{b(8c^2d+9e)\,x\left(-1+c^2x^2\right)^{3/2}}{72c^7\sqrt{c^2x^2}}\\ &-\frac{b(4c^2d+9e)\,x\left(-1+c^2x^2\right)^{5/2}}{120c^7\sqrt{c^2x^2}}-\frac{bex\left(-1+c^2x^2\right)^{7/2}}{56c^7\sqrt{c^2x^2}}\\ &+\frac{1}{6}dx^6\left(a+b\sec^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\sec^{-1}(cx)\right)\end{aligned}$$

output

```
1/6*d*x^6*(a+b*arcsec(c*x))+1/8*e*x^8*(a+b*arcsec(c*x))-1/72*b*(8*c^2*d+9*
e)*x*(c^2*x^2-1)^(3/2)/c^7/(c^2*x^2)^(1/2)-1/120*b*(4*c^2*d+9*e)*x*(c^2*x^
2-1)^(5/2)/c^7/(c^2*x^2)^(1/2)-1/56*b*e*x*(c^2*x^2-1)^(7/2)/c^7/(c^2*x^2)^
(1/2)-1/24*b*(4*c^2*d+3*e)*x*(c^2*x^2-1)^(1/2)/c^7/(c^2*x^2)^(1/2)
```

3.76.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.60

$$\begin{aligned}
&\int x^5\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx \\
&= \frac{1}{24}ax^6\left(4d+3ex^2\right) \\
&\quad - \frac{b\sqrt{1-\frac{1}{c^2x^2}}x\left(144e+8c^2(28d+9ex^2)+2c^4(56dx^2+27ex^4)+c^6(84dx^4+45ex^6)\right)}{2520c^7} \\
&\quad + \frac{1}{24}bx^6\left(4d+3ex^2\right)\sec^{-1}(cx)
\end{aligned}$$

input
```
Integrate[x^5*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

output
```
(a*x^6*(4*d + 3*e*x^2))/24 - (b*Sqrt[1 - 1/(c^2*x^2)]*x*(144*e + 8*c^2*(28
*d + 9*e*x^2) + 2*c^4*(56*d*x^2 + 27*e*x^4) + c^6*(84*d*x^4 + 45*e*x^6)))/
(2520*c^7) + (b*x^6*(4*d + 3*e*x^2)*ArcSec[c*x])/24
```

3.76.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5761, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\downarrow 5761$$

$$-\frac{bcx\int\frac{x^5\left(3ex^2+4d\right)}{24\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{1}{6}dx^6\left(a+b\sec^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 27$$

$$-\frac{bcx\int\frac{x^5\left(3ex^2+4d\right)}{\sqrt{c^2x^2-1}}dx}{24\sqrt{c^2x^2}}+\frac{1}{6}dx^6\left(a+b\sec^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 354$$

$$-\frac{bcx\int\frac{x^4(3ex^2+4d)}{\sqrt{c^2x^2-1}}dx^2}{48\sqrt{c^2x^2}}+\frac{1}{6}dx^6\left(a+b\sec^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 86$$

$$-\frac{bcx\int\left(\frac{3e\left(c^2x^2-1\right)^{5/2}}{c^6}+\frac{\left(4dc^2+9e\right)\left(c^2x^2-1\right)^{3/2}}{c^6}+\frac{\left(8dc^2+9e\right)\sqrt{c^2x^2-1}}{c^6}+\frac{4dc^2+3e}{c^6\sqrt{c^2x^2-1}}\right)dx^2}{48\sqrt{c^2x^2}}+$$
$$\frac{1}{6}dx^6\left(a+b\sec^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 2009$$

$$\frac{1}{6}dx^6\left(a+b\sec^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\sec^{-1}(cx)\right)-$$
$$\frac{bcx\left(\frac{2\left(c^2x^2-1\right)^{5/2}\left(4c^2d+9e\right)}{5c^8}+\frac{2\left(c^2x^2-1\right)^{3/2}\left(8c^2d+9e\right)}{3c^8}+\frac{2\sqrt{c^2x^2-1}\left(4c^2d+3e\right)}{c^8}+\frac{6e\left(c^2x^2-1\right)^{7/2}}{7c^8}\right)}{48\sqrt{c^2x^2}}$$

input
```
Int[x^5*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

output
```
-1/48*(b*c*x*((2*(4*c^2*d + 3*e)*Sqrt[-1 + c^2*x^2])/c^8 + (2*(8*c^2*d + 9
*e)*(-1 + c^2*x^2)^(3/2))/(3*c^8) + (2*(4*c^2*d + 9*e)*(-1 + c^2*x^2)^(5/2
))/(5*c^8) + (6*e*(-1 + c^2*x^2)^(7/2))/(7*c^8)))/Sqrt[c^2*x^2] + (d*x^6*(
a + b*ArcSec[c*x]))/6 + (e*x^8*(a + b*ArcSec[c*x]))/8
```

3.76.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])  u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])  Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.76.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.71

method	result
parts	$a\left(\frac{1}{8}e\,x^8+\frac{1}{6}d\,x^6\right)+\frac{b\left(\frac{c^6\,\mathrm{arcsec}(cx)e\,x^8}{8}+\frac{\mathrm{arcsec}(cx)d\,x^6c^6}{6}-\frac{\left(c^2x^2-1\right)\left(45c^6e\,x^6+84c^6d\,x^4+54c^4e\,x^4+112c^4d\,x^2+72c^2e\,x^2+2\right.}{2520c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}\right.}{c^6}$
derivativedivides	$\frac{a\left(\frac{1}{6}c^8d\,x^6+\frac{1}{8}e\,c^8x^8\right)}{c^2}+\frac{b\left(\frac{\mathrm{arcsec}(cx)d\,c^8x^6}{6}+\frac{\mathrm{arcsec}(cx)e\,c^8x^8}{8}-\frac{\left(c^2x^2-1\right)\left(45c^6e\,x^6+84c^6d\,x^4+54c^4e\,x^4+112c^4d\,x^2+72c^2e\,x^2+2\right.}{2520\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right.}{c^2}}{c^6}$
default	$\frac{a\left(\frac{1}{6}c^8d\,x^6+\frac{1}{8}e\,c^8x^8\right)}{c^2}+\frac{b\left(\frac{\mathrm{arcsec}(cx)d\,c^8x^6}{6}+\frac{\mathrm{arcsec}(cx)e\,c^8x^8}{8}-\frac{\left(c^2x^2-1\right)\left(45c^6e\,x^6+84c^6d\,x^4+54c^4e\,x^4+112c^4d\,x^2+72c^2e\,x^2+2\right.}{2520\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right.}{c^2}}{c^6}$

input
```
int(x^5*(e*x^2+d)*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/8*e*x^8+1/6*d*x^6)+b/c^6*(1/8*c^6*arcsec(c*x)*e*x^8+1/6*arcsec(c*x)*d
*x^6*c^6-1/2520/c^3*(c^2*x^2-1)*(45*c^6*e*x^6+84*c^6*d*x^4+54*c^4*e*x^4+11
2*c^4*d*x^2+72*c^2*e*x^2+224*c^2*d+144*e)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x)
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.65

$$\int x^5\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{315\,ac^8ex^8+420\,ac^8dx^6+105\left(3\,bc^8ex^8+4\,bc^8dx^6\right)\operatorname{arcsec}(cx)-\left(45\,bc^6ex^6+6\left(14\,bc^6d+9\,bc^4e\right)x^4+22\right.}{2520\,c^8}$$

input
```
integrate(x^5*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/2520*(315*a*c^8*e*x^8 + 420*a*c^8*d*x^6 + 105*(3*b*c^8*e*x^8 + 4*b*c^8*d
*x^6)*arcsec(c*x) - (45*b*c^6*e*x^6 + 6*(14*b*c^6*d + 9*b*c^4*e)*x^4 + 224
*b*c^2*d + 8*(14*b*c^4*d + 9*b*c^2*e)*x^2 + 144*b*e)*sqrt(c^2*x^2 - 1))/c^
8
```

3.76.6 Sympy [A] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.86

$$\int x^5\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{adx^6}{6}+\frac{aex^8}{8}+\frac{bdx^6\operatorname{asec}(cx)}{6}+\frac{bex^8\operatorname{asec}(cx)}{8}$$
$$-\frac{bd\left(\begin{cases}\frac{x^4\sqrt{c^2x^2-1}}{5c}+\frac{4x^2\sqrt{c^2x^2-1}}{15c^3}+\frac{8\sqrt{c^2x^2-1}}{15c^5} & \text{for } |c^2x^2|>1\\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c}+\frac{4ix^2\sqrt{-c^2x^2+1}}{15c^3}+\frac{8i\sqrt{-c^2x^2+1}}{15c^5} & \text{otherwise}\end{cases}\right)}{6c}$$
$$-\frac{be\left(\begin{cases}\frac{x^6\sqrt{c^2x^2-1}}{7c}+\frac{6x^4\sqrt{c^2x^2-1}}{35c^3}+\frac{8x^2\sqrt{c^2x^2-1}}{35c^5}+\frac{16\sqrt{c^2x^2-1}}{35c^7} & \text{for } |c^2x^2|>1\\ \frac{ix^6\sqrt{-c^2x^2+1}}{7c}+\frac{6ix^4\sqrt{-c^2x^2+1}}{35c^3}+\frac{8ix^2\sqrt{-c^2x^2+1}}{35c^5}+\frac{16i\sqrt{-c^2x^2+1}}{35c^7} & \text{otherwise}\end{cases}\right)}{8c}$$

input
```
integrate(x**5*(e*x**2+d)*(a+b*asec(c*x)),x)
```

output
```
a*d*x**6/6 + a*e*x**8/8 + b*d*x**6*asec(c*x)/6 + b*e*x**8*asec(c*x)/8 - b*
d*Piecewise((x**4*sqrt(c**2*x**2 - 1)/(5*c) + 4*x**2*sqrt(c**2*x**2 - 1)/(
15*c**3) + 8*sqrt(c**2*x**2 - 1)/(15*c**5), Abs(c**2*x**2) > 1), (I*x**4*s
qrt(-c**2*x**2 + 1)/(5*c) + 4*I*x**2*sqrt(-c**2*x**2 + 1)/(15*c**3) + 8*I*
sqrt(-c**2*x**2 + 1)/(15*c**5), True))/(6*c) - b*e*Piecewise((x**6*sqrt(c*
*2*x**2 - 1)/(7*c) + 6*x**4*sqrt(c**2*x**2 - 1)/(35*c**3) + 8*x**2*sqrt(c*
*2*x**2 - 1)/(35*c**5) + 16*sqrt(c**2*x**2 - 1)/(35*c**7), Abs(c**2*x**2)
> 1), (I*x**6*sqrt(-c**2*x**2 + 1)/(7*c) + 6*I*x**4*sqrt(-c**2*x**2 + 1)/(
35*c**3) + 8*I*x**2*sqrt(-c**2*x**2 + 1)/(35*c**5) + 16*I*sqrt(-c**2*x**2
+ 1)/(35*c**7), True))/(8*c)
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.94

$$\int x^5\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{1}{8}aex^8+\frac{1}{6}adx^6$$
$$+\frac{1}{90}\left(15\,x^6\operatorname{arcsec}(cx)-\frac{3\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+10\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^5}\right)bd$$
$$+\frac{1}{280}\left(35\,x^8\operatorname{arcsec}(cx)-\frac{5\,c^6x^7\left(-\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}}+21\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+35\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+35\,x\sqrt{-}\right.$$
$$\left.c^7\right.$$

input
```
integrate(x^5*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/8*a*e*x^8 + 1/6*a*d*x^6 + 1/90*(15*x^6*arcsec(c*x) - (3*c^4*x^5*(-1/(c^2
*x^2) + 1)^(5/2) + 10*c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(-1/(c^2
*x^2) + 1))/c^5)*b*d + 1/280*(35*x^8*arcsec(c*x) - (5*c^6*x^7*(-1/(c^2*x^2
) + 1)^(7/2) + 21*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2) + 35*c^2*x^3*(-1/(c^2*x
^2) + 1)^(3/2) + 35*x*sqrt(-1/(c^2*x^2) + 1))/c^7)*b*e
```

3.76.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13018 vs. 2(168) = 336.

Time = 0.47 (sec) , antiderivative size = 13018, normalized size of antiderivative = 66.42

$$\int x^5\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
1/2520*(420*b*c^2*d*arccos(1/(c*x))/(c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x
) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2
) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56
*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/
(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x
^2) - 1)^8/(1/(c*x) + 1)^16) + 420*a*c^2*d/(c^9 + 8*c^9*(1/(c^2*x^2) - 1)/
(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(
c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)
^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1
)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1
/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16) - 1680*b*c^2*d*(1/(c^2*x^2) - 1)*arcco
s(1/(c*x))/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^
2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6
 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5
/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1
/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)
^16)*(1/(c*x) + 1)^2) - 840*b*c^2*d*sqrt(-1/(c^2*x^2) + 1)/((c^9 + 8*c^9*(
1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1
)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1
)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*...
```

3.76.9 Mupad [F(-1)]

Timed out.

$$\int x^5\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx = \int x^5\left(e\,x^2+d\right)\left(a+b\,\text{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^5*(d + e*x^2)*(a + b*acos(1/(c*x))),x)
```

output
```
int(x^5*(d + e*x^2)*(a + b*acos(1/(c*x))), x)
```

3.77 $\int x^3(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$

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3.77.1 Optimal result

Integrand size = 19, antiderivative size = 153

$$\int x^3(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = -\frac{b(3c^2d+2e)\,x\sqrt{-1+c^2x^2}}{12c^5\sqrt{c^2x^2}} - \frac{b(3c^2d+4e)\,x(-1+c^2x^2)^{3/2}}{36c^5\sqrt{c^2x^2}} - \frac{bex(-1+c^2x^2)^{5/2}}{30c^5\sqrt{c^2x^2}} + \frac{1}{4}dx^4\left(a+b\sec^{-1}(cx)\right) + \frac{1}{6}ex^6\left(a+b\sec^{-1}(cx)\right)$$

output
```
1/4*d*x^4*(a+b*arcsec(c*x))+1/6*e*x^6*(a+b*arcsec(c*x))-1/36*b*(3*c^2*d+4*
e)*x*(c^2*x^2-1)^(3/2)/c^5/(c^2*x^2)^(1/2)-1/30*b*e*x*(c^2*x^2-1)^(5/2)/c^
5/(c^2*x^2)^(1/2)-1/12*b*(3*c^2*d+2*e)*x*(c^2*x^2-1)^(1/2)/c^5/(c^2*x^2)^(
1/2)
```

3.77.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.64

$$\int x^3(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{1}{180}x\left(15ax^3\left(3d+2ex^2\right) - \frac{b\sqrt{1-\frac{1}{c^2x^2}}\left(16e+c^2(30d+8ex^2)+3c^4(5dx^2+2ex^4)\right)}{c^5} + 15bx^3\left(3d+2ex^2\right)\sec^{-1}(cx)\right)$$

input
```
Integrate[x^3*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

output
```
(x*(15*a*x^3*(3*d + 2*e*x^2) - (b*Sqrt[1 - 1/(c^2*x^2)]*(16*e + c^2*(30*d
+ 8*e*x^2) + 3*c^4*(5*d*x^2 + 2*e*x^4)))/c^5 + 15*b*x^3*(3*d + 2*e*x^2)*Ar
cSec[c*x]))/180
```

3.77.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5761, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\downarrow 5761$$

$$-\frac{bcx\int \frac{x^3(2ex^2+3d)}{12\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{1}{4}dx^4\left(a+b\sec^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 27$$

$$-\frac{bcx\int \frac{x^3(2ex^2+3d)}{\sqrt{c^2x^2-1}}dx}{12\sqrt{c^2x^2}}+\frac{1}{4}dx^4\left(a+b\sec^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 354$$

$$-\frac{bcx\int \frac{x^2(2ex^2+3d)}{\sqrt{c^2x^2-1}}dx^2}{24\sqrt{c^2x^2}}+\frac{1}{4}dx^4\left(a+b\sec^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 86$$

$$-\frac{bcx\int \left(\frac{2e\left(c^2x^2-1\right)^{3/2}}{c^4}+\frac{(3dc^2+4e)\sqrt{c^2x^2-1}}{c^4}+\frac{3dc^2+2e}{c^4\sqrt{c^2x^2-1}}\right)dx^2}{24\sqrt{c^2x^2}}+\frac{1}{4}dx^4\left(a+b\sec^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 2009$$

$$\frac{\frac{1}{4}dx^4\left(a+b\sec^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\sec^{-1}(cx)\right)-\frac{bcx\left(\frac{2\left(c^2x^2-1\right)^{3/2}\left(3c^2d+4e\right)}{3c^6}+\frac{2\sqrt{c^2x^2-1}\left(3c^2d+2e\right)}{c^6}+\frac{4e\left(c^2x^2-1\right)^{5/2}}{5c^6}\right)}{24\sqrt{c^2x^2}}}{}$$

input
```
Int[x^3*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

output
```
-1/24*(b*c*x*((2*(3*c^2*d + 2*e)*Sqrt[-1 + c^2*x^2])/c^6 + (2*(3*c^2*d + 4
*e)*(-1 + c^2*x^2)^(3/2))/(3*c^6) + (4*e*(-1 + c^2*x^2)^(5/2))/(5*c^6)))/S
qrt[c^2*x^2] + (d*x^4*(a + b*ArcSec[c*x]))/4 + (e*x^6*(a + b*ArcSec[c*x]))
/6
```

3.77.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5761

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])  u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])  Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.77.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.79

method	result
parts	$a\left(\frac{1}{6}e\,x^6+\frac{1}{4}d\,x^4\right)+\frac{b\left(\frac{c^4\,\mathrm{arcsec}(cx)e\,x^6}{6}+\frac{\mathrm{arcsec}(cx)x^4c^4d}{4}-\frac{\left(c^2x^2-1\right)\left(6c^4e\,x^4+15c^4d\,x^2+8c^2e\,x^2+30c^2d+16e\right)}{180c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}\right)}{c^4}$
derivativedivides	$\frac{-\frac{a\left(\frac{c^2d\left(c^2e\,x^2+c^2d\right)^2}{2}-\frac{\left(c^2e\,x^2+c^2d\right)^3}{3}\right)}{2c^2e^2}-\frac{b\,c^4\,\mathrm{arcsec}(cx)d^3}{12e^2}+\frac{b\,\mathrm{arcsec}(cx)d\,c^4x^4}{4}+\frac{b\,c^4e\,\mathrm{arcsec}(cx)x^6}{6}-\frac{b\,c^3\sqrt{c^2x^2-1}\,d^3\arctan}{12e^2\sqrt{\frac{c^2x^2-}{c^2x^2}}}}{c^4}$
default	$\frac{-\frac{a\left(\frac{c^2d\left(c^2e\,x^2+c^2d\right)^2}{2}-\frac{\left(c^2e\,x^2+c^2d\right)^3}{3}\right)}{2c^2e^2}-\frac{b\,c^4\,\mathrm{arcsec}(cx)d^3}{12e^2}+\frac{b\,\mathrm{arcsec}(cx)d\,c^4x^4}{4}+\frac{b\,c^4e\,\mathrm{arcsec}(cx)x^6}{6}-\frac{b\,c^3\sqrt{c^2x^2-1}\,d^3\arctan}{12e^2\sqrt{\frac{c^2x^2-}{c^2x^2}}}}{c^4}$

input

```
int(x^3*(e*x^2+d)*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output

```
a*(1/6*e*x^6+1/4*d*x^4)+b/c^4*(1/6*c^4*arcsec(c*x)*e*x^6+1/4*arcsec(c*x)*x
^4*c^4*d-1/180/c^3*(c^2*x^2-1)*(6*c^4*e*x^4+15*c^4*d*x^2+8*c^2*e*x^2+30*c^
2*d+16*e)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x)
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.70

$$\int x^3\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$=\frac{30\,ac^6ex^6+45\,ac^6dx^4+15\left(2\,bc^6ex^6+3\,bc^6dx^4\right)\mathrm{arcsec}\left(cx\right)-\left(6\,bc^4ex^4+30\,bc^2d+\left(15\,bc^4d+8\,bc^2e\right)x^2-\right.}{180\,c^6}$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/180*(30*a*c^6*e*x^6 + 45*a*c^6*d*x^4 + 15*(2*b*c^6*e*x^6 + 3*b*c^6*d*x^4
)*arcsec(c*x) - (6*b*c^4*e*x^4 + 30*b*c^2*d + (15*b*c^4*d + 8*b*c^2*e)*x^2
 + 16*b*e)*sqrt(c^2*x^2 - 1))/c^6
```

3.77.6 Sympy [A] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.78

$$\int x^3\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$= \frac{adx^4}{4}+\frac{aex^6}{6}+\frac{bdx^4\operatorname{asec}(cx)}{4}+\frac{bex^6\operatorname{asec}(cx)}{6}$$
$$-\frac{bd\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{4c}$$
$$-\frac{be\left(\begin{cases}\frac{x^4\sqrt{c^2x^2-1}}{5c}+\frac{4x^2\sqrt{c^2x^2-1}}{15c^3}+\frac{8\sqrt{c^2x^2-1}}{15c^5} & \text{for } |c^2x^2|>1\\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c}+\frac{4ix^2\sqrt{-c^2x^2+1}}{15c^3}+\frac{8i\sqrt{-c^2x^2+1}}{15c^5} & \text{otherwise}\end{cases}\right)}{6c}$$

input
```
integrate(x**3*(e*x**2+d)*(a+b*asec(c*x)),x)
```

output
```
a*d*x**4/4 + a*e*x**6/6 + b*d*x**4*asec(c*x)/4 + b*e*x**6*asec(c*x)/6 - b*
d*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c) + 2*sqrt(c**2*x**2 - 1)/(3*c**
3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*x**2 + 1)/(3*c) + 2*I*sqrt(-c*
*2*x**2 + 1)/(3*c**3), True))/(4*c) - b*e*Piecewise((x**4*sqrt(c**2*x**2 -
 1)/(5*c) + 4*x**2*sqrt(c**2*x**2 - 1)/(15*c**3) + 8*sqrt(c**2*x**2 - 1)/(
15*c**5), Abs(c**2*x**2) > 1), (I*x**4*sqrt(-c**2*x**2 + 1)/(5*c) + 4*I*x*
*2*sqrt(-c**2*x**2 + 1)/(15*c**3) + 8*I*sqrt(-c**2*x**2 + 1)/(15*c**5), Tr
ue))/(6*c)
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.94

$$\int x^3\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{1}{6}aex^6+\frac{1}{4}adx^4+\frac{1}{12}\left(3\,x^4\operatorname{arcsec}(cx)-\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)bd$$
$$+\frac{1}{90}\left(15\,x^6\operatorname{arcsec}(cx)-\frac{3\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+10\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^5}\right)be$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*e*x^6 + 1/4*a*d*x^4 + 1/12*(3*x^4*arcsec(c*x) - (c^2*x^3*(-1/(c^2*x^
2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b*d + 1/90*(15*x^6*arcsec
(c*x) - (3*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2) + 10*c^2*x^3*(-1/(c^2*x^2) + 1
)^(3/2) + 15*x*sqrt(-1/(c^2*x^2) + 1))/c^5)*b*e
```

3.77.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7820 vs. $2(131) = 262$.

Time = 0.37 (sec) , antiderivative size = 7820, normalized size of antiderivative = 51.11

$$\int x^3\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output

```
1/180*(45*b*c^2*d*arccos(1/(c*x))/(c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x)
+ 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2)
- 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^
7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x)
+ 1)^12) + 45*a*c^2*d/(c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*
c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c
*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^
2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12) - 9
0*b*c^2*d*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^2*x^2) - 1
)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1
/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) +
1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^
6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^2) - 90*b*c^2*d*sqrt(-1/(c^2*x^2) + 1)/(
(c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^
2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1
/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1
)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)) - 90*a*c^2
*d*(1/(c^2*x^2) - 1)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*
c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c
*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2...
```

3.77.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \left(d + e x^2\right) \left(a + b \sec^{-1}(c x)\right)\, dx = \int x^3 \left(e\, x^2 + d\right) \left(a + b \operatorname{acos}\left(\frac{1}{c\, x}\right)\right)\, dx$$

input

```
int(x^3*(d + e*x^2)*(a + b*acos(1/(c*x))),x)
```

output

```
int(x^3*(d + e*x^2)*(a + b*acos(1/(c*x))), x)
```

3.78 $\int x(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$

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3.78.1 Optimal result

Integrand size = 17, antiderivative size = 138

$$\int x(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = -\frac{b(2c^2d+e)\,x\sqrt{-1+c^2x^2}}{4c^3\sqrt{c^2x^2}} - \frac{bex(-1+c^2x^2)^{3/2}}{12c^3\sqrt{c^2x^2}} + \frac{(d+ex^2)^2\left(a+b\sec^{-1}(cx)\right)}{4e} - \frac{bcd^2x\arctan\left(\sqrt{-1+c^2x^2}\right)}{4e\sqrt{c^2x^2}}$$

output
```
1/4*(e*x^2+d)^2*(a+b*arcsec(c*x))/e-1/12*b*e*x*(c^2*x^2-1)^(3/2)/c^3/(c^2*
x^2)^(1/2)-1/4*b*c*d^2*x*arctan((c^2*x^2-1)^(1/2))/e/(c^2*x^2)^(1/2)-1/4*b
*(2*c^2*d+e)*x*(c^2*x^2-1)^(1/2)/c^3/(c^2*x^2)^(1/2)
```

3.78.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.57

$$\int x(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{x\left(3ac^3x(2d+ex^2) - b\sqrt{1-\frac{1}{c^2x^2}}(2e+c^2(6d+ex^2)) + 3bc^3x(2d+ex^2)\sec^{-1}(cx)\right)}{12c^3}$$

input
```
Integrate[x*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

output
```
(x*(3*a*c^3*x*(2*d + e*x^2) - b*Sqrt[1 - 1/(c^2*x^2)]*(2*e + c^2*(6*d + e*
x^2)) + 3*b*c^3*x*(2*d + e*x^2)*ArcSec[c*x]))/(12*c^3)
```

3.78.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {5759, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\downarrow 5759$$

$$\frac{\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)}{4e}-\frac{bcx\int\frac{\left(ex^2+d\right)^2}{x\sqrt{c^2x^2-1}}dx}{4e\sqrt{c^2x^2}}$$

$$\downarrow 354$$

$$\frac{\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)}{4e}-\frac{bcx\int\frac{\left(ex^2+d\right)^2}{x^2\sqrt{c^2x^2-1}}dx^2}{8e\sqrt{c^2x^2}}$$

$$\downarrow 99$$

$$\frac{\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)}{4e}-\frac{bcx\int\left(\frac{d^2}{x^2\sqrt{c^2x^2-1}}+\frac{e^2\sqrt{c^2x^2-1}}{c^2}+\frac{e\left(2dc^2+e\right)}{c^2\sqrt{c^2x^2-1}}\right)dx^2}{8e\sqrt{c^2x^2}}$$

$$\downarrow 2009$$

$$\frac{\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)}{4e}-\frac{bcx\left(2d^2\arctan\left(\sqrt{c^2x^2-1}\right)+\frac{2e\sqrt{c^2x^2-1}\left(2c^2d+e\right)}{c^4}+\frac{2e^2\left(c^2x^2-1\right)^{3/2}}{3c^4}\right)}{8e\sqrt{c^2x^2}}$$

input
```
Int[x*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

output
```
((d + e*x^2)^2*(a + b*ArcSec[c*x]))/(4*e) - (b*c*x*((2*e*(2*c^2*d + e)*Sqr
t[-1 + c^2*x^2])/c^4 + (2*e^2*(-1 + c^2*x^2)^(3/2))/(3*c^4) + 2*d^2*ArcTan
[Sqrt[-1 + c^2*x^2]]))/(8*e*Sqrt[c^2*x^2])
```

3.78.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5759
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSec[c*x])/(2*e*(p + 1))), x
] - Simp[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2]))   Int[(d + e*x^2)^(p + 1)/(x*S
qrt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

3.78.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.57

method	result
parts	$\frac{a(ex^2+d)^2}{4e}+\frac{b\,\mathrm{arcsec}(cx)ex^4}{4}+\frac{b\,\mathrm{arcsec}(cx)x^2d}{2}+\frac{b\,\mathrm{arcsec}(cx)d^2}{4e}-\frac{b(c^2x^2-1)xe}{12c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\sqrt{c^2x^2-1}\,d^2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{4ce\sqrt{\frac{c^2x^2-1}{c^2x^2}}}$
derivativedivides	$\frac{\frac{a\left(c^2ex^2+c^2d\right)^2}{4e}+\frac{bc^2\,\mathrm{arcsec}(cx)d^2}{4e}+\frac{b\,\mathrm{arcsec}(cx)dc^2x^2}{2}+\frac{bc^2e\,\mathrm{arcsec}(cx)x^4}{4}+\frac{bc\sqrt{c^2x^2-1}\,d^2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{4e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}-\frac{b\left(c^2x^2-1\right)c}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}}{c^2}$
default	$\frac{\frac{a\left(c^2ex^2+c^2d\right)^2}{4e}+\frac{bc^2\,\mathrm{arcsec}(cx)d^2}{4e}+\frac{b\,\mathrm{arcsec}(cx)dc^2x^2}{2}+\frac{bc^2e\,\mathrm{arcsec}(cx)x^4}{4}+\frac{bc\sqrt{c^2x^2-1}\,d^2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{4e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}-\frac{b\left(c^2x^2-1\right)c}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}}{c^2}$

input
```
int(x*(e*x^2+d)*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*(e*x^2+d)^2/e+1/4*b*arcsec(c*x)*e*x^4+1/2*b*arcsec(c*x)*x^2*d+1/4*b/
e*arcsec(c*x)*d^2-1/12*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x*e+1
/4*b/c/e*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^2*arctan(1/(c^2
*x^2-1)^(1/2))-1/2*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d-1/6*b
/c^5*e*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.62

$$\int x\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{3\,ac^4ex^4+6\,ac^4dx^2+3\left(bc^4ex^4+2\,bc^4dx^2\right)\operatorname{arcsec}\left(cx\right)-\left(bc^2ex^2+6\,bc^2d+2\,be\right)\sqrt{c^2x^2-1}}{12\,c^4}$$

input
```
integrate(x*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/12*(3*a*c^4*e*x^4 + 6*a*c^4*d*x^2 + 3*(b*c^4*e*x^4 + 2*b*c^4*d*x^2)*arcs
ec(c*x) - (b*c^2*e*x^2 + 6*b*c^2*d + 2*b*e)*sqrt(c^2*x^2 - 1))/c^4
```

3.78.6 Sympy [A] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.28

$$\int x\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{adx^2}{2}+\frac{aex^4}{4}+\frac{bdx^2\operatorname{asec}\left(cx\right)}{2}+\frac{bex^4\operatorname{asec}\left(cx\right)}{4}$$
$$-\frac{bd\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c}$$
$$-\frac{be\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{4c}$$

input
```
integrate(x*(e*x**2+d)*(a+b*asec(c*x)),x)
```

output
```
a*d*x**2/2 + a*e*x**4/4 + b*d*x**2*asec(c*x)/2 + b*e*x**4*asec(c*x)/4 - b*
d*Piecewise((sqrt(c**2*x**2 - 1)/c, Abs(c**2*x**2) > 1), (I*sqrt(-c**2*x**
2 + 1)/c, True))/(2*c) - b*e*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c) + 2
*sqrt(c**2*x**2 - 1)/(3*c**3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*x**
2 + 1)/(3*c) + 2*I*sqrt(-c**2*x**2 + 1)/(3*c**3), True))/(4*c)
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.72

$$\begin{aligned}&\int x\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx\\&=\frac{1}{4}\,aex^4+\frac{1}{2}\,adx^2+\frac{1}{2}\left(x^2\operatorname{arcsec}\left(cx\right)-\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)bd\\&\quad+\frac{1}{12}\left(3\,x^4\operatorname{arcsec}\left(cx\right)-\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)be\end{aligned}$$

input
```
integrate(x*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/4*a*e*x^4 + 1/2*a*d*x^2 + 1/2*(x^2*arcsec(c*x) - x*sqrt(-1/(c^2*x^2) + 1
)/c)*b*d + 1/12*(3*x^4*arcsec(c*x) - (c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 3
*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b*e
```

3.78.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3346 vs. $2(118) = 236$.

Time = 0.33 (sec) , antiderivative size = 3346, normalized size of antiderivative = 24.25

$$\int x\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output

```
1/12*(6*b*c^2*d*arccos(1/(c*x))/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) +
1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)
^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) + 6*a*c^2*d/
(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2
/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2
*x^2) - 1)^4/(1/(c*x) + 1)^8) - 12*b*c^2*d*sqrt(-1/(c^2*x^2) + 1)/((c^5 +
4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*
x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) -
 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)) + 3*b*e*arccos(1/(c*x))/(c^5 + 4*c^5
*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) +
1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4
/(1/(c*x) + 1)^8) - 12*b*c^2*d*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^5 +
 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c
*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2)
- 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) + 36*b*c^2*d*(-1/(c^2*x^2) + 1)^(
3/2)/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2)
- 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(
1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^3) + 3*a*e/(c^5 + 4*c^5*
(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1
)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)...
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int x\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx=\int x\left(e\,x^2+d\right)\left(a+b\,\text{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input

```
int(x*(d + e*x^2)*(a + b*acos(1/(c*x))),x)
```

output

```
int(x*(d + e*x^2)*(a + b*acos(1/(c*x))), x)
```

3.79 $\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x}\,dx$

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3.79.1 Optimal result

Integrand size = 19, antiderivative size = 124

$$\begin{aligned}\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x}\,dx = &-\frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} - \frac{1}{2}ibd\csc^{-1}(cx)^2 \\ &+ \frac{1}{2}ex^2\left(a+b\sec^{-1}(cx)\right) \\ &+ bd\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) \\ &- bd\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - d\left(a+b\sec^{-1}(cx)\right)\log\left(\frac{1}{x}\right) \\ &- \frac{1}{2}ibd\,\mathrm{PolyLog}\left(2, e^{2i\csc^{-1}(cx)}\right)\end{aligned}$$

output

```
-1/2*I*b*d*arccsc(c*x)^2+1/2*e*x^2*(a+b*arcsec(c*x))+b*d*arccsc(c*x)*ln(1-
(I/c/x+(1-1/c^2/x^2)^(1/2))^2)-b*d*arccsc(c*x)*ln(1/x)-d*(a+b*arcsec(c*x))
*ln(1/x)-1/2*I*b*d*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)-1/2*b*e*x*(1-1
/c^2/x^2)^(1/2)/c
```

3.79.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x}\,dx = \frac{1}{2}aex^2 - \frac{bex\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{2c} + \frac{1}{2}bex^2\sec^{-1}(cx) + \frac{1}{2}ibd\sec^{-1}(cx)^2 - bd\sec^{-1}(cx)\log\left(1+e^{2i\sec^{-1}(cx)}\right) + ad\log(x) + \frac{1}{2}ibd\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcSec[c*x]))/x,x]
```

output
```
(a*e*x^2)/2 - (b*e*x*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(2*c) + (b*e*x^2*ArcS
ec[c*x])/2 + (I/2)*b*d*ArcSec[c*x]^2 - b*d*ArcSec[c*x]*Log[1 + E^((2*I)*Ar
cSec[c*x])] + a*d*Log[x] + (I/2)*b*d*PolyLog[2, -E^((2*I)*ArcSec[c*x])]
```

3.79.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5763, 5231, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x}\,dx$$

$$\downarrow 5763$$

$$-\int \left(\frac{d}{x^2}+e\right)x^3\left(a+b\arccos\left(\frac{1}{cx}\right)\right)d\frac{1}{x}$$

$$\downarrow 5231$$

$$-\frac{b\int -\frac{ex^2-2d\log\left(\frac{1}{x}\right)}{2\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{c} - d\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right) + \frac{1}{2}ex^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)$$

$$\downarrow 27$$

$$\frac{b\int \frac{ex^2-2d\log\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{2c}-d\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+\frac{1}{2}ex^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)$$

$$\downarrow 7293$$

$$\frac{b\int \left(\frac{ex^2}{\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2d\log\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{c^2x^2}}}\right)d\frac{1}{x}}{2c}-d\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+\frac{1}{2}ex^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)$$

$$\downarrow 2009$$

$$-d\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+\frac{1}{2}ex^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+$$
$$\frac{b\left(-icd\,\mathrm{PolyLog}\left(2,e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)-icd\arcsin\left(\frac{1}{cx}\right)^2+2cd\arcsin\left(\frac{1}{cx}\right)\log\left(1-e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)-2cd\log\left(\frac{1}{x}\right)\arcsin\left(\right.\right.}{2c}$$

input
```
Int[((d + e*x^2)*(a + b*ArcSec[c*x]))/x,x]
```

output
```
(e*x^2*(a + b*ArcCos[1/(c*x)]))/2 - d*(a + b*ArcCos[1/(c*x)])*Log[x^(-1)]
+ (b*(-(e*Sqrt[1 - 1/(c^2*x^2)]*x) - I*c*d*ArcSin[1/(c*x)]^2 + 2*c*d*ArcSi
n[1/(c*x)]*Log[1 - E^((2*I)*ArcSin[1/(c*x)])] - 2*c*d*ArcSin[1/(c*x)]*Log[
x^(-1)] - I*c*d*PolyLog[2, E^((2*I)*ArcSin[1/(c*x)])]))/(2*c)
```

3.79.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.79.4 Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10

method	result
parts	$\frac{aex^2}{2}+ad\ln(x)+b\left(\frac{i\operatorname{arcsec}(cx)^2d}{2}+\frac{e\left(c^2x^2\operatorname{arcsec}(cx)-xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}-i\right)}{2c^2}-d\operatorname{arcsec}(cx)\ln\left(1+\right.\right.$
derivativedivides	$\frac{aex^2}{2}+ad\ln(cx)+\frac{b\left(\frac{ic^2d\operatorname{arcsec}(cx)^2}{2}+\frac{e\left(c^2x^2\operatorname{arcsec}(cx)-xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}-i\right)}{2}-\ln\left(1+\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2\right)c^2d\right.}{c^2}$
default	$\frac{aex^2}{2}+ad\ln(cx)+\frac{b\left(\frac{ic^2d\operatorname{arcsec}(cx)^2}{2}+\frac{e\left(c^2x^2\operatorname{arcsec}(cx)-xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}-i\right)}{2}-\ln\left(1+\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2\right)c^2d\right.}{c^2}$

input
```
int((e*x^2+d)*(a+b*arcsec(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*e*x^2+a*d*ln(x)+b*(1/2*I*arcsec(c*x)^2*d+1/2*e*(c^2*x^2*arcsec(c*x)-
x*c*((c^2*x^2-1)/c^2/x^2)^(1/2)-I)/c^2-d*arcsec(c*x)*ln(1+(1/c/x+I*(1-1/c^
2/x^2)^(1/2))^2)+1/2*I*d*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2))
```

3.79.5 Fricas [F]

$$\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x}\,dx=\int \frac{(ex^2+d)(b\operatorname{arcsec}(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsec(c*x))/x, x)
```

3.79.6 Sympy [F]

$$\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x}\,dx = \int \frac{(a+b\,\text{asec}\,(cx))\,(d+ex^2)}{x}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*asec(c*x))/x,x)
```

output
```
Integral((a + b*asec(c*x))*(d + e*x**2)/x, x)
```

3.79.7 Maxima [F]

$$\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x}\,dx = \int \frac{(ex^2+d)(b\,\text{arcsec}\,(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x,x, algorithm="maxima")
```

output
```
1/2*a*e*x^2 + a*d*log(x) - 1/4*(-2*I*b*c^2*e*x^2*log(c) - 2*I*b*c^2*d*log(
-c*x + 1)*log(x) - 2*I*b*c^2*d*log(x)^2 - 2*I*b*c^2*d*dilog(c*x) - 2*I*b*c
^2*d*dilog(-c*x) + I*(b*e*(log(c*x + 1)/c^2 + log(c*x - 1)/c^2) + 8*b*d*in
tegrate(1/2*log(x)/(c^2*x^3 - x), x))*c^2 + 4*c^2*integrate(1/2*(b*e*x^2 +
 2*b*d*log(x))*sqrt(c*x + 1)*sqrt(c*x - 1)/(c^2*x^3 - x), x) - I*b*e*log(c
*x - 1) - 2*(b*c^2*e*x^2 + 2*b*c^2*d*log(x))*arctan(sqrt(c*x + 1)*sqrt(c*x
 - 1)) + (I*b*c^2*e*x^2 + 2*I*b*c^2*d*log(x))*log(c^2*x^2) + (-2*I*b*c^2*d
*log(x) - I*b*e)*log(c*x + 1) - 2*(I*b*c^2*e*x^2 + 2*I*b*c^2*d*log(c))*log
(x))/c^2
```

3.79.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Limit: Max order reached or unable to make series expan
sion Error: Bad Argument Value
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{(e\,x^2+d)\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

input
```
int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x,x)
```

output
```
int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x, x)
```

3.80 $\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x^3}\,dx$

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3.80.7 Maxima [F] . . . 598
3.80.8 Giac [F] . . . 599
3.80.9 Mupad [F(-1)] . . . 599

3.80.1 Optimal result

Integrand size = 19, antiderivative size = 137

$$\begin{aligned}\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x^3}\,dx = {}& \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d\csc^{-1}(cx) - \frac{1}{2}ibe\csc^{-1}(cx)^2 \\ & - \frac{d(a+b\sec^{-1}(cx))}{2x^2} + be\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) \\ & - be\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - e\left(a+b\sec^{-1}(cx)\right)\log\left(\frac{1}{x}\right) \\ & - \frac{1}{2}ibe\,\mathrm{PolyLog}\left(2, e^{2i\csc^{-1}(cx)}\right)\end{aligned}$$

output

```
-1/4*b*c^2*d*arccsc(c*x)-1/2*I*b*e*arccsc(c*x)^2-1/2*d*(a+b*arcsec(c*x))/x
^2+b*e*arccsc(c*x)*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)-b*e*arccsc(c*x)*ln(
1/x)-e*(a+b*arcsec(c*x))*ln(1/x)-1/2*I*b*e*polylog(2,(I/c/x+(1-1/c^2/x^2)^
(1/2))^2)+1/4*b*c*d*(1-1/c^2/x^2)^(1/2)/x
```

3.80.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^3}\,dx = -\frac{ad}{2x^2} + \frac{bcd\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{4x} - \frac{bd\sec^{-1}(cx)}{2x^2} + \frac{1}{2}ibe\sec^{-1}(cx)^2 - \frac{1}{4}bc^2d\arcsin\left(\frac{1}{cx}\right) - be\sec^{-1}(cx)\log\left(1+e^{2i\sec^{-1}(cx)}\right) + ae\log(x) + \frac{1}{2}ibe\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^3,x]
```

output
```
-1/2*(a*d)/x^2 + (b*c*d*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(4*x) - (b*d*ArcSe
c[c*x])/(2*x^2) + (I/2)*b*e*ArcSec[c*x]^2 - (b*c^2*d*ArcSin[1/(c*x)])/4 -
b*e*ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] + a*e*Log[x] + (I/2)*b*e*Po
lyLog[2, -E^((2*I)*ArcSec[c*x])]
```

3.80.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.28, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5763, 5231, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^3}\,dx$$

$$\downarrow 5763$$

$$-\int \left(\frac{d}{x^2}+e\right)x\left(a+b\arccos\left(\frac{1}{cx}\right)\right)d\frac{1}{x}$$

$$\downarrow 5231$$

$$-\frac{b\int \frac{\frac{d}{x^2}+2e\log\left(\frac{1}{x}\right)}{2\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{c} - \frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{2x^2} - e\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)$$

$$\downarrow 27$$

$$-\frac{b\int \frac{\frac{d}{x^2}+2e\log\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{2c}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{2x^2}-e\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)$$

$$\downarrow 7293$$

$$-\frac{b\int \left(\frac{d}{\sqrt{1-\frac{1}{c^2x^2}}x^2}+\frac{2e\log\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{c^2x^2}}}\right)d\frac{1}{x}}{2c}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{2x^2}-e\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)$$

$$\downarrow 2009$$

$$-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{2x^2}-e\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)-$$

$$\frac{b\left(\frac{1}{2}c^3d\arcsin\left(\frac{1}{cx}\right)+ice\,\mathrm{PolyLog}\left(2,e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)+ice\arcsin\left(\frac{1}{cx}\right)^2-2ce\arcsin\left(\frac{1}{cx}\right)\log\left(1-e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)+2c\right.}{2c}$$

input
```
Int[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^3,x]
```

output
```
-1/2*(d*(a + b*ArcCos[1/(c*x)]))/x^2 - e*(a + b*ArcCos[1/(c*x)])*Log[x^(-1
)] - (b*(-1/2*(c^2*d*Sqrt[1 - 1/(c^2*x^2)])/x + (c^3*d*ArcSin[1/(c*x)])/2
+ I*c*e*ArcSin[1/(c*x)]^2 - 2*c*e*ArcSin[1/(c*x)]*Log[1 - E^((2*I)*ArcSin[
1/(c*x)])] + 2*c*e*ArcSin[1/(c*x)]*Log[x^(-1)] + I*c*e*PolyLog[2, E^((2*I)
*ArcSin[1/(c*x)])]))/(2*c)
```

3.80.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.80.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04

method	result
parts	$-\frac{ad}{2x^2}+ae\ln(x)+\frac{ib\,\mathrm{arcsec}(cx)^2e}{2}+\frac{b\,c^2d\,\mathrm{arcsec}(cx)}{4}+\frac{bcd\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4x}-\frac{bd\,\mathrm{arcsec}(cx)}{2x^2}-be\,\mathrm{arcsec}(cx$
derivativedivides	$c^2\left(-\frac{ad}{2c^2x^2}+\frac{ae\ln(cx)}{c^2}+\frac{ibe\,\mathrm{arcsec}(cx)^2}{2c^2}+\frac{bd\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4cx}+\frac{bd\,\mathrm{arcsec}(cx)}{4}-\frac{b\,\mathrm{arcsec}(cx)d}{2c^2x^2}-\frac{be\,\mathrm{arcsec}(cx)}{}\right.$
default	$c^2\left(-\frac{ad}{2c^2x^2}+\frac{ae\ln(cx)}{c^2}+\frac{ibe\,\mathrm{arcsec}(cx)^2}{2c^2}+\frac{bd\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4cx}+\frac{bd\,\mathrm{arcsec}(cx)}{4}-\frac{b\,\mathrm{arcsec}(cx)d}{2c^2x^2}-\frac{be\,\mathrm{arcsec}(cx)}{}\right.$

input
```
int((e*x^2+d)*(a+b*arcsec(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*d/x^2+a*e*ln(x)+1/2*I*b*arcsec(c*x)^2*e+1/4*b*c^2*d*arcsec(c*x)+1/4
*b*c*d/x*((c^2*x^2-1)/c^2/x^2)^(1/2)-1/2*b*d/x^2*arcsec(c*x)-b*e*arcsec(c*
x)*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)+1/2*I*b*e*polylog(2,-(1/c/x+I*(1-
1/c^2/x^2)^(1/2))^2)
```

3.80.5 Fricas [F]

$$\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x^3}\,dx = \int \frac{(ex^2+d)(b\operatorname{arcsec}(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsec(c*x))/x^3, x)
```

3.80.6 Sympy [F]

$$\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x^3}\,dx = \int \frac{(a+b\operatorname{asec}(cx))\,(d+ex^2)}{x^3}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*asec(c*x))/x**3,x)
```

output
```
Integral((a + b*asec(c*x))*(d + e*x**2)/x**3, x)
```

3.80.7 Maxima [F]

$$\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x^3}\,dx = \int \frac{(ex^2+d)(b\operatorname{arcsec}(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^3,x, algorithm="maxima")
```

output
```
-(c^2*integrate(sqrt(c*x + 1)*sqrt(c*x - 1)*log(x)/(c^4*x^3 - c^2*x), x) -
 arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(x))*b*e - 1/4*b*d*((c^4*x*sqrt(-1
/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) - c^3*arctan(c*x*sqrt(-1/(
c^2*x^2) + 1)))/c + 2*arcsec(c*x)/x^2) + a*e*log(x) - 1/2*a*d/x^2
```

3.80.8 Giac [F]

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{(ex^2+d)(b\,\text{arcsec}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^3,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arcsec(c*x) + a)/x^3, x)
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{(e\,x^2+d)\,\left(a+b\,\text{acos}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

input
```
int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x^3,x)
```

output
```
int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x^3, x)
```

3.81 $\int x^2(d+ex^2)^2\left(a+b\sec^{-1}(cx)\right)\,dx$

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3.81.9 Mupad [F(-1)] . . . 608

3.81.1 Optimal result

Integrand size = 21, antiderivative size = 252

$$\begin{aligned}\int x^2(d+ex^2)^2\left(a+b\sec^{-1}(cx)\right)\,dx = &-\frac{b(280c^4d^2+252c^2de+75e^2)\,x^2\sqrt{-1+c^2x^2}}{1680c^5\sqrt{c^2x^2}}\\ &-\frac{be(84c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}}\\ &-\frac{be^2x^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\sec^{-1}(cx)\right)\\ &+\frac{2}{5}dex^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\sec^{-1}(cx)\right)\\ &-\frac{b(280c^4d^2+252c^2de+75e^2)\,x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{1680c^6\sqrt{c^2x^2}}\end{aligned}$$

output

```
1/3*d^2*x^3*(a+b*arcsec(c*x))+2/5*d*e*x^5*(a+b*arcsec(c*x))+1/7*e^2*x^7*(a
+b*arcsec(c*x))-1/1680*b*(280*c^4*d^2+252*c^2*d*e+75*e^2)*x*arctanh(c*x/(c
^2*x^2-1)^(1/2))/c^6/(c^2*x^2)^(1/2)-1/1680*b*(280*c^4*d^2+252*c^2*d*e+75*
e^2)*x^2*(c^2*x^2-1)^(1/2)/c^5/(c^2*x^2)^(1/2)-1/840*b*e*(84*c^2*d+25*e)*x
^4*(c^2*x^2-1)^(1/2)/c^3/(c^2*x^2)^(1/2)-1/42*b*e^2*x^6*(c^2*x^2-1)^(1/2)/
c/(c^2*x^2)^(1/2)
```

3.81.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.74

$$\int x^2\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{c^2x^2\Big(16ac^5x(35d^2+42dex^2+15e^2x^4)-b\sqrt{1-\frac{1}{c^2x^2}}(75e^2+2c^2e(126d+25ex^2)+8c^4(35d^2+21dex^2+$$

input
```
Integrate[x^2*(d + e*x^2)^2*(a + b*ArcSec[c*x]),x]
```

output
```
(c^2*x^2*(16*a*c^5*x*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4) - b*Sqrt[1 - 1/(c^
2*x^2)]*(75*e^2 + 2*c^2*e*(126*d + 25*e*x^2) + 8*c^4*(35*d^2 + 21*d*e*x^2
+ 5*e^2*x^4))) + 16*b*c^7*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4)*ArcSec[c*
x] - b*(280*c^4*d^2 + 252*c^2*d*e + 75*e^2)*Log[(1 + Sqrt[1 - 1/(c^2*x^2)]
)*x])/(1680*c^7)
```

3.81.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.85, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {5761, 27, 1590, 27, 363, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$\downarrow 5761$$
$$-\frac{bcx\int\frac{x^2\left(15e^2x^4+42dex^2+35d^2\right)}{105\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\sec^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\sec^{-1}(cx)\right)$$
$$\downarrow 27$$
$$-\frac{bcx\int\frac{x^2\left(15e^2x^4+42dex^2+35d^2\right)}{\sqrt{c^2x^2-1}}dx}{105\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\sec^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 1590$$

$$-\frac{bcx\left(\frac{\int \frac{3x^2\left(70c^2d^2+e\left(84dc^2+25e\right)x^2\right)}{\sqrt{c^2x^2-1}}dx}{6c^2}+\frac{5e^2x^5\sqrt{c^2x^2-1}}{2c^2}\right)}{105\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\sec^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 27$$

$$-\frac{bcx\left(\frac{\int \frac{x^2\left(70c^2d^2+e\left(84dc^2+25e\right)x^2\right)}{\sqrt{c^2x^2-1}}dx}{2c^2}+\frac{5e^2x^5\sqrt{c^2x^2-1}}{2c^2}\right)}{105\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\sec^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 363$$

$$-\frac{bcx\left(\frac{\left(280c^4d^2+252c^2de+75e^2\right)\int \frac{x^2}{\sqrt{c^2x^2-1}}dx}{4c^2}+\frac{ex^3\sqrt{c^2x^2-1}\left(84c^2d+25e\right)}{4c^2}+\frac{5e^2x^5\sqrt{c^2x^2-1}}{2c^2}\right)}{105\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\sec^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 262$$

$$-\frac{bcx\left(\frac{\left(280c^4d^2+252c^2de+75e^2\right)\left(\frac{\int \frac{1}{\sqrt{c^2x^2-1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)}{4c^2}+\frac{ex^3\sqrt{c^2x^2-1}\left(84c^2d+25e\right)}{4c^2}+\frac{5e^2x^5\sqrt{c^2x^2-1}}{2c^2}\right)}{105\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\sec^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 224$$

$$-\frac{bcx\left(\frac{\left(280c^4d^2+252c^2de+75e^2\right)\left(\frac{\int \frac{1}{1-\frac{c^2x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}}{2c^2}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)}{4c^2}+\frac{ex^3\sqrt{c^2x^2-1}\left(84c^2d+25e\right)}{4c^2}+\frac{5e^2x^5\sqrt{c^2x^2-1}}{2c^2}\right)}{105\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\sec^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 219$$

$$\frac{1}{3}d^2x^3\left(a+b\sec^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\sec^{-1}(cx)\right)-$$

$$bcx\left(\frac{\left(\frac{\operatorname{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{2c^3}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)\left(280c^4d^2+252c^2de+75e^2\right)}{4c^2}+\frac{ex^3\sqrt{c^2x^2-1}\left(84c^2d+25e\right)}{4c^2}+\frac{5e^2x^5\sqrt{c^2x^2-1}}{2c^2}\right)$$

$$\frac{}{105\sqrt{c^2x^2}}$$

input
```
Int[x^2*(d + e*x^2)^2*(a + b*ArcSec[c*x]),x]
```

output
```
(d^2*x^3*(a + b*ArcSec[c*x]))/3 + (2*d*e*x^5*(a + b*ArcSec[c*x]))/5 + (e^2
*x^7*(a + b*ArcSec[c*x]))/7 - (b*c*x*((5*e^2*x^5*Sqrt[-1 + c^2*x^2])/(2*c^
2) + ((e*(84*c^2*d + 25*e)*x^3*Sqrt[-1 + c^2*x^2])/(4*c^2) + ((280*c^4*d^2
 + 252*c^2*d*e + 75*e^2)*((x*Sqrt[-1 + c^2*x^2])/(2*c^2) + ArcTanh[(c*x)/S
qrt[-1 + c^2*x^2]]/(2*c^3)))/(4*c^2))/(2*c^2)))/(105*Sqrt[c^2*x^2])
```

3.81.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.81.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(222) = 444$.

Time = 0.77 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.82

method	result
parts	$a\left(\frac{1}{7}e^2x^7+\frac{2}{5}de\,x^5+\frac{1}{3}x^3d^2\right)+\frac{b\,\mathrm{arcsec}(cx)e^2x^7}{7}+\frac{2b\,\mathrm{arcsec}(cx)de\,x^5}{5}+\frac{b\,\mathrm{arcsec}(cx)d^2x^3}{3}-\frac{b(c^2x^2-1)}{42c^3\sqrt{\frac{c^2x}{c^2}}}$
derivativedivides	$\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}d\,c^7e\,x^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\,\mathrm{arcsec}(cx)d^2c^3x^3}{3}+\frac{2b\,c^3\,\mathrm{arcsec}(cx)de\,x^5}{5}+\frac{b\,c^3\,\mathrm{arcsec}(cx)e^2x^7}{7}-\frac{b\left(c^2x^2-1\right)d^2}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b\left(c^2x^2-1\right.}{10\sqrt{\frac{c^2x}{c^2}}}$
default	$\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}d\,c^7e\,x^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\,\mathrm{arcsec}(cx)d^2c^3x^3}{3}+\frac{2b\,c^3\,\mathrm{arcsec}(cx)de\,x^5}{5}+\frac{b\,c^3\,\mathrm{arcsec}(cx)e^2x^7}{7}-\frac{b\left(c^2x^2-1\right)d^2}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b\left(c^2x^2-1\right.}{10\sqrt{\frac{c^2x}{c^2}}}$

input
```
int(x^2*(e*x^2+d)^2*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/7*e^2*x^7+2/5*d*e*x^5+1/3*x^3*d^2)+1/7*b*arcsec(c*x)*e^2*x^7+2/5*b*ar
csec(c*x)*d*e*x^5+1/3*b*arcsec(c*x)*d^2*x^3-1/42*b/c^3*(c^2*x^2-1)/((c^2*x
^2-1)/c^2/x^2)^(1/2)*x^4*e^2-1/10*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^
(1/2)*x^2*d*e-5/168*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^2*e^2-
1/6*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*d^2-3/20*b/c^5*(c^2*x^2-
1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*d*e-1/6*b/c^4*(c^2*x^2-1)^(1/2)/((c^2*x^2-1
)/c^2/x^2)^(1/2)/x*d^2*ln(c*x+(c^2*x^2-1)^(1/2))-5/112*b/c^7*(c^2*x^2-1)/(
(c^2*x^2-1)/c^2/x^2)^(1/2)*e^2-3/20*b/c^6*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c
^2/x^2)^(1/2)/x*d*e*ln(c*x+(c^2*x^2-1)^(1/2))-5/112*b/c^8*(c^2*x^2-1)^(1/2
)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*e^2*ln(c*x+(c^2*x^2-1)^(1/2))
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.08

$$\int x^2\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{240\,ac^7e^2x^7+672\,ac^7dex^5+560\,ac^7d^2x^3+16\left(15\,bc^7e^2x^7+42\,bc^7dex^5+35\,bc^7d^2x^3-35\,bc^7d^2-42\,bc^7de\right.}{}$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/1680*(240*a*c^7*e^2*x^7 + 672*a*c^7*d*e*x^5 + 560*a*c^7*d^2*x^3 + 16*(15
*b*c^7*e^2*x^7 + 42*b*c^7*d*e*x^5 + 35*b*c^7*d^2*x^3 - 35*b*c^7*d^2 - 42*b
*c^7*d*e - 15*b*c^7*e^2)*arcsec(c*x) + 32*(35*b*c^7*d^2 + 42*b*c^7*d*e + 1
5*b*c^7*e^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)) + (280*b*c^4*d^2 + 252*b*c^2
*d*e + 75*b*e^2)*log(-c*x + sqrt(c^2*x^2 - 1)) - (40*b*c^5*e^2*x^5 + 2*(84
*b*c^5*d*e + 25*b*c^3*e^2)*x^3 + (280*b*c^5*d^2 + 252*b*c^3*d*e + 75*b*c*e
^2)*x)*sqrt(c^2*x^2 - 1))/c^7
```

3.81.6 Sympy [A] (verification not implemented)

Time = 11.90 (sec) , antiderivative size = 542, normalized size of antiderivative = 2.15

$$\int x^2\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$= \frac{ad^2x^3}{3}+\frac{2adex^5}{5}+\frac{ae^2x^7}{7}+\frac{bd^2x^3\,\text{asec}\,(cx)}{3}+\frac{2bdex^5\,\text{asec}\,(cx)}{5}+\frac{be^2x^7\,\text{asec}\,(cx)}{7}$$
$$-\frac{bd^2\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\text{acosh}\,(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\,\text{asin}\,(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}$$
$$-\frac{2bde\left(\begin{cases}\frac{cx^5}{4\sqrt{c^2x^2-1}}+\frac{x^3}{8c\sqrt{c^2x^2-1}}-\frac{3x}{8c^3\sqrt{c^2x^2-1}}+\frac{3\,\text{acosh}\,(cx)}{8c^4} & \text{for } |c^2x^2|>1\\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}}-\frac{ix^3}{8c\sqrt{-c^2x^2+1}}+\frac{3ix}{8c^3\sqrt{-c^2x^2+1}}-\frac{3i\,\text{asin}\,(cx)}{8c^4} & \text{otherwise}\end{cases}\right)}{5c}$$
$$-\frac{be^2\left(\begin{cases}\frac{cx^7}{6\sqrt{c^2x^2-1}}+\frac{x^5}{24c\sqrt{c^2x^2-1}}+\frac{5x^3}{48c^3\sqrt{c^2x^2-1}}-\frac{5x}{16c^5\sqrt{c^2x^2-1}}+\frac{5\,\text{acosh}\,(cx)}{16c^6} & \text{for } |c^2x^2|>1\\ -\frac{icx^7}{6\sqrt{-c^2x^2+1}}-\frac{ix^5}{24c\sqrt{-c^2x^2+1}}-\frac{5ix^3}{48c^3\sqrt{-c^2x^2+1}}+\frac{5ix}{16c^5\sqrt{-c^2x^2+1}}-\frac{5i\,\text{asin}\,(cx)}{16c^6} & \text{otherwise}\end{cases}\right)}{7c}$$

input
```
integrate(x**2*(e*x**2+d)**2*(a+b*asec(c*x)),x)
```

output
```
a*d**2*x**3/3 + 2*a*d*e*x**5/5 + a*e**2*x**7/7 + b*d**2*x**3*asec(c*x)/3 +
 2*b*d*e*x**5*asec(c*x)/5 + b*e**2*x**7*asec(c*x)/7 - b*d**2*Piecewise((x*
sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-I*
c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin(
c*x)/(2*c**2), True))/(3*c) - 2*b*d*e*Piecewise((c*x**5/(4*sqrt(c**2*x**2
 - 1)) + x**3/(8*c*sqrt(c**2*x**2 - 1)) - 3*x/(8*c**3*sqrt(c**2*x**2 - 1))
 + 3*acosh(c*x)/(8*c**4), Abs(c**2*x**2) > 1), (-I*c*x**5/(4*sqrt(-c**2*x**
2 + 1)) - I*x**3/(8*c*sqrt(-c**2*x**2 + 1)) + 3*I*x/(8*c**3*sqrt(-c**2*x**
2 + 1)) - 3*I*asin(c*x)/(8*c**4), True))/(5*c) - b*e**2*Piecewise((c*x**7/
(6*sqrt(c**2*x**2 - 1)) + x**5/(24*c*sqrt(c**2*x**2 - 1)) + 5*x**3/(48*c**
3*sqrt(c**2*x**2 - 1)) - 5*x/(16*c**5*sqrt(c**2*x**2 - 1)) + 5*acosh(c*x)/
(16*c**6), Abs(c**2*x**2) > 1), (-I*c*x**7/(6*sqrt(-c**2*x**2 + 1)) - I*x*
*5/(24*c*sqrt(-c**2*x**2 + 1)) - 5*I*x**3/(48*c**3*sqrt(-c**2*x**2 + 1)) +
 5*I*x/(16*c**5*sqrt(-c**2*x**2 + 1)) - 5*I*asin(c*x)/(16*c**6), True))/(7
*c)
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.61

$$\int x^2\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{1}{7}ae^2x^7+\frac{2}{5}adex^5+\frac{1}{3}ad^2x^3$$
$$+\frac{1}{12}\left(4x^3\operatorname{arcsec}(cx)-\frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}-\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bd^2$$
$$+\frac{1}{40}\left(16x^5\operatorname{arcsec}(cx)+\frac{\frac{2\left(3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2c^4\left(\frac{1}{c^2x^2}-1\right)+c^4}-\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)bde$$
$$+\frac{1}{672}\left(96x^7\operatorname{arcsec}(cx)-\frac{\frac{2\left(15\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-40\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+33\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^6\left(\frac{1}{c^2x^2}-1\right)^3+3c^6\left(\frac{1}{c^2x^2}-1\right)^2+3c^6\left(\frac{1}{c^2x^2}-1\right)+c^6}+\frac{15\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^6}-\frac{15\log\left(\sqrt{-\frac{1}{c^2x^2}+1}\right.\right.}{c}\right.$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*e^2*x^7 + 2/5*a*d*e*x^5 + 1/3*a*d^2*x^3 + 1/12*(4*x^3*arcsec(c*x) -
(2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2
*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d^2 + 1/40
*(16*x^5*arcsec(c*x) + (2*(3*(-1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(-1/(c^2*x^2
) + 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^4) - 3*log(
sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^4)/c
)*b*d*e + 1/672*(96*x^7*arcsec(c*x) - (2*(15*(-1/(c^2*x^2) + 1)^(5/2) - 40
*(-1/(c^2*x^2) + 1)^(3/2) + 33*sqrt(-1/(c^2*x^2) + 1))/(c^6*(1/(c^2*x^2) -
 1)^3 + 3*c^6*(1/(c^2*x^2) - 1)^2 + 3*c^6*(1/(c^2*x^2) - 1) + c^6) + 15*lo
g(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 - 15*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^6
)/c)*b*e^2
```

3.81.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4760 vs. 2(222) = 444.

Time = 6.39 (sec) , antiderivative size = 4760, normalized size of antiderivative = 18.89

$$\int x^2\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
1/1680*(560*b*c^4*d^2*arccos(1/(c*x)) - 280*b*c^4*d^2*log(abs(sqrt(-1/(c^2
*x^2) + 1) + 1/(c*x) + 1)) + 280*b*c^4*d^2*log(abs(sqrt(-1/(c^2*x^2) + 1)
- 1/(c*x) - 1)) + 560*a*c^4*d^2 + 560*b*c^4*d^2*(1/(c^2*x^2) - 1)*arccos(1
/(c*x))/(1/(c*x) + 1)^2 - 1960*b*c^4*d^2*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1
/(c^2*x^2) + 1) + 1/(c*x) + 1))/(1/(c*x) + 1)^2 + 1960*b*c^4*d^2*(1/(c^2*x
^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(1/(c*x) + 1)^2 -
560*b*c^4*d^2*sqrt(-1/(c^2*x^2) + 1)/(1/(c*x) + 1) + 560*a*c^4*d^2*(1/(c^2
*x^2) - 1)/(1/(c*x) + 1)^2 + 672*b*c^2*d*e*arccos(1/(c*x)) - 1680*b*c^4*d^
2*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/(1/(c*x) + 1)^4 - 252*b*c^2*d*e*log(
abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1)) - 5880*b*c^4*d^2*(1/(c^2*x^2) -
 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(1/(c*x) + 1)^4 + 252
*b*c^2*d*e*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1)) + 5880*b*c^4*d^2
*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(1/(c*
x) + 1)^4 + 2240*b*c^4*d^2*(-1/(c^2*x^2) + 1)^(3/2)/(1/(c*x) + 1)^3 + 672*
a*c^2*d*e - 1680*a*c^4*d^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 - 2016*b*c^
2*d*e*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/(1/(c*x) + 1)^2 - 1680*b*c^4*d^2*(
1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/(1/(c*x) + 1)^6 - 1764*b*c^2*d*e*(1/(c^
2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(1/(c*x) + 1)^2
 - 9800*b*c^4*d^2*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(
c*x) + 1))/(1/(c*x) + 1)^6 + 1764*b*c^2*d*e*(1/(c^2*x^2) - 1)*log(abs(s...
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int x^2\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx = \int x^2\left(e\,x^2+d\right)^2\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^2*(d + e*x^2)^2*(a + b*acos(1/(c*x))),x)
```

output
```
int(x^2*(d + e*x^2)^2*(a + b*acos(1/(c*x))), x)
```

3.82 $\int (d+ex^2)^2 (a+b\sec^{-1}(cx))\,dx$

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3.82.1 Optimal result

Integrand size = 18, antiderivative size = 191

$$\begin{aligned}\int (d+ex^2)^2 (a+b\sec^{-1}(cx))\,dx = &-\frac{be(40c^2d+9e)\,x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}}\\ &-\frac{be^2x^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+d^2x\left(a+b\sec^{-1}(cx)\right)\\ &+\frac{2}{3}dex^3\left(a+b\sec^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\sec^{-1}(cx)\right)\\ &-\frac{b(120c^4d^2+40c^2de+9e^2)\,x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{120c^4\sqrt{c^2x^2}}\end{aligned}$$

output

```
d^2*x*(a+b*arcsec(c*x))+2/3*d*e*x^3*(a+b*arcsec(c*x))+1/5*e^2*x^5*(a+b*arc
sec(c*x))-1/120*b*(120*c^4*d^2+40*c^2*d*e+9*e^2)*x*arctanh(c*x/(c^2*x^2-1)
^(1/2))/c^4/(c^2*x^2)^(1/2)-1/120*b*e*(40*c^2*d+9*e)*x^2*(c^2*x^2-1)^(1/2)
/c^3/(c^2*x^2)^(1/2)-1/20*b*e^2*x^4*(c^2*x^2-1)^(1/2)/c/(c^2*x^2)^(1/2)
```

3.82.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.80

$$\int \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx$$
$$= \frac{c^2x\left(8ac^3(15d^2+10dex^2+3e^2x^4) - be\sqrt{1-\frac{1}{c^2x^2}}x(9e+c^2(40d+6ex^2))\right) + 8bc^5x(15d^2+10dex^2+3e^2x$$
$$120c^5$$

input
```
Integrate[(d + e*x^2)^2*(a + b*ArcSec[c*x]),x]
```

output
```
(c^2*x*(8*a*c^3*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - b*e*Sqrt[1 - 1/(c^2*x^
2)]*x*(9*e + c^2*(40*d + 6*e*x^2))) + 8*b*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e
^2*x^4)*ArcSec[c*x] - b*(120*c^4*d^2 + 40*c^2*d*e + 9*e^2)*Log[(1 + Sqrt[1
 - 1/(c^2*x^2)])*x])/(120*c^5)
```

3.82.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5751, 27, 1473, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx$$
$$\downarrow 5751$$
$$-\frac{bcx\int \frac{3e^2x^4+10dex^2+15d^2}{15\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}} + d^2x\left(a+b\sec^{-1}(cx)\right) + \frac{2}{3}dex^3\left(a+b\sec^{-1}(cx)\right) + \frac{1}{5}e^2x^5\left(a+b\sec^{-1}(cx)\right)$$
$$\downarrow 27$$
$$-\frac{bcx\int \frac{3e^2x^4+10dex^2+15d^2}{\sqrt{c^2x^2-1}}dx}{15\sqrt{c^2x^2}} + d^2x\left(a+b\sec^{-1}(cx)\right) + \frac{2}{3}dex^3\left(a+b\sec^{-1}(cx)\right) + \frac{1}{5}e^2x^5\left(a+b\sec^{-1}(cx)\right)$$
$$\downarrow 1473$$

$$-\frac{bcx\left(\frac{\int \frac{60c^2d^2+e\left(40dc^2+9e\right)x^2}{\sqrt{c^2x^2-1}}dx}{4c^2}+\frac{3e^2x^3\sqrt{c^2x^2-1}}{4c^2}\right)}{15\sqrt{c^2x^2}}+d^2x\left(a+b\sec^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sec^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 299$$

$$-\frac{bcx\left(\frac{\frac{\left(120c^4d^2+40c^2de+9e^2\right)\int \frac{1}{\sqrt{c^2x^2-1}}dx}{2c^2}+\frac{ex\sqrt{c^2x^2-1}\left(40c^2d+9e\right)}{2c^2}}{4c^2}+\frac{3e^2x^3\sqrt{c^2x^2-1}}{4c^2}\right)}{15\sqrt{c^2x^2}}+d^2x\left(a+b\sec^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sec^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 224$$

$$-\frac{bcx\left(\frac{\frac{\left(120c^4d^2+40c^2de+9e^2\right)\int \frac{1}{1-\frac{c^2x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}}{2c^2}+\frac{ex\sqrt{c^2x^2-1}\left(40c^2d+9e\right)}{2c^2}}{4c^2}+\frac{3e^2x^3\sqrt{c^2x^2-1}}{4c^2}\right)}{15\sqrt{c^2x^2}}+d^2x\left(a+b\sec^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sec^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 219$$

$$d^2x\left(a+b\sec^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sec^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\sec^{-1}(cx)\right)-\frac{bcx\left(\frac{\frac{\arctanh\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)\left(120c^4d^2+40c^2de+9e^2\right)}{2c^3}+\frac{ex\sqrt{c^2x^2-1}\left(40c^2d+9e\right)}{2c^2}}{4c^2}+\frac{3e^2x^3\sqrt{c^2x^2-1}}{4c^2}\right)}{15\sqrt{c^2x^2}}$$

input
```
Int[(d + e*x^2)^2*(a + b*ArcSec[c*x]),x]
```

output
```
d^2*x*(a + b*ArcSec[c*x]) + (2*d*e*x^3*(a + b*ArcSec[c*x]))/3 + (e^2*x^5*(
a + b*ArcSec[c*x]))/5 - (b*c*x*((3*e^2*x^3*Sqrt[-1 + c^2*x^2])/(4*c^2) + (
(e*(40*c^2*d + 9*e)*x*Sqrt[-1 + c^2*x^2])/(2*c^2) + ((120*c^4*d^2 + 40*c^2
*d*e + 9*e^2)*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(2*c^3))/(4*c^2)))/(15*Sq
rt[c^2*x^2])
```

3.82.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 5751
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSec[c*x])   u,
x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 -
 1]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1
/2, 0])
```

3.82.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(169) = 338.

Time = 0.42 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.78

method	result
parts	$a\left(\frac{1}{5}e^2x^5+\frac{2}{3}de\,x^3+d^2x\right)+\frac{b\,\mathrm{arcsec}(cx)e^2x^5}{5}+\frac{2b\,\mathrm{arcsec}(cx)de\,x^3}{3}+b\,\mathrm{arcsec}\left(cx\right)d^2x-\frac{b(c^2x^2-1}{20c^3\sqrt{\frac{c^2}{}}}$
derivativedivides	$\frac{\frac{a\left(d^2c^5x+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+b\,\mathrm{arcsec}(cx)d^2cx+\frac{2bc\,\mathrm{arcsec}(cx)de\,x^3}{3}+\frac{bc\,\mathrm{arcsec}(cx)e^2x^5}{5}-\frac{b\sqrt{c^2x^2-1}\,d^2\ln\left(cx+\sqrt{c^2x^2-1}\right)}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{c}$
default	$\frac{\frac{a\left(d^2c^5x+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+b\,\mathrm{arcsec}(cx)d^2cx+\frac{2bc\,\mathrm{arcsec}(cx)de\,x^3}{3}+\frac{bc\,\mathrm{arcsec}(cx)e^2x^5}{5}-\frac{b\sqrt{c^2x^2-1}\,d^2\ln\left(cx+\sqrt{c^2x^2-1}\right)}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{c}$

input
```
int((e*x^2+d)^2*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*e^2*x^5+2/3*d*e*x^3+d^2*x)+1/5*b*arcsec(c*x)*e^2*x^5+2/3*b*arcsec(c
*x)*d*e*x^3+b*arcsec(c*x)*d^2*x-1/20*b/c^3*(c^2*x^2-1)*x^2/((c^2*x^2-1)/c^
2/x^2)^(1/2)*e^2-1/3*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*d*e-b/c
^2*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^2*ln(c*x+(c^2*x^2-1)^
(1/2))-3/40*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*e^2-1/3*b/c^4*(c
^2*x^2-1)^(1/2)/x/((c^2*x^2-1)/c^2/x^2)^(1/2)*d*e*ln(c*x+(c^2*x^2-1)^(1/2)
)-3/40*b/c^6*(c^2*x^2-1)^(1/2)/x/((c^2*x^2-1)/c^2/x^2)^(1/2)*e^2*ln(c*x+(c
^2*x^2-1)^(1/2))
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.24

$$\int \left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$=\frac{24\,ac^5e^2x^5+80\,ac^5dex^3+120\,ac^5d^2x+8\left(3\,bc^5e^2x^5+10\,bc^5dex^3+15\,bc^5d^2x-15\,bc^5d^2-10\,bc^5de-3\,bc\right.}{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/120*(24*a*c^5*e^2*x^5 + 80*a*c^5*d*e*x^3 + 120*a*c^5*d^2*x + 8*(3*b*c^5*
e^2*x^5 + 10*b*c^5*d*e*x^3 + 15*b*c^5*d^2*x - 15*b*c^5*d^2 - 10*b*c^5*d*e
- 3*b*c^5*e^2)*arcsec(c*x) + 16*(15*b*c^5*d^2 + 10*b*c^5*d*e + 3*b*c^5*e^2
)*arctan(-c*x + sqrt(c^2*x^2 - 1)) + (120*b*c^4*d^2 + 40*b*c^2*d*e + 9*b*e
^2)*log(-c*x + sqrt(c^2*x^2 - 1)) - (6*b*c^3*e^2*x^3 + (40*b*c^3*d*e + 9*b
*c*e^2)*x)*sqrt(c^2*x^2 - 1))/c^5
```

3.82.6 Sympy [A] (verification not implemented)

Time = 5.83 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.86

$$\int \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx$$

$$= ad^2x + \frac{2adex^3}{3} + \frac{ae^2x^5}{5} + bd^2x\,\text{asec}\left(cx\right) + \frac{2bdex^3\,\text{asec}\left(cx\right)}{3}$$

$$+ \frac{be^2x^5\,\text{asec}\left(cx\right)}{5} - \frac{bd^2\left(\begin{cases}\text{acosh}\left(cx\right) & \text{for } |c^2x^2| > 1\\ -i\,\text{asin}\left(cx\right) & \text{otherwise}\end{cases}\right)}{c}$$

$$- \frac{2bde\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c} + \frac{\text{acosh}\left(cx\right)}{2c^2} & \text{for } |c^2x^2| > 1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}} + \frac{ix}{2c\sqrt{-c^2x^2+1}} - \frac{i\,\text{asin}\left(cx\right)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}$$

$$- \frac{be^2\left(\begin{cases}\frac{cx^5}{4\sqrt{c^2x^2-1}} + \frac{x^3}{8c\sqrt{c^2x^2-1}} - \frac{3x}{8c^3\sqrt{c^2x^2-1}} + \frac{3\,\text{acosh}\left(cx\right)}{8c^4} & \text{for } |c^2x^2| > 1\\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}} - \frac{ix^3}{8c\sqrt{-c^2x^2+1}} + \frac{3ix}{8c^3\sqrt{-c^2x^2+1}} - \frac{3i\,\text{asin}\left(cx\right)}{8c^4} & \text{otherwise}\end{cases}\right)}{5c}$$

input
```
integrate((e*x**2+d)**2*(a+b*asec(c*x)),x)
```

output
```
a*d**2*x + 2*a*d*e*x**3/3 + a*e**2*x**5/5 + b*d**2*x*asec(c*x) + 2*b*d*e*x
**3*asec(c*x)/3 + b*e**2*x**5*asec(c*x)/5 - b*d**2*Piecewise((acosh(c*x),
Abs(c**2*x**2) > 1), (-I*asin(c*x), True))/c - 2*b*d*e*Piecewise((x*sqrt(c
**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-I*c*x**3
/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin(c*x)/(
2*c**2), True))/(3*c) - b*e**2*Piecewise((c*x**5/(4*sqrt(c**2*x**2 - 1)) +
 x**3/(8*c*sqrt(c**2*x**2 - 1)) - 3*x/(8*c**3*sqrt(c**2*x**2 - 1)) + 3*aco
sh(c*x)/(8*c**4), Abs(c**2*x**2) > 1), (-I*c*x**5/(4*sqrt(-c**2*x**2 + 1))
 - I*x**3/(8*c*sqrt(-c**2*x**2 + 1)) + 3*I*x/(8*c**3*sqrt(-c**2*x**2 + 1))
 - 3*I*asin(c*x)/(8*c**4), True))/(5*c)
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.55

$$\int \left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{1}{5}ae^2x^5+\frac{2}{3}adex^3$$
$$+\frac{1}{6}\left(4x^3\operatorname{arcsec}(cx)-\frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}-\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bde$$
$$+\frac{1}{80}\left(16x^5\operatorname{arcsec}(cx)+\frac{\frac{2\left(3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2c^4\left(\frac{1}{c^2x^2}-1\right)+c^4}-\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)be^2$$
$$+ad^2x+\frac{\left(2cx\operatorname{arcsec}(cx)-\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)+\log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bd^2}{2c}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*e^2*x^5 + 2/3*a*d*e*x^3 + 1/6*(4*x^3*arcsec(c*x) - (2*sqrt(-1/(c^2*x
^2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c
^2 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d*e + 1/80*(16*x^5*arcsec(c
*x) + (2*(3*(-1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(-1/(c^2*x^2) + 1))/(c^4*(1/(
c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^4) - 3*log(sqrt(-1/(c^2*x^2)
 + 1) + 1)/c^4 + 3*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*e^2 + a*d^2*x
 + 1/2*(2*c*x*arcsec(c*x) - log(sqrt(-1/(c^2*x^2) + 1) + 1) + log(-sqrt(-1
/(c^2*x^2) + 1) + 1))*b*d^2/c
```

3.82.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14166 vs. 2(169) = 338.

Time = 4.60 (sec) , antiderivative size = 14166, normalized size of antiderivative = 74.17

$$\int \left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
1/120*(120*b*c^4*d^2*arccos(1/(c*x))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*
x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^
2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^
6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10) - 120*b*c^4*d^2*log(abs(sqrt(-1/(c
^2*x^2) + 1) + 1/(c*x) + 1))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^
2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^
3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^
2*x^2) - 1)^5/(1/(c*x) + 1)^10) + 120*b*c^4*d^2*log(abs(sqrt(-1/(c^2*x^2)
+ 1) - 1/(c*x) - 1))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c
^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*
x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) -
 1)^5/(1/(c*x) + 1)^10) + 120*a*c^4*d^2/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/
(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2
*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 +
 c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10) + 360*b*c^4*d^2*(1/(c^2*x^2) -
1)*arccos(1/(c*x))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^
6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x
) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) -
1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^2) - 600*b*c^4*d^2*(1/(c^2*x^2) - 1)*
log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^...
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx = \int \left(e\,x^2+d\right)^2 \left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int((d + e*x^2)^2*(a + b*acos(1/(c*x))),x)
```

output
```
int((d + e*x^2)^2*(a + b*acos(1/(c*x))), x)
```

3.83 $\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^2}\,dx$

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3.83.1 Optimal result

Integrand size = 21, antiderivative size = 162

$$\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^2}\,dx = \frac{bcd^2\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} - \frac{be^2x^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{x} + 2dex(a+b\sec^{-1}(cx)) + \frac{1}{3}e^2x^3(a+b\sec^{-1}(cx)) - \frac{be(12c^2d+e)\,x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{6c^2\sqrt{c^2x^2}}$$

output

```
-d^2*(a+b*arcsec(c*x))/x+2*d*e*x*(a+b*arcsec(c*x))+1/3*e^2*x^3*(a+b*arcsec
(c*x))-1/6*b*e*(12*c^2*d+e)*x*arctanh(c*x/(c^2*x^2-1)^(1/2))/c^2/(c^2*x^2)
^(1/2)+b*c*d^2*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)-1/6*b*e^2*x^2*(c^2*x^2-1)
^(1/2)/c/(c^2*x^2)^(1/2)
```

3.83.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x^2}\,dx$$
$$= \frac{c^2\left(b\sqrt{1-\frac{1}{c^2x^2}}x(6c^2d^2-e^2x^2)+2ac(-3d^2+6dex^2+e^2x^4)\right)+2bc^3(-3d^2+6dex^2+e^2x^4)\sec^{-1}(cx)-b}{6c^3x}$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x^2,x]
```

output
```
(c^2*(b*Sqrt[1 - 1/(c^2*x^2)]*x*(6*c^2*d^2 - e^2*x^2) + 2*a*c*(-3*d^2 + 6*
d*e*x^2 + e^2*x^4)) + 2*b*c^3*(-3*d^2 + 6*d*e*x^2 + e^2*x^4)*ArcSec[c*x] -
 b*e*(12*c^2*d + e)*x*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/(6*c^3*x)
```

3.83.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.90, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {5761, 27, 1588, 25, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x^2}\,dx$$
$$\downarrow 5761$$
$$-\frac{bcx\int -\frac{-e^2x^4-6dex^2+3d^2}{3x^2\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}} - \frac{d^2\left(a+b\sec^{-1}(cx)\right)}{x} + 2dex\left(a+b\sec^{-1}(cx)\right) + \frac{1}{3}e^2x^3\left(a+b\sec^{-1}(cx)\right)$$
$$\downarrow 27$$
$$\frac{bcx\int \frac{-e^2x^4-6dex^2+3d^2}{x^2\sqrt{c^2x^2-1}}dx}{3\sqrt{c^2x^2}} - \frac{d^2\left(a+b\sec^{-1}(cx)\right)}{x} + 2dex\left(a+b\sec^{-1}(cx)\right) + \frac{1}{3}e^2x^3\left(a+b\sec^{-1}(cx)\right)$$
$$\downarrow 1588$$

$$\frac{bcx\left(\int -\frac{e(ex^2+6d)}{\sqrt{c^2x^2-1}}dx+\frac{3d^2\sqrt{c^2x^2-1}}{x}\right)}{3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{x}+2dex\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 25$$

$$\frac{bcx\left(\frac{3d^2\sqrt{c^2x^2-1}}{x}-\int\frac{e(ex^2+6d)}{\sqrt{c^2x^2-1}}dx\right)}{3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{x}+2dex\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 27$$

$$\frac{bcx\left(\frac{3d^2\sqrt{c^2x^2-1}}{x}-e\int\frac{ex^2+6d}{\sqrt{c^2x^2-1}}dx\right)}{3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{x}+2dex\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 299$$

$$\frac{bcx\left(\frac{3d^2\sqrt{c^2x^2-1}}{x}-e\left(\frac{\left(12c^2d+e\right)\int\frac{1}{\sqrt{c^2x^2-1}}dx}{2c^2}+\frac{ex\sqrt{c^2x^2-1}}{2c^2}\right)\right)}{3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{x}+2dex\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 224$$

$$\frac{bcx\left(\frac{3d^2\sqrt{c^2x^2-1}}{x}-e\left(\frac{\left(12c^2d+e\right)\int\frac{1}{1-\frac{c^2x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}}{2c^2}+\frac{ex\sqrt{c^2x^2-1}}{2c^2}\right)\right)}{3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{x}+2dex\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 219$$

$$-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{x}+2dex\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\sec^{-1}(cx)\right)+\frac{bcx\left(\frac{3d^2\sqrt{c^2x^2-1}}{x}-e\left(\frac{\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)\left(12c^2d+e\right)}{2c^3}+\frac{ex\sqrt{c^2x^2-1}}{2c^2}\right)\right)}{3\sqrt{c^2x^2}}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x^2,x]
```

output
```
-((d^2*(a + b*ArcSec[c*x]))/x) + 2*d*e*x*(a + b*ArcSec[c*x]) + (e^2*x^3*(a
 + b*ArcSec[c*x]))/3 + (b*c*x*((3*d^2*Sqrt[-1 + c^2*x^2])/x - e*((e*x*Sqrt
[-1 + c^2*x^2])/(2*c^2) + ((12*c^2*d + e)*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]
])/(2*c^3))))/(3*Sqrt[c^2*x^2])
```

3.83.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 5761

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])  u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])  Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.83.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.54

method	result
parts	$a\left(\frac{e^2x^3}{3}+2dex-\frac{d^2}{x}\right)+\frac{b\,e^2\operatorname{arcsec}(cx)x^3}{3}+2be\operatorname{arcsec}(cx)xd-\frac{b\operatorname{arcsec}(cx)d^2}{x}-\frac{b(c^2x^2-1)e^2}{6c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+$
derivativedivides	$c\left(\frac{a\left(2c^3dex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4}+\frac{2b\operatorname{arcsec}(cx)dex}{c}+\frac{b\operatorname{arcsec}(cx)e^2x^3}{3c}-\frac{b\operatorname{arcsec}(cx)d^2}{cx}+\frac{b(c^2x^2-1)d^2}{c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{2b}{}\right.$
default	$c\left(\frac{a\left(2c^3dex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4}+\frac{2b\operatorname{arcsec}(cx)dex}{c}+\frac{b\operatorname{arcsec}(cx)e^2x^3}{3c}-\frac{b\operatorname{arcsec}(cx)d^2}{cx}+\frac{b(c^2x^2-1)d^2}{c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{2b}{}\right.$

input

```
int((e*x^2+d)^2*(a+b*arcsec(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output

```
a*(1/3*e^2*x^3+2*d*e*x-d^2/x)+1/3*b*e^2*arcsec(c*x)*x^3+2*b*e*arcsec(c*x)*
x*d-b*arcsec(c*x)*d^2/x-1/6*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*
e^2+b/c*(c^2*x^2-1)/x^2/((c^2*x^2-1)/c^2/x^2)^(1/2)*d^2-2*b/c^2*(c^2*x^2-1
)^(1/2)/x/((c^2*x^2-1)/c^2/x^2)^(1/2)*d*e*ln(c*x+(c^2*x^2-1)^(1/2))-1/6*b/
c^4*(c^2*x^2-1)^(1/2)/x/((c^2*x^2-1)/c^2/x^2)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/
2))*e^2
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.42

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x^2}\,dx$$

$$=\frac{2\,ac^3e^2x^4+6\,bc^4d^2x+12\,ac^3dex^2-6\,ac^3d^2-4\,(3\,bc^3d^2-6\,bc^3de-bc^3e^2)x\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+}{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^2,x, algorithm="fricas")
```

output
```
1/6*(2*a*c^3*e^2*x^4 + 6*b*c^4*d^2*x + 12*a*c^3*d*e*x^2 - 6*a*c^3*d^2 - 4*
(3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x*arctan(-c*x + sqrt(c^2*x^2 - 1))
 + (12*b*c^2*d*e + b*e^2)*x*log(-c*x + sqrt(c^2*x^2 - 1)) + 2*(b*c^3*e^2*x
^4 + 6*b*c^3*d*e*x^2 - 3*b*c^3*d^2 + (3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^
2)*x)*arcsec(c*x) + (6*b*c^3*d^2 - b*c*e^2*x^2)*sqrt(c^2*x^2 - 1))/(c^3*x)
```

3.83.6 Sympy [A] (verification not implemented)

Time = 4.36 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x^2}\,dx$$
$$= -\frac{ad^2}{x} + 2adex + \frac{ae^2x^3}{3} + bcd^2\sqrt{1-\frac{1}{c^2x^2}} - \frac{bd^2\operatorname{asec}(cx)}{x} + 2bdex\operatorname{asec}(cx)$$
$$+ \frac{be^2x^3\operatorname{asec}(cx)}{3} - \frac{2bde\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c}$$
$$- \frac{be^2\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}$$

input
```
integrate((e*x**2+d)**2*(a+b*asec(c*x))/x**2,x)
```

output
```
-a*d**2/x + 2*a*d*e*x + a*e**2*x**3/3 + b*c*d**2*sqrt(1 - 1/(c**2*x**2)) -
 b*d**2*asec(c*x)/x + 2*b*d*e*x*asec(c*x) + b*e**2*x**3*asec(c*x)/3 - 2*b*
d*e*Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x), True))/c -
b*e**2*Piecewise((x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c
**2*x**2) > 1), (-I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*
x**2 + 1)) - I*asin(c*x)/(2*c**2), True))/(3*c)
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.22

$$\begin{aligned}
&\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x^2}\,dx \\
&= \frac{1}{3}\,ae^2x^3 + \left(c\sqrt{-\frac{1}{c^2x^2}+1} - \frac{\operatorname{arcsec}(cx)}{x}\right)bd^2 \\
&\quad + \frac{1}{12}\left(4\,x^3\operatorname{arcsec}(cx) - \frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)be^2 \\
&\quad + 2\,adex \\
&\quad + \frac{\left(2\,cx\operatorname{arcsec}(cx) - \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) + \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bde}{c} - \frac{ad^2}{x}
\end{aligned}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^2,x, algorithm="maxima")
```

output
```
1/3*a*e^2*x^3 + (c*sqrt(-1/(c^2*x^2) + 1) - arcsec(c*x)/x)*b*d^2 + 1/12*(4
*x^3*arcsec(c*x) - (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2)
 + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c
^2)/c)*b*e^2 + 2*a*d*e*x + (2*c*x*arcsec(c*x) - log(sqrt(-1/(c^2*x^2) + 1)
 + 1) + log(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*d*e/c - a*d^2/x
```

3.83.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6018 vs. $2(144) = 288$.

Time = 2.47 (sec) , antiderivative size = 6018, normalized size of antiderivative = 37.15

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^2,x, algorithm="giac")
```

output
```
-1/6*(6*b*c^4*d^2*arccos(1/(c*x))/(c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x)
+ 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)
^4/(1/(c*x) + 1)^8) + 6*a*c^4*d^2/(c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x)
+ 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)
^4/(1/(c*x) + 1)^8) + 24*b*c^4*d^2*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^4
 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/
(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) -
 12*b*c^4*d^2*sqrt(-1/(c^2*x^2) + 1)/((c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c
*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2)
- 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)) + 24*a*c^4*d^2*(1/(c^2*x^2) - 1)/((
c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/
(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2
) - 12*b*c^2*d*e*arccos(1/(c*x))/(c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) +
 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^
4/(1/(c*x) + 1)^8) + 36*b*c^4*d^2*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^
4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1
/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4)
+ 12*b*c^2*d*e*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(c^4 + 2*c^4
*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) +
1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) - 12*b*c^2*d*e*log(abs(...
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x^2}\,dx = \int \frac{(e\,x^2+d)^2\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

input
```
int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^2,x)
```

output
```
int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^2, x)
```

3.84 $\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^4}\,dx$

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3.84.1 Optimal result

Integrand size = 21, antiderivative size = 158

$$\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^4}\,dx = \frac{2bcd(c^2d+9e)\sqrt{-1+c^2x^2}}{9\sqrt{c^2x^2}} + \frac{bcd^2\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{3x^3} - \frac{2de(a+b\sec^{-1}(cx))}{x} + e^2x(a+b\sec^{-1}(cx)) - \frac{be^2x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{c^2x^2}}$$

output
```
-1/3*d^2*(a+b*arcsec(c*x))/x^3-2*d*e*(a+b*arcsec(c*x))/x+e^2*x*(a+b*arcsec
(c*x))-b*e^2*x*arctanh(c*x/(c^2*x^2-1)^(1/2))/(c^2*x^2)^(1/2)+2/9*b*c*d*(c
^2*d+9*e)*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)+1/9*b*c*d^2*(c^2*x^2-1)^(1/2)/
x^2/(c^2*x^2)^(1/2)
```

3.84.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^4}\,dx = \frac{c\left(bcd\sqrt{1-\frac{1}{c^2x^2}}x(d+2c^2dx^2+18ex^2)-3a(d^2+6dex^2-3e^2x^4)\right)-3bc(d^2+6dex^2-3e^2x^4)\sec^{-1}(cx)}{9cx^3}$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x^4,x]
```

output
```
(c*(b*c*d*Sqrt[1 - 1/(c^2*x^2)]*x*(d + 2*c^2*d*x^2 + 18*e*x^2) - 3*a*(d^2
+ 6*d*e*x^2 - 3*e^2*x^4)) - 3*b*c*(d^2 + 6*d*e*x^2 - 3*e^2*x^4)*ArcSec[c*x
] - 9*b*e^2*x^3*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/(9*c*x^3)
```

3.84.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5761, 27, 1588, 358, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)}{x^4}\,dx$$

$$\downarrow 5761$$

$$-\frac{bcx\int -\frac{-3e^2x^4+6dex^2+d^2}{3x^4\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\sec^{-1}(cx)\right)}{x}+e^2x\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 27$$

$$\frac{bcx\int \frac{-3e^2x^4+6dex^2+d^2}{x^4\sqrt{c^2x^2-1}}dx}{3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\sec^{-1}(cx)\right)}{x}+e^2x\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 1588$$

$$\frac{bcx\left(\frac{1}{3}\int \frac{2d(dc^2+9e)-9e^2x^2}{x^2\sqrt{c^2x^2-1}}dx+\frac{d^2\sqrt{c^2x^2-1}}{3x^3}\right)}{3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\sec^{-1}(cx)\right)}{x}+e^2x\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 358$$

$$\frac{bcx\left(\frac{1}{3}\left(\frac{2d\sqrt{c^2x^2-1}(c^2d+9e)}{x}-9e^2\int \frac{1}{\sqrt{c^2x^2-1}}dx\right)+\frac{d^2\sqrt{c^2x^2-1}}{3x^3}\right)}{3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\sec^{-1}(cx)\right)}{x}+e^2x\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 224$$

$$\frac{bcx\left(\frac{1}{3}\left(\frac{2d\sqrt{c^2x^2-1}(c^2d+9e)}{x}-9e^2\int\frac{1}{1-\frac{c^2x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}\right)+\frac{d^2\sqrt{c^2x^2-1}}{3x^3}\right)}{3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{3x^3}-$$
$$\frac{2de\left(a+b\sec^{-1}(cx)\right)}{x}+e^2x\left(a+b\sec^{-1}(cx)\right)$$
$$\downarrow 219$$
$$-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\sec^{-1}(cx)\right)}{x}+e^2x\left(a+b\sec^{-1}(cx)\right)+$$
$$\frac{bcx\left(\frac{1}{3}\left(\frac{2d\sqrt{c^2x^2-1}(c^2d+9e)}{x}-\frac{9e^2\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{c}\right)+\frac{d^2\sqrt{c^2x^2-1}}{3x^3}\right)}{3\sqrt{c^2x^2}}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x^4,x]
```

output
```
-1/3*(d^2*(a + b*ArcSec[c*x]))/x^3 - (2*d*e*(a + b*ArcSec[c*x]))/x + e^2*x
*(a + b*ArcSec[c*x]) + (b*c*x*((d^2*Sqrt[-1 + c^2*x^2])/(3*x^3) + ((2*d*(c
^2*d + 9*e)*Sqrt[-1 + c^2*x^2])/x - (9*e^2*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2
]])/c)/3))/(3*Sqrt[c^2*x^2])
```

3.84.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 1588

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 5761

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.84.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.46

method	result
parts	$a\left(e^2x-\frac{2de}{x}-\frac{d^2}{3x^3}\right)+b\,\operatorname{arcsec}(cx)\,e^2x-\frac{2b\,\operatorname{arcsec}(cx)de}{x}-\frac{b\,\operatorname{arcsec}(cx)d^2}{3x^3}+\frac{2bc(c^2x^2-1)d^2}{9\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^2}+\frac{2b}{c}$
derivativedivides	$c^3\left(\frac{a\left(e^2cx-\frac{cd^2}{3x^3}-\frac{2cde}{x}\right)}{c^4}+\frac{b\,\operatorname{arcsec}(cx)e^2x}{c^3}-\frac{b\,\operatorname{arcsec}(cx)d^2}{3c^3x^3}-\frac{2b\,\operatorname{arcsec}(cx)de}{c^3x}+\frac{2b(c^2x^2-1)d^2}{9c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b(c^2x^2}{9\sqrt{\frac{c^2x^2}{c^2}}}\right.$
default	$c^3\left(\frac{a\left(e^2cx-\frac{cd^2}{3x^3}-\frac{2cde}{x}\right)}{c^4}+\frac{b\,\operatorname{arcsec}(cx)e^2x}{c^3}-\frac{b\,\operatorname{arcsec}(cx)d^2}{3c^3x^3}-\frac{2b\,\operatorname{arcsec}(cx)de}{c^3x}+\frac{2b(c^2x^2-1)d^2}{9c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b(c^2x^2}{9\sqrt{\frac{c^2x^2}{c^2}}}\right.$

input

```
int((e*x^2+d)^2*(a+b*arcsec(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output

```
a*(e^2*x-2*d*e/x-1/3*d^2/x^3)+b*arcsec(c*x)*e^2*x-2*b*arcsec(c*x)*d*e/x-1/
3*b*arcsec(c*x)*d^2/x^3+2/9*b*c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^
2*d^2+2*b/c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^2*d*e-b/c^2*(c^2*x^2
-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*e^2*ln(c*x+(c^2*x^2-1)^(1/2))+1/9*
b/c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^4*d^2
```

3.84.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.40

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)}{x^4}\,dx$$
$$= \frac{9\,ace^2x^4+9\,be^2x^3\log\left(-cx+\sqrt{c^2x^2-1}\right)-18\,acdex^2-6\left(bcd^2+6\,bcde-3\,bce^2\right)x^3\arctan\left(-cx+\sqrt{c^2x}\right.}{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^4,x, algorithm="fricas")
```

output
```
1/9*(9*a*c*e^2*x^4 + 9*b*e^2*x^3*log(-c*x + sqrt(c^2*x^2 - 1)) - 18*a*c*d*
e*x^2 - 6*(b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3*arctan(-c*x + sqrt(c^2*x^2
 - 1)) - 3*a*c*d^2 + 2*(b*c^4*d^2 + 9*b*c^2*d*e)*x^3 + 3*(3*b*c*e^2*x^4 -
6*b*c*d*e*x^2 - b*c*d^2 + (b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3)*arcsec(c*
x) + (b*c*d^2 + 2*(b*c^3*d^2 + 9*b*c*d*e)*x^2)*sqrt(c^2*x^2 - 1))/(c*x^3)
```

3.84.6 Sympy [A] (verification not implemented)

Time = 4.02 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.34

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)}{x^4}\,dx = -\frac{ad^2}{3x^3}-\frac{2ade}{x}+ae^2x+2bcde\sqrt{1-\frac{1}{c^2x^2}}$$
$$-\frac{bd^2\operatorname{asec}(cx)}{3x^3}-\frac{2bde\operatorname{asec}(cx)}{x}+be^2x\operatorname{asec}(cx)$$
$$+\frac{bd^2\left(\begin{cases}\frac{2c^3\sqrt{c^2x^2-1}}{3x}+\frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2|>1\\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x}+\frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)}{3c}$$
$$-\frac{be^2\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c}$$

input
```
integrate((e*x**2+d)**2*(a+b*asec(c*x))/x**4,x)
```

output
```
-a*d**2/(3*x**3) - 2*a*d*e/x + a*e**2*x + 2*b*c*d*e*sqrt(1 - 1/(c**2*x**2)
) - b*d**2*asec(c*x)/(3*x**3) - 2*b*d*e*asec(c*x)/x + b*e**2*x*asec(c*x) +
 b*d**2*Piecewise((2*c**3*sqrt(c**2*x**2 - 1)/(3*x) + c*sqrt(c**2*x**2 - 1
)/(3*x**3), Abs(c**2*x**2) > 1), (2*I*c**3*sqrt(-c**2*x**2 + 1)/(3*x) + I*
c*sqrt(-c**2*x**2 + 1)/(3*x**3), True))/(3*c) - b*e**2*Piecewise((acosh(c*
x), Abs(c**2*x**2) > 1), (-I*asin(c*x), True))/c
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.01

$$
\begin{aligned}
&\int \frac{\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)}{x^4}\,dx \\
&= 2\left(c\sqrt{-\frac{1}{c^2x^2}+1}-\frac{\operatorname{arcsec}(cx)}{x}\right)bde+ae^2x \\
&\quad -\frac{1}{9}bd^2\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{3\operatorname{arcsec}(cx)}{x^3}\right) \\
&\quad +\frac{\left(2\,cx\operatorname{arcsec}(cx)-\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)+\log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)be^2}{2\,c} \\
&\quad -\frac{2\,ade}{x}-\frac{ad^2}{3\,x^3}
\end{aligned}
$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^4,x, algorithm="maxima")
```

output
```
2*(c*sqrt(-1/(c^2*x^2) + 1) - arcsec(c*x)/x)*b*d*e + a*e^2*x - 1/9*b*d^2*(
(c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c + 3*arcsec
(c*x)/x^3) + 1/2*(2*c*x*arcsec(c*x) - log(sqrt(-1/(c^2*x^2) + 1) + 1) + lo
g(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*e^2/c - 2*a*d*e/x - 1/3*a*d^2/x^3
```

3.84.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4968 vs. 2(140) = 280.

Time = 99.04 (sec) , antiderivative size = 4968, normalized size of antiderivative = 31.44

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^4,x, algorithm="giac")
```

output
```
-1/9*(3*b*c^4*d^2*arccos(1/(c*x))/(c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x)
+ 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)
^4/(1/(c*x) + 1)^8) + 3*a*c^4*d^2/(c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x)
+ 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)
^4/(1/(c*x) + 1)^8) + 12*b*c^4*d^2*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^2
 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/
(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) -
 6*b*c^4*d^2*sqrt(-1/(c^2*x^2) + 1)/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*
x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) -
 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)) + 12*a*c^4*d^2*(1/(c^2*x^2) - 1)/((c
^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(
1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2)
 + 18*b*c^2*d*e*arccos(1/(c*x))/(c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) +
1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4
/(1/(c*x) + 1)^8) + 18*b*c^4*d^2*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^2
 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/
(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) +
 2*b*c^4*d^2*(-1/(c^2*x^2) + 1)^(3/2)/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(
c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2)
 - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^3) + 18*a*c^2*d*e/(c^2 - 2*c^2*(...
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x^4}\,dx = \int \frac{(e\,x^2+d)^2\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

input
```
int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^4,x)
```

output
```
int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^4, x)
```

3.85 $\int \frac{\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)}{x^6}\,dx$

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3.85.1 Optimal result

Integrand size = 21, antiderivative size = 183

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)}{x^6}\,dx = \frac{bc(24c^4d^2+100c^2de+225e^2)\sqrt{-1+c^2x^2}}{225\sqrt{c^2x^2}} + \frac{bcd^2\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}} + \frac{2bcd(6c^2d+25e)\sqrt{-1+c^2x^2}}{225x^2\sqrt{c^2x^2}} - \frac{d^2\left(a+b\sec^{-1}(cx)\right)}{5x^5} - \frac{2de\left(a+b\sec^{-1}(cx)\right)}{3x^3} - \frac{e^2\left(a+b\sec^{-1}(cx)\right)}{x}$$

output

```
-1/5*d^2*(a+b*arcsec(c*x))/x^5-2/3*d*e*(a+b*arcsec(c*x))/x^3-e^2*(a+b*arcs
ec(c*x))/x+1/225*b*c*(24*c^4*d^2+100*c^2*d*e+225*e^2)*(c^2*x^2-1)^(1/2)/(c
^2*x^2)^(1/2)+1/25*b*c*d^2*(c^2*x^2-1)^(1/2)/x^4/(c^2*x^2)^(1/2)+2/225*b*c
*d*(6*c^2*d+25*e)*(c^2*x^2-1)^(1/2)/x^2/(c^2*x^2)^(1/2)
```

3.85.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x^6}\,dx$$
$$= \frac{-15a(3d^2+10dex^2+15e^2x^4)+bc\sqrt{1-\frac{1}{c^2x^2}}x(225e^2x^4+50dex^2(1+2c^2x^2)+3d^2(3+4c^2x^2+8c^4x^4))-}{225x^5}$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x^6,x]
```

output
```
(-15*a*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(22
5*e^2*x^4 + 50*d*e*x^2*(1 + 2*c^2*x^2) + 3*d^2*(3 + 4*c^2*x^2 + 8*c^4*x^4)
) - 15*b*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4)*ArcSec[c*x])/(225*x^5)
```

3.85.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5761, 27, 1588, 359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x^6}\,dx$$
$$\downarrow 5761$$
$$-\frac{bcx\int -\frac{15e^2x^4+10dex^2+3d^2}{15x^6\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}} - \frac{d^2\left(a+b\sec^{-1}(cx)\right)}{5x^5} - \frac{2de\left(a+b\sec^{-1}(cx)\right)}{3x^3} - \frac{e^2\left(a+b\sec^{-1}(cx)\right)}{x}$$
$$\downarrow 27$$
$$\frac{bcx\int \frac{15e^2x^4+10dex^2+3d^2}{x^6\sqrt{c^2x^2-1}}dx}{15\sqrt{c^2x^2}} - \frac{d^2\left(a+b\sec^{-1}(cx)\right)}{5x^5} - \frac{2de\left(a+b\sec^{-1}(cx)\right)}{3x^3} - \frac{e^2\left(a+b\sec^{-1}(cx)\right)}{x}$$
$$\downarrow 1588$$
$$\frac{bcx\left(\frac{1}{5}\int \frac{75e^2x^2+2d(6dc^2+25e)}{x^4\sqrt{c^2x^2-1}}dx + \frac{3d^2\sqrt{c^2x^2-1}}{5x^5}\right)}{15\sqrt{c^2x^2}} - \frac{d^2\left(a+b\sec^{-1}(cx)\right)}{5x^5} - \frac{2de\left(a+b\sec^{-1}(cx)\right)}{3x^3} - \frac{e^2\left(a+b\sec^{-1}(cx)\right)}{x}$$

$$\downarrow 359$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(24c^4d^2+100c^2de+225e^2\right)\int\frac{1}{x^2\sqrt{c^2x^2-1}}dx+\frac{2d\sqrt{c^2x^2-1}\left(6c^2d+25e\right)}{3x^3}\right)+\frac{3d^2\sqrt{c^2x^2-1}}{5x^5}\right)}{15\sqrt{c^2x^2}}-$$
$$\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\sec^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\sec^{-1}(cx)\right)}{x}$$

$$\downarrow 242$$

$$-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\sec^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\sec^{-1}(cx)\right)}{x}+$$
$$\frac{bcx\left(\frac{3d^2\sqrt{c^2x^2-1}}{5x^5}+\frac{1}{5}\left(\frac{2d\sqrt{c^2x^2-1}\left(6c^2d+25e\right)}{3x^3}+\frac{\sqrt{c^2x^2-1}\left(24c^4d^2+100c^2de+225e^2\right)}{3x}\right)\right)}{15\sqrt{c^2x^2}}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x^6,x]
```

output
```
(b*c*x*((3*d^2*Sqrt[-1 + c^2*x^2])/(5*x^5) + ((2*d*(6*c^2*d + 25*e)*Sqrt[-
1 + c^2*x^2])/(3*x^3) + ((24*c^4*d^2 + 100*c^2*d*e + 225*e^2)*Sqrt[-1 + c^
2*x^2])/(3*x))/5))/(15*Sqrt[c^2*x^2]) - (d^2*(a + b*ArcSec[c*x]))/(5*x^5)
- (2*d*e*(a + b*ArcSec[c*x]))/(3*x^3) - (e^2*(a + b*ArcSec[c*x]))/x
```

3.85.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.85.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.96

method	result
parts	$a\left(-\frac{e^2}{x}-\frac{d^2}{5x^5}-\frac{2de}{3x^3}\right)+b\,c^5\left(-\frac{\operatorname{arcsec}(cx)e^2}{c^5x}-\frac{\operatorname{arcsec}(cx)d^2}{5x^5c^5}-\frac{2\operatorname{arcsec}(cx)de}{3c^5x^3}+\frac{(c^2x^2-1)(24c^8d^2x^4+1}{}\right.$
derivativedivides	$c^5\left(\frac{a\left(-\frac{e^2}{cx}-\frac{d^2}{5c\,x^5}-\frac{2de}{3c\,x^3}\right)}{c^4}+\frac{b\left(-\frac{\operatorname{arcsec}(cx)e^2}{cx}-\frac{\operatorname{arcsec}(cx)d^2}{5c\,x^5}-\frac{2\operatorname{arcsec}(cx)de}{3c\,x^3}+\frac{(c^2x^2-1)(24c^8d^2x^4+100c^6de\,x^4+12c^6d^2}{225\sqrt{\frac{c^2x^2-}{c^2x^2}}}\right.}{c^4}\right.$
default	$c^5\left(\frac{a\left(-\frac{e^2}{cx}-\frac{d^2}{5c\,x^5}-\frac{2de}{3c\,x^3}\right)}{c^4}+\frac{b\left(-\frac{\operatorname{arcsec}(cx)e^2}{cx}-\frac{\operatorname{arcsec}(cx)d^2}{5c\,x^5}-\frac{2\operatorname{arcsec}(cx)de}{3c\,x^3}+\frac{(c^2x^2-1)(24c^8d^2x^4+100c^6de\,x^4+12c^6d^2}{225\sqrt{\frac{c^2x^2-}{c^2x^2}}}\right.}{c^4}\right.$

input
```
int((e*x^2+d)^2*(a+b*arcsec(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
a*(-e^2/x-1/5*d^2/x^5-2/3*d*e/x^3)+b*c^5*(-1/c^5*arcsec(c*x)*e^2/x-1/5*arc
sec(c*x)*d^2/x^5/c^5-2/3/c^5*arcsec(c*x)*d*e/x^3+1/225/c^10*(c^2*x^2-1)*(2
4*c^8*d^2*x^4+100*c^6*d*e*x^4+12*c^6*d^2*x^2+225*c^4*e^2*x^4+50*c^4*d*e*x^
2+9*c^4*d^2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^6)
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.69

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)}{x^6}\,dx =$$
$$-\frac{225\,ae^2x^4+150\,adex^2+45\,ad^2+15\left(15\,be^2x^4+10\,bdex^2+3\,bd^2\right)\operatorname{arcsec}(cx)-\left(\left(24\,bc^4d^2+100\,bc^2de\right.\right.}{225\,x^5}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^6,x, algorithm="fricas")
```

output
```
-1/225*(225*a*e^2*x^4 + 150*a*d*e*x^2 + 45*a*d^2 + 15*(15*b*e^2*x^4 + 10*b
*d*e*x^2 + 3*b*d^2)*arcsec(c*x) - ((24*b*c^4*d^2 + 100*b*c^2*d*e + 225*b*e
^2)*x^4 + 9*b*d^2 + 2*(6*b*c^2*d^2 + 25*b*d*e)*x^2)*sqrt(c^2*x^2 - 1))/x^5
```

3.85.6 Sympy [A] (verification not implemented)

Time = 5.19 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.82

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)}{x^6}\,dx$$
$$= -\frac{ad^2}{5x^5}-\frac{2ade}{3x^3}-\frac{ae^2}{x}+bce^2\sqrt{1-\frac{1}{c^2x^2}}-\frac{bd^2\operatorname{asec}(cx)}{5x^5}-\frac{2bde\operatorname{asec}(cx)}{3x^3}$$
$$-\frac{be^2\operatorname{asec}(cx)}{x}+\frac{bd^2\left(\begin{cases}\frac{8c^5\sqrt{c^2x^2-1}}{15x}+\frac{4c^3\sqrt{c^2x^2-1}}{15x^3}+\frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for } |c^2x^2|>1\\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x}+\frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3}+\frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise}\end{cases}\right)}{5c}$$
$$+\frac{2bde\left(\begin{cases}\frac{2c^3\sqrt{c^2x^2-1}}{3x}+\frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2|>1\\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x}+\frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)}{3c}$$

input
```
integrate((e*x**2+d)**2*(a+b*asec(c*x))/x**6,x)
```

output
```
-a*d**2/(5*x**5) - 2*a*d*e/(3*x**3) - a*e**2/x + b*c*e**2*sqrt(1 - 1/(c**2
*x**2)) - b*d**2*asec(c*x)/(5*x**5) - 2*b*d*e*asec(c*x)/(3*x**3) - b*e**2*
asec(c*x)/x + b*d**2*Piecewise((8*c**5*sqrt(c**2*x**2 - 1)/(15*x) + 4*c**3
*sqrt(c**2*x**2 - 1)/(15*x**3) + c*sqrt(c**2*x**2 - 1)/(5*x**5), Abs(c**2*
x**2) > 1), (8*I*c**5*sqrt(-c**2*x**2 + 1)/(15*x) + 4*I*c**3*sqrt(-c**2*x*
*2 + 1)/(15*x**3) + I*c*sqrt(-c**2*x**2 + 1)/(5*x**5), True))/(5*c) + 2*b*
d*e*Piecewise((2*c**3*sqrt(c**2*x**2 - 1)/(3*x) + c*sqrt(c**2*x**2 - 1)/(3
*x**3), Abs(c**2*x**2) > 1), (2*I*c**3*sqrt(-c**2*x**2 + 1)/(3*x) + I*c*sq
rt(-c**2*x**2 + 1)/(3*x**3), True))/(3*c)
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.99

$$\begin{aligned}
&\int \frac{\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)}{x^6}\,dx \\
&= \left(c\sqrt{-\frac{1}{c^2x^2}+1}-\frac{\operatorname{arcsec}(cx)}{x}\right)be^2 \\
&\quad+\frac{1}{75}bd^2\left(\frac{3\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{15\,\operatorname{arcsec}(cx)}{x^5}\right) \\
&\quad-\frac{2}{9}bde\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{3\,\operatorname{arcsec}(cx)}{x^3}\right)-\frac{ae^2}{x}-\frac{2\,ade}{3\,x^3}-\frac{ad^2}{5\,x^5}
\end{aligned}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^6,x, algorithm="maxima")
```

output
```
(c*sqrt(-1/(c^2*x^2) + 1) - arcsec(c*x)/x)*b*e^2 + 1/75*b*d^2*((3*c^6*(-1/
(c^2*x^2) + 1)^(5/2) - 10*c^6*(-1/(c^2*x^2) + 1)^(3/2) + 15*c^6*sqrt(-1/(c
^2*x^2) + 1))/c - 15*arcsec(c*x)/x^5) - 2/9*b*d*e*((c^4*(-1/(c^2*x^2) + 1)
^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c + 3*arcsec(c*x)/x^3) - a*e^2/x -
2/3*a*d*e/x^3 - 1/5*a*d^2/x^5
```

3.85.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x^6}\,dx$$

$$= \frac{1}{225}\left(24\,bc^4d^2\sqrt{-\frac{1}{c^2x^2}+1}+100\,bc^2de\sqrt{-\frac{1}{c^2x^2}+1}+225\,be^2\sqrt{-\frac{1}{c^2x^2}+1}+\frac{12\,bc^2d^2\sqrt{-\frac{1}{c^2x^2}+1}}{x^2}-\frac{22}{}\right.$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^6,x, algorithm="giac")
```

output
```
1/225*(24*b*c^4*d^2*sqrt(-1/(c^2*x^2) + 1) + 100*b*c^2*d*e*sqrt(-1/(c^2*x^
2) + 1) + 225*b*e^2*sqrt(-1/(c^2*x^2) + 1) + 12*b*c^2*d^2*sqrt(-1/(c^2*x^2
) + 1)/x^2 - 225*b*e^2*arccos(1/(c*x))/(c*x) + 50*b*d*e*sqrt(-1/(c^2*x^2)
+ 1)/x^2 - 225*a*e^2/(c*x) - 150*b*d*e*arccos(1/(c*x))/(c*x^3) + 9*b*d^2*s
qrt(-1/(c^2*x^2) + 1)/x^4 - 150*a*d*e/(c*x^3) - 45*b*d^2*arccos(1/(c*x))/(
c*x^5) - 45*a*d^2/(c*x^5))*c
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x^6}\,dx = \int \frac{(e\,x^2+d)^2\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

input
```
int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^6,x)
```

output
```
int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^6, x)
```

3.86 $\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^8}\,dx$

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3.86.1 Optimal result

Integrand size = 21, antiderivative size = 241

$$\begin{aligned}\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^8}\,dx = {}& \frac{2bc^3(360c^4d^2+1176c^2de+1225e^2)\sqrt{-1+c^2x^2}}{11025\sqrt{c^2x^2}} \\ &+ \frac{bcd^2\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} + \frac{2bcd(15c^2d+49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}} \\ &+ \frac{bc(360c^4d^2+1176c^2de+1225e^2)\sqrt{-1+c^2x^2}}{11025x^2\sqrt{c^2x^2}} \\ &- \frac{d^2(a+b\sec^{-1}(cx))}{7x^7} \\ &- \frac{2de(a+b\sec^{-1}(cx))}{5x^5} - \frac{e^2(a+b\sec^{-1}(cx))}{3x^3}\end{aligned}$$

output

```
-1/7*d^2*(a+b*arcsec(c*x))/x^7-2/5*d*e*(a+b*arcsec(c*x))/x^5-1/3*e^2*(a+b*
arcsec(c*x))/x^3+2/11025*b*c^3*(360*c^4*d^2+1176*c^2*d*e+1225*e^2)*(c^2*x^
2-1)^(1/2)/(c^2*x^2)^(1/2)+1/49*b*c*d^2*(c^2*x^2-1)^(1/2)/x^6/(c^2*x^2)^(1
/2)+2/1225*b*c*d*(15*c^2*d+49*e)*(c^2*x^2-1)^(1/2)/x^4/(c^2*x^2)^(1/2)+1/1
1025*b*c*(360*c^4*d^2+1176*c^2*d*e+1225*e^2)*(c^2*x^2-1)^(1/2)/x^2/(c^2*x^
2)^(1/2)
```

3.86.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.63

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x^8}\,dx$$
$$= \frac{-105a(15d^2+42dex^2+35e^2x^4)+bc\sqrt{1-\frac{1}{c^2x^2}}x(1225e^2x^4(1+2c^2x^2)+294dex^2(3+4c^2x^2+8c^4x^4)+4}{11025x^7}$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x^8,x]
```

output
```
(-105*a*(15*d^2 + 42*d*e*x^2 + 35*e^2*x^4) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(
1225*e^2*x^4*(1 + 2*c^2*x^2) + 294*d*e*x^2*(3 + 4*c^2*x^2 + 8*c^4*x^4) + 4
5*d^2*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 16*c^6*x^6)) - 105*b*(15*d^2 + 42*d*e*x
^2 + 35*e^2*x^4)*ArcSec[c*x])/(11025*x^7)
```

3.86.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5761, 27, 1588, 359, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x^8}\,dx$$
$$\downarrow 5761$$
$$-\frac{bcx\int -\frac{35e^2x^4+42dex^2+15d^2}{105x^8\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{7x^7} - \frac{2de(a+b\sec^{-1}(cx))}{5x^5} - \frac{e^2(a+b\sec^{-1}(cx))}{3x^3}$$
$$\downarrow 27$$
$$\frac{bcx\int \frac{35e^2x^4+42dex^2+15d^2}{x^8\sqrt{c^2x^2-1}}dx}{105\sqrt{c^2x^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{7x^7} - \frac{2de(a+b\sec^{-1}(cx))}{5x^5} - \frac{e^2(a+b\sec^{-1}(cx))}{3x^3}$$
$$\downarrow 1588$$

$$\frac{bcx\left(\frac{1}{7}\int\frac{245e^2x^2+6d\left(15dc^2+49e\right)}{x^6\sqrt{c^2x^2-1}}dx+\frac{15d^2\sqrt{c^2x^2-1}}{7x^7}\right)}{105\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\sec^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\sec^{-1}(cx)\right)}{3x^3}$$

$$\downarrow 359$$

$$\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(360c^4d^2+1176c^2de+1225e^2\right)\int\frac{1}{x^4\sqrt{c^2x^2-1}}dx+\frac{6d\sqrt{c^2x^2-1}\left(15c^2d+49e\right)}{5x^5}\right)+\frac{15d^2\sqrt{c^2x^2-1}}{7x^7}\right)}{105\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\sec^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\sec^{-1}(cx)\right)}{3x^3}$$

$$\downarrow 245$$

$$\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(360c^4d^2+1176c^2de+1225e^2\right)\left(\frac{2}{3}c^2\int\frac{1}{x^2\sqrt{c^2x^2-1}}dx+\frac{\sqrt{c^2x^2-1}}{3x^3}\right)+\frac{6d\sqrt{c^2x^2-1}\left(15c^2d+49e\right)}{5x^5}\right)+\frac{15d^2\sqrt{c^2x^2-1}}{7x^7}\right)}{105\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\sec^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\sec^{-1}(cx)\right)}{3x^3}$$

$$\downarrow 242$$

$$-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\sec^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\sec^{-1}(cx)\right)}{3x^3}+\frac{bcx\left(\frac{15d^2\sqrt{c^2x^2-1}}{7x^7}+\frac{1}{7}\left(\frac{6d\sqrt{c^2x^2-1}\left(15c^2d+49e\right)}{5x^5}+\frac{1}{5}\left(\frac{2c^2\sqrt{c^2x^2-1}}{3x}+\frac{\sqrt{c^2x^2-1}}{3x^3}\right)\left(360c^4d^2+1176c^2de+1225e^2\right)\right)\right)}{105\sqrt{c^2x^2}}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x^8,x]
```

output
```
(b*c*x*((15*d^2*Sqrt[-1 + c^2*x^2])/(7*x^7) + ((6*d*(15*c^2*d + 49*e)*Sqrt
[-1 + c^2*x^2])/(5*x^5) + ((360*c^4*d^2 + 1176*c^2*d*e + 1225*e^2)*(Sqrt[-
1 + c^2*x^2]/(3*x^3) + (2*c^2*Sqrt[-1 + c^2*x^2])/(3*x)))/5)/7))/(105*Sqrt
[c^2*x^2]) - (d^2*(a + b*ArcSec[c*x]))/(7*x^7) - (2*d*e*(a + b*ArcSec[c*x]
))/(5*x^5) - (e^2*(a + b*ArcSec[c*x]))/(3*x^3)
```

3.86.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.86.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.86

method	result
parts	$a\left(-\frac{d^2}{7x^7}-\frac{2de}{5x^5}-\frac{e^2}{3x^3}\right)+b\,c^7\left(-\frac{\operatorname{arcsec}(cx)d^2}{7x^7c^7}-\frac{2\operatorname{arcsec}(cx)de}{5c^7x^5}-\frac{\operatorname{arcsec}(cx)e^2}{3c^7x^3}+\frac{(c^2x^2-1)(720c^{10}d^2x^6$
derivativedivides	$c^7\left(\frac{a\left(-\frac{e^2}{3c^3x^3}-\frac{d^2}{7c^3x^7}-\frac{2de}{5c^3x^5}\right)}{c^4}+\frac{b\left(-\frac{\operatorname{arcsec}(cx)e^2}{3c^3x^3}-\frac{\operatorname{arcsec}(cx)d^2}{7c^3x^7}-\frac{2\operatorname{arcsec}(cx)de}{5c^3x^5}+\frac{\left(c^2x^2-1\right)\left(720c^{10}d^2x^6+2352c^8dex^6\right.}{}\right.}{}\right.$
default	$c^7\left(\frac{a\left(-\frac{e^2}{3c^3x^3}-\frac{d^2}{7c^3x^7}-\frac{2de}{5c^3x^5}\right)}{c^4}+\frac{b\left(-\frac{\operatorname{arcsec}(cx)e^2}{3c^3x^3}-\frac{\operatorname{arcsec}(cx)d^2}{7c^3x^7}-\frac{2\operatorname{arcsec}(cx)de}{5c^3x^5}+\frac{\left(c^2x^2-1\right)\left(720c^{10}d^2x^6+2352c^8dex^6\right.}{}\right.}{}\right.$

input
```
int((e*x^2+d)^2*(a+b*arcsec(c*x))/x^8,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/7*d^2/x^7-2/5*d*e/x^5-1/3*e^2/x^3)+b*c^7*(-1/7*arcsec(c*x)*d^2/x^7/c
^7-2/5/c^7*arcsec(c*x)*d*e/x^5-1/3/c^7*arcsec(c*x)*e^2/x^3+1/11025/c^12*(c
^2*x^2-1)*(720*c^10*d^2*x^6+2352*c^8*d*e*x^6+360*c^8*d^2*x^4+2450*c^6*e^2*
x^6+1176*c^6*d*e*x^4+270*c^6*d^2*x^2+1225*c^4*e^2*x^4+882*c^4*d*e*x^2+225*
c^4*d^2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^8)
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.66

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)}{x^8}\,dx =$$
$$-\frac{3675\,ae^2x^4+4410\,adex^2+1575\,ad^2+105\left(35\,be^2x^4+42\,bdex^2+15\,bd^2\right)\operatorname{arcsec}(cx)-\left(2\left(360\,bc^6d^2+\right.\right.}{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^8,x, algorithm="fricas")
```

output
```
-1/11025*(3675*a*e^2*x^4 + 4410*a*d*e*x^2 + 1575*a*d^2 + 105*(35*b*e^2*x^4
 + 42*b*d*e*x^2 + 15*b*d^2)*arcsec(c*x) - (2*(360*b*c^6*d^2 + 1176*b*c^4*d
*e + 1225*b*c^2*e^2)*x^6 + (360*b*c^4*d^2 + 1176*b*c^2*d*e + 1225*b*e^2)*x
^4 + 225*b*d^2 + 18*(15*b*c^2*d^2 + 49*b*d*e)*x^2)*sqrt(c^2*x^2 - 1))/x^7
```

3.86.6 Sympy [A] (verification not implemented)

Time = 30.11 (sec) , antiderivative size = 508, normalized size of antiderivative = 2.11

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)}{x^8}\,dx$$

$$= -\frac{ad^2}{7x^7} - \frac{2ade}{5x^5} - \frac{ae^2}{3x^3} - \frac{bd^2\,\text{asec}\,(cx)}{7x^7} - \frac{2bde\,\text{asec}\,(cx)}{5x^5} - \frac{be^2\,\text{asec}\,(cx)}{3x^3}$$
$$+ \frac{bd^2\left(\begin{cases} \frac{16c^7\sqrt{c^2x^2-1}}{35x} + \frac{8c^5\sqrt{c^2x^2-1}}{35x^3} + \frac{6c^3\sqrt{c^2x^2-1}}{35x^5} + \frac{c\sqrt{c^2x^2-1}}{7x^7} & \text{for } |c^2x^2| > 1 \\ \frac{16ic^7\sqrt{-c^2x^2+1}}{35x} + \frac{8ic^5\sqrt{-c^2x^2+1}}{35x^3} + \frac{6ic^3\sqrt{-c^2x^2+1}}{35x^5} + \frac{ic\sqrt{-c^2x^2+1}}{7x^7} & \text{otherwise} \end{cases}\right)}{7c}$$
$$+ \frac{2bde\left(\begin{cases} \frac{8c^5\sqrt{c^2x^2-1}}{15x} + \frac{4c^3\sqrt{c^2x^2-1}}{15x^3} + \frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for } |c^2x^2| > 1 \\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x} + \frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3} + \frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise} \end{cases}\right)}{5c}$$
$$+ \frac{be^2\left(\begin{cases} \frac{2c^3\sqrt{c^2x^2-1}}{3x} + \frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2| > 1 \\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x} + \frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise} \end{cases}\right)}{3c}$$

input
```
integrate((e*x**2+d)**2*(a+b*asec(c*x))/x**8,x)
```

output
```
-a*d**2/(7*x**7) - 2*a*d*e/(5*x**5) - a*e**2/(3*x**3) - b*d**2*asec(c*x)/(
7*x**7) - 2*b*d*e*asec(c*x)/(5*x**5) - b*e**2*asec(c*x)/(3*x**3) + b*d**2*
Piecewise((16*c**7*sqrt(c**2*x**2 - 1)/(35*x) + 8*c**5*sqrt(c**2*x**2 - 1)
/(35*x**3) + 6*c**3*sqrt(c**2*x**2 - 1)/(35*x**5) + c*sqrt(c**2*x**2 - 1)/
(7*x**7), Abs(c**2*x**2) > 1), (16*I*c**7*sqrt(-c**2*x**2 + 1)/(35*x) + 8*
I*c**5*sqrt(-c**2*x**2 + 1)/(35*x**3) + 6*I*c**3*sqrt(-c**2*x**2 + 1)/(35*
x**5) + I*c*sqrt(-c**2*x**2 + 1)/(7*x**7), True))/(7*c) + 2*b*d*e*Piecewis
e((8*c**5*sqrt(c**2*x**2 - 1)/(15*x) + 4*c**3*sqrt(c**2*x**2 - 1)/(15*x**3
) + c*sqrt(c**2*x**2 - 1)/(5*x**5), Abs(c**2*x**2) > 1), (8*I*c**5*sqrt(-c
**2*x**2 + 1)/(15*x) + 4*I*c**3*sqrt(-c**2*x**2 + 1)/(15*x**3) + I*c*sqrt(
-c**2*x**2 + 1)/(5*x**5), True))/(5*c) + b*e**2*Piecewise((2*c**3*sqrt(c**
2*x**2 - 1)/(3*x) + c*sqrt(c**2*x**2 - 1)/(3*x**3), Abs(c**2*x**2) > 1), (
2*I*c**3*sqrt(-c**2*x**2 + 1)/(3*x) + I*c*sqrt(-c**2*x**2 + 1)/(3*x**3), T
rue))/(3*c)
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x^8}\,dx =$$
$$-\frac{1}{245}\,bd^2\left(\frac{5\,c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}}-21\,c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+35\,c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-35\,c^8\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{35\ \text{arcsec}\,(}{x^7}\right.$$
$$+\frac{2}{75}\,bde\left(\frac{3\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{15\ \text{arcsec}\,(cx)}{x^5}\right)$$
$$-\frac{1}{9}\,be^2\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{3\ \text{arcsec}\,(cx)}{x^3}\right)-\frac{ae^2}{3\,x^3}-\frac{2\,ade}{5\,x^5}-\frac{ad^2}{7\,x^7}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^8,x, algorithm="maxima")
```

output
```
-1/245*b*d^2*((5*c^8*(-1/(c^2*x^2) + 1)^(7/2) - 21*c^8*(-1/(c^2*x^2) + 1)^
(5/2) + 35*c^8*(-1/(c^2*x^2) + 1)^(3/2) - 35*c^8*sqrt(-1/(c^2*x^2) + 1))/c
 + 35*arcsec(c*x)/x^7) + 2/75*b*d*e*((3*c^6*(-1/(c^2*x^2) + 1)^(5/2) - 10*
c^6*(-1/(c^2*x^2) + 1)^(3/2) + 15*c^6*sqrt(-1/(c^2*x^2) + 1))/c - 15*arcse
c(c*x)/x^5) - 1/9*b*e^2*((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^
2*x^2) + 1))/c + 3*arcsec(c*x)/x^3) - 1/3*a*e^2/x^3 - 2/5*a*d*e/x^5 - 1/7*
a*d^2/x^7
```

3.86.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x^8}\,dx$$
$$=\frac{1}{11025}\left(720\,bc^6d^2\sqrt{-\frac{1}{c^2x^2}+1}+2352\,bc^4de\sqrt{-\frac{1}{c^2x^2}+1}+2450\,bc^2e^2\sqrt{-\frac{1}{c^2x^2}+1}+\frac{360\,bc^4d^2\sqrt{-\frac{1}{c^2x^2}}}{x^2}\right.$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^8,x, algorithm="giac")
```

output
```
1/11025*(720*b*c^6*d^2*sqrt(-1/(c^2*x^2) + 1) + 2352*b*c^4*d*e*sqrt(-1/(c^
2*x^2) + 1) + 2450*b*c^2*e^2*sqrt(-1/(c^2*x^2) + 1) + 360*b*c^4*d^2*sqrt(-
1/(c^2*x^2) + 1)/x^2 + 1176*b*c^2*d*e*sqrt(-1/(c^2*x^2) + 1)/x^2 + 270*b*c
^2*d^2*sqrt(-1/(c^2*x^2) + 1)/x^4 + 1225*b*e^2*sqrt(-1/(c^2*x^2) + 1)/x^2
- 3675*b*e^2*arccos(1/(c*x))/(c*x^3) + 882*b*d*e*sqrt(-1/(c^2*x^2) + 1)/x^
4 - 3675*a*e^2/(c*x^3) - 4410*b*d*e*arccos(1/(c*x))/(c*x^5) + 225*b*d^2*sq
rt(-1/(c^2*x^2) + 1)/x^6 - 4410*a*d*e/(c*x^5) - 1575*b*d^2*arccos(1/(c*x))
/(c*x^7) - 1575*a*d^2/(c*x^7))*c
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x^8}\,dx = \int \frac{(e\,x^2+d)^2\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

input
```
int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^8,x)
```

output
```
int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^8, x)
```

3.87 $\int x^3(d+ex^2)^2\left(a+b\sec^{-1}(cx)\right)\,dx$

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3.87.1 Optimal result

Integrand size = 21, antiderivative size = 242

$$\begin{aligned}
&\int x^3(d+ex^2)^2\left(a+b\sec^{-1}(cx)\right)\,dx \\
&= -\frac{b(6c^4d^2+8c^2de+3e^2)\,x\sqrt{-1+c^2x^2}}{24c^7\sqrt{c^2x^2}} - \frac{b(6c^4d^2+16c^2de+9e^2)\,x(-1+c^2x^2)^{3/2}}{72c^7\sqrt{c^2x^2}} \\
&\quad - \frac{be(8c^2d+9e)\,x(-1+c^2x^2)^{5/2}}{120c^7\sqrt{c^2x^2}} - \frac{be^2x(-1+c^2x^2)^{7/2}}{56c^7\sqrt{c^2x^2}} \\
&\quad + \frac{1}{4}d^2x^4\left(a+b\sec^{-1}(cx)\right) + \frac{1}{3}dex^6\left(a+b\sec^{-1}(cx)\right) + \frac{1}{8}e^2x^8\left(a+b\sec^{-1}(cx)\right)
\end{aligned}$$

output

```
1/4*d^2*x^4*(a+b*arcsec(c*x))+1/3*d*e*x^6*(a+b*arcsec(c*x))+1/8*e^2*x^8*(a
+b*arcsec(c*x))-1/72*b*(6*c^4*d^2+16*c^2*d*e+9*e^2)*x*(c^2*x^2-1)^(3/2)/c^
7/(c^2*x^2)^(1/2)-1/120*b*e*(8*c^2*d+9*e)*x*(c^2*x^2-1)^(5/2)/c^7/(c^2*x^2
)^(1/2)-1/56*b*e^2*x*(c^2*x^2-1)^(7/2)/c^7/(c^2*x^2)^(1/2)-1/24*b*(6*c^4*d
^2+8*c^2*d*e+3*e^2)*x*(c^2*x^2-1)^(1/2)/c^7/(c^2*x^2)^(1/2)
```

3.87.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.67

$$\int x^3\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{1}{24}ax^4\left(6d^2+8dex^2+3e^2x^4\right)$$
$$-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x\left(144e^2+8c^2e(56d+9ex^2)+c^4(420d^2+224dex^2+54e^2x^4)+3c^6(70d^2x^2+56dex^4+15e^{\cdots}\right.}{2520c^7}$$
$$+\frac{1}{24}bx^4\left(6d^2+8dex^2+3e^2x^4\right)\sec^{-1}(cx)$$

input
```
Integrate[x^3*(d + e*x^2)^2*(a + b*ArcSec[c*x]),x]
```

output
```
(a*x^4*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4))/24 - (b*Sqrt[1 - 1/(c^2*x^2)]*x*(1
44*e^2 + 8*c^2*e*(56*d + 9*e*x^2) + c^4*(420*d^2 + 224*d*e*x^2 + 54*e^2*x^
4) + 3*c^6*(70*d^2*x^2 + 56*d*e*x^4 + 15*e^2*x^6)))/(2520*c^7) + (b*x^4*(6
*d^2 + 8*d*e*x^2 + 3*e^2*x^4)*ArcSec[c*x])/24
```

3.87.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5761, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$\downarrow 5761$$
$$-\frac{bcx\int\frac{x^3\left(3e^2x^4+8dex^2+6d^2\right)}{24\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{1}{4}d^2x^4\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\sec^{-1}(cx)\right)+$$
$$\frac{1}{8}e^2x^8\left(a+b\sec^{-1}(cx)\right)$$
$$\downarrow 27$$
$$-\frac{bcx\int\frac{x^3\left(3e^2x^4+8dex^2+6d^2\right)}{\sqrt{c^2x^2-1}}dx}{24\sqrt{c^2x^2}}+\frac{1}{4}d^2x^4\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\sec^{-1}(cx)\right)+$$
$$\frac{1}{8}e^2x^8\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 1578$$

$$-\frac{bcx\int\frac{x^2\left(3e^2x^4+8dex^2+6d^2\right)}{\sqrt{c^2x^2-1}}dx^2}{48\sqrt{c^2x^2}}+\frac{1}{4}d^2x^4\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\sec^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 1195$$

$$-\frac{bcx\int\left(\frac{3e^2\left(c^2x^2-1\right)^{5/2}}{c^6}+\frac{e\left(8dc^2+9e\right)\left(c^2x^2-1\right)^{3/2}}{c^6}+\frac{\left(6d^2c^4+16dec^2+9e^2\right)\sqrt{c^2x^2-1}}{c^6}+\frac{6d^2c^4+8dec^2+3e^2}{c^6\sqrt{c^2x^2-1}}\right)dx^2}{48\sqrt{c^2x^2}}+\frac{1}{4}d^2x^4\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\sec^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\sec^{-1}(cx)\right)$$

$$\downarrow 2009$$

$$\frac{1}{4}d^2x^4\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\sec^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\sec^{-1}(cx)\right)-\frac{bcx\left(\frac{2e\left(c^2x^2-1\right)^{5/2}\left(8c^2d+9e\right)}{5c^8}+\frac{6e^2\left(c^2x^2-1\right)^{7/2}}{7c^8}+\frac{2\left(c^2x^2-1\right)^{3/2}\left(6c^4d^2+16c^2de+9e^2\right)}{3c^8}+\frac{2\sqrt{c^2x^2-1}\left(6c^4d^2+8c^2de+3e^2\right)}{c^8}\right)}{48\sqrt{c^2x^2}}$$

input
```
Int[x^3*(d + e*x^2)^2*(a + b*ArcSec[c*x]),x]
```

output
```
-1/48*(b*c*x*((2*(6*c^4*d^2 + 8*c^2*d*e + 3*e^2)*Sqrt[-1 + c^2*x^2])/c^8 +
 (2*(6*c^4*d^2 + 16*c^2*d*e + 9*e^2)*(-1 + c^2*x^2)^(3/2))/(3*c^8) + (2*e*
(8*c^2*d + 9*e)*(-1 + c^2*x^2)^(5/2))/(5*c^8) + (6*e^2*(-1 + c^2*x^2)^(7/2
))/(7*c^8)))/Sqrt[c^2*x^2] + (d^2*x^4*(a + b*ArcSec[c*x]))/4 + (d*e*x^6*(a
 + b*ArcSec[c*x]))/3 + (e^2*x^8*(a + b*ArcSec[c*x]))/8
```

3.87.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.87.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.82

method	result
parts	$a\left(\frac{1}{8}e^2x^8+\frac{1}{3}de\,x^6+\frac{1}{4}x^4d^2\right)+\frac{b\left(\frac{c^4\operatorname{arcsec}(cx)e^2x^8}{8}+\frac{c^4\operatorname{arcsec}(cx)de\,x^6}{3}+\frac{\operatorname{arcsec}(cx)d^2x^4c^4}{4}-\frac{\left(c^2x^2-1\right)\left(45c^6e^2x^6\right.}{}\right.}{}$
derivativedivides	$\frac{-\frac{a\left(\frac{c^2d\left(c^2ex^2+c^2d\right)^3}{3}-\frac{\left(c^2ex^2+c^2d\right)^4}{4}\right)}{2c^4e^2}-\frac{bc^4\operatorname{arcsec}(cx)d^4}{24e^2}+\frac{b\operatorname{arcsec}(cx)d^2c^4x^4}{4}+\frac{bc^4e\operatorname{arcsec}(cx)dx^6}{3}+\frac{bc^4e^2\operatorname{arcsec}(cx)x^8}{8}-}{}$
default	$\frac{-\frac{a\left(\frac{c^2d\left(c^2ex^2+c^2d\right)^3}{3}-\frac{\left(c^2ex^2+c^2d\right)^4}{4}\right)}{2c^4e^2}-\frac{bc^4\operatorname{arcsec}(cx)d^4}{24e^2}+\frac{b\operatorname{arcsec}(cx)d^2c^4x^4}{4}+\frac{bc^4e\operatorname{arcsec}(cx)dx^6}{3}+\frac{bc^4e^2\operatorname{arcsec}(cx)x^8}{8}-}{}$

input
```
int(x^3*(e*x^2+d)^2*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/8*e^2*x^8+1/3*d*e*x^6+1/4*x^4*d^2)+b/c^4*(1/8*c^4*arcsec(c*x)*e^2*x^8
+1/3*c^4*arcsec(c*x)*d*e*x^6+1/4*arcsec(c*x)*d^2*x^4*c^4-1/2520/c^5*(c^2*x
^2-1)*(45*c^6*e^2*x^6+168*c^6*d*e*x^4+210*c^6*d^2*x^2+54*c^4*e^2*x^4+224*c
^4*d*e*x^2+420*c^4*d^2+72*c^2*e^2*x^2+448*c^2*d*e+144*e^2)/((c^2*x^2-1)/c^
2/x^2)^(1/2)/x)
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.77

$$\int x^3 \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx$$
$$= \frac{315\,ac^8e^2x^8+840\,ac^8dex^6+630\,ac^8d^2x^4+105\left(3\,bc^8e^2x^8+8\,bc^8dex^6+6\,bc^8d^2x^4\right)\operatorname{arcsec}(cx)-(45\,bc^6e^2$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/2520*(315*a*c^8*e^2*x^8 + 840*a*c^8*d*e*x^6 + 630*a*c^8*d^2*x^4 + 105*(3
*b*c^8*e^2*x^8 + 8*b*c^8*d*e*x^6 + 6*b*c^8*d^2*x^4)*arcsec(c*x) - (45*b*c^
6*e^2*x^6 + 420*b*c^4*d^2 + 448*b*c^2*d*e + 6*(28*b*c^6*d*e + 9*b*c^4*e^2)
*x^4 + 144*b*e^2 + 2*(105*b*c^6*d^2 + 112*b*c^4*d*e + 36*b*c^2*e^2)*x^2)*s
qrt(c^2*x^2 - 1))/c^8
```

3.87.6 Sympy [A] (verification not implemented)

Time = 4.38 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.04

$$\int x^3\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$= \frac{ad^2x^4}{4} + \frac{adex^6}{3} + \frac{ae^2x^8}{8} + \frac{bd^2x^4\,\mathrm{asec}\,(cx)}{4} + \frac{bdex^6\,\mathrm{asec}\,(cx)}{3}$$
$$+ \frac{be^2x^8\,\mathrm{asec}\,(cx)}{8} - \frac{bd^2\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{4c}$$
$$- \frac{bde\left(\begin{cases}\frac{x^4\sqrt{c^2x^2-1}}{5c}+\frac{4x^2\sqrt{c^2x^2-1}}{15c^3}+\frac{8\sqrt{c^2x^2-1}}{15c^5} & \text{for } |c^2x^2|>1\\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c}+\frac{4ix^2\sqrt{-c^2x^2+1}}{15c^3}+\frac{8i\sqrt{-c^2x^2+1}}{15c^5} & \text{otherwise}\end{cases}\right)}{3c}$$
$$- \frac{be^2\left(\begin{cases}\frac{x^6\sqrt{c^2x^2-1}}{7c}+\frac{6x^4\sqrt{c^2x^2-1}}{35c^3}+\frac{8x^2\sqrt{c^2x^2-1}}{35c^5}+\frac{16\sqrt{c^2x^2-1}}{35c^7} & \text{for } |c^2x^2|>1\\ \frac{ix^6\sqrt{-c^2x^2+1}}{7c}+\frac{6ix^4\sqrt{-c^2x^2+1}}{35c^3}+\frac{8ix^2\sqrt{-c^2x^2+1}}{35c^5}+\frac{16i\sqrt{-c^2x^2+1}}{35c^7} & \text{otherwise}\end{cases}\right)}{8c}$$

input
```
integrate(x**3*(e*x**2+d)**2*(a+b*asec(c*x)),x)
```

output
```
a*d**2*x**4/4 + a*d*e*x**6/3 + a*e**2*x**8/8 + b*d**2*x**4*asec(c*x)/4 + b
*d*e*x**6*asec(c*x)/3 + b*e**2*x**8*asec(c*x)/8 - b*d**2*Piecewise((x**2*s
qrt(c**2*x**2 - 1)/(3*c) + 2*sqrt(c**2*x**2 - 1)/(3*c**3), Abs(c**2*x**2)
> 1), (I*x**2*sqrt(-c**2*x**2 + 1)/(3*c) + 2*I*sqrt(-c**2*x**2 + 1)/(3*c**
3), True))/(4*c) - b*d*e*Piecewise((x**4*sqrt(c**2*x**2 - 1)/(5*c) + 4*x**
2*sqrt(c**2*x**2 - 1)/(15*c**3) + 8*sqrt(c**2*x**2 - 1)/(15*c**5), Abs(c**
2*x**2) > 1), (I*x**4*sqrt(-c**2*x**2 + 1)/(5*c) + 4*I*x**2*sqrt(-c**2*x**
2 + 1)/(15*c**3) + 8*I*sqrt(-c**2*x**2 + 1)/(15*c**5), True))/(3*c) - b*e*
*2*Piecewise((x**6*sqrt(c**2*x**2 - 1)/(7*c) + 6*x**4*sqrt(c**2*x**2 - 1)/
(35*c**3) + 8*x**2*sqrt(c**2*x**2 - 1)/(35*c**5) + 16*sqrt(c**2*x**2 - 1)/
(35*c**7), Abs(c**2*x**2) > 1), (I*x**6*sqrt(-c**2*x**2 + 1)/(7*c) + 6*I*x
**4*sqrt(-c**2*x**2 + 1)/(35*c**3) + 8*I*x**2*sqrt(-c**2*x**2 + 1)/(35*c**
5) + 16*I*sqrt(-c**2*x**2 + 1)/(35*c**7), True))/(8*c)
```

3.87.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.06

$$\int x^3\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{1}{8}ae^2x^8+\frac{1}{3}adex^6+\frac{1}{4}ad^2x^4$$
$$+\frac{1}{12}\left(3\,x^4\operatorname{arcsec}(cx)-\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)bd^2$$
$$+\frac{1}{45}\left(15\,x^6\operatorname{arcsec}(cx)-\frac{3\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+10\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^5}\right)bde$$
$$+\frac{1}{280}\left(35\,x^8\operatorname{arcsec}(cx)-\frac{5\,c^6x^7\left(-\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}}+21\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+35\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+35\,x\sqrt{-}\right.$$
$$\left.c^7\right.$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/8*a*e^2*x^8 + 1/3*a*d*e*x^6 + 1/4*a*d^2*x^4 + 1/12*(3*x^4*arcsec(c*x) -
(c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b*d^2
 + 1/45*(15*x^6*arcsec(c*x) - (3*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2) + 10*c^2
*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(-1/(c^2*x^2) + 1))/c^5)*b*d*e +
1/280*(35*x^8*arcsec(c*x) - (5*c^6*x^7*(-1/(c^2*x^2) + 1)^(7/2) + 21*c^4*x
^5*(-1/(c^2*x^2) + 1)^(5/2) + 35*c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 35*x*s
qrt(-1/(c^2*x^2) + 1))/c^7)*b*e^2
```

3.87.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17666 vs. 2(212) = 424.

Time = 0.50 (sec) , antiderivative size = 17666, normalized size of antiderivative = 73.00

$$\int x^3\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output

```
1/2520*(630*b*c^4*d^2*arccos(1/(c*x))/(c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c
*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x
^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 +
56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(
1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2
*x^2) - 1)^8/(1/(c*x) + 1)^16) + 630*a*c^4*d^2/(c^9 + 8*c^9*(1/(c^2*x^2) -
 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*
(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x)
+ 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2)
 - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^
9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16) - 1260*b*c^4*d^2*sqrt(-1/(c^2*x^2)
 + 1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2
) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70
*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(
c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2
*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*
(1/(c*x) + 1)) + 840*b*c^2*d*e*arccos(1/(c*x))/(c^9 + 8*c^9*(1/(c^2*x^2) -
 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*
(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x)
+ 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x...
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx = \int x^3 \left(e\,x^2+d\right)^2 \left(a+b\,\text{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input

```
int(x^3*(d + e*x^2)^2*(a + b*acos(1/(c*x))),x)
```

output

```
int(x^3*(d + e*x^2)^2*(a + b*acos(1/(c*x))), x)
```

3.88 $\int x(d+ex^2)^2\left(a+b\sec^{-1}(cx)\right)\,dx$

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3.88.1 Optimal result

Integrand size = 19, antiderivative size = 195

$$\begin{aligned}\int x(d+ex^2)^2\left(a+b\sec^{-1}(cx)\right)\,dx = &-\frac{b(3c^4d^2+3c^2de+e^2)\,x\sqrt{-1+c^2x^2}}{6c^5\sqrt{c^2x^2}}\\ &-\frac{be(3c^2d+2e)\,x(-1+c^2x^2)^{3/2}}{18c^5\sqrt{c^2x^2}}\\ &-\frac{be^2x(-1+c^2x^2)^{5/2}}{30c^5\sqrt{c^2x^2}}+\frac{(d+ex^2)^3\left(a+b\sec^{-1}(cx)\right)}{6e}\\ &-\frac{bcd^3x\arctan\left(\sqrt{-1+c^2x^2}\right)}{6e\sqrt{c^2x^2}}\end{aligned}$$

output

```
1/6*(e*x^2+d)^3*(a+b*arcsec(c*x))/e-1/18*b*e*(3*c^2*d+2*e)*x*(c^2*x^2-1)^(
3/2)/c^5/(c^2*x^2)^(1/2)-1/30*b*e^2*x*(c^2*x^2-1)^(5/2)/c^5/(c^2*x^2)^(1/2
)-1/6*b*c*d^3*x*arctan((c^2*x^2-1)^(1/2))/e/(c^2*x^2)^(1/2)-1/6*b*(3*c^4*d
^2+3*c^2*d*e+e^2)*x*(c^2*x^2-1)^(1/2)/c^5/(c^2*x^2)^(1/2)
```

3.88.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.64

$$\begin{aligned}&\int x\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx\\&=\frac{1}{90}x\left(15ax\left(3d^2+3dex^2+e^2x^4\right)\right.\\&\qquad-\frac{b\sqrt{1-\frac{1}{c^2x^2}}\left(8e^2+2c^2e(15d+2ex^2)+3c^4(15d^2+5dex^2+e^2x^4)\right)}{c^5}\\&\qquad\left.+15bx\left(3d^2+3dex^2+e^2x^4\right)\sec^{-1}(cx)\right)\end{aligned}$$

input
```
Integrate[x*(d + e*x^2)^2*(a + b*ArcSec[c*x]),x]
```

output
```
(x*(15*a*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4) - (b*Sqrt[1 - 1/(c^2*x^2)]*(8*e^2
 + 2*c^2*e*(15*d + 2*e*x^2) + 3*c^4*(15*d^2 + 5*d*e*x^2 + e^2*x^4)))/c^5 +
 15*b*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4)*ArcSec[c*x]))/90
```

3.88.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.82, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {5759, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\downarrow 5759$$

$$\frac{\left(d+ex^2\right)^3\left(a+b\sec^{-1}(cx)\right)}{6e}-\frac{bcx\int\frac{\left(ex^2+d\right)^3}{x\sqrt{c^2x^2-1}}dx}{6e\sqrt{c^2x^2}}$$

$$\downarrow 354$$

$$\frac{\left(d+ex^2\right)^3\left(a+b\sec^{-1}(cx)\right)}{6e}-\frac{bcx\int\frac{\left(ex^2+d\right)^3}{x^2\sqrt{c^2x^2-1}}dx^2}{12e\sqrt{c^2x^2}}$$

$$\downarrow 99$$

$$\frac{(d+ex^2)^3\left(a+b\sec^{-1}(cx)\right)}{6e} - \frac{bcx\int\left(\frac{d^3}{x^2\sqrt{c^2x^2-1}}+\frac{e^3(c^2x^2-1)^{3/2}}{c^4}+\frac{e^2(3dc^2+2e)\sqrt{c^2x^2-1}}{c^4}+\frac{e(3d^2c^4+3dec^2+e^2)}{c^4\sqrt{c^2x^2-1}}\right)dx^2}{12e\sqrt{c^2x^2}}$$

$$\downarrow 2009$$

$$\frac{(d+ex^2)^3\left(a+b\sec^{-1}(cx)\right)}{6e} - \frac{bcx\left(2d^3\arctan\left(\sqrt{c^2x^2-1}\right)+\frac{2e^2(c^2x^2-1)^{3/2}(3c^2d+2e)}{3c^6}+\frac{2e^3(c^2x^2-1)^{5/2}}{5c^6}+\frac{2e\sqrt{c^2x^2-1}(3c^4d^2+3c^2de+e^2)}{c^6}\right)}{12e\sqrt{c^2x^2}}$$

input
```
Int[x*(d + e*x^2)^2*(a + b*ArcSec[c*x]),x]
```

output
```
((d + e*x^2)^3*(a + b*ArcSec[c*x]))/(6*e) - (b*c*x*((2*e*(3*c^4*d^2 + 3*c^
2*d*e + e^2)*Sqrt[-1 + c^2*x^2])/c^6 + (2*e^2*(3*c^2*d + 2*e)*(-1 + c^2*x^
2)^(3/2))/(3*c^6) + (2*e^3*(-1 + c^2*x^2)^(5/2))/(5*c^6) + 2*d^3*ArcTan[Sq
rt[-1 + c^2*x^2]]))/(12*e*Sqrt[c^2*x^2])
```

3.88.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5759

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSec[c*x])/(2*e*(p + 1))), x
] - Simp[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2]))   Int[(d + e*x^2)^(p + 1)/(x*S
qrt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

3.88.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(169) = 338$.

Time = 0.82 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.81

method	result
parts	$\frac{a\left(e\,x^2+d\right)^3}{6e}+\frac{b\,\mathrm{arcsec}(cx)e^2x^6}{6}+\frac{b\,\mathrm{arcsec}(cx)de\,x^4}{2}+\frac{b\,\mathrm{arcsec}(cx)d^2x^2}{2}+\frac{b\,\mathrm{arcsec}(cx)d^3}{6e}-\frac{b(c^2x^2-1)x^3e^2}{30c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-$
derivativedivides	$\frac{a\left(c^2e\,x^2+c^2d\right)^3}{6c^4e}+\frac{b\,c^2\,\mathrm{arcsec}(cx)d^3}{6e}+\frac{b\,\mathrm{arcsec}(cx)d^2c^2x^2}{2}+\frac{b\,c^2e\,\mathrm{arcsec}(cx)d\,x^4}{2}+\frac{b\,c^2e^2\,\mathrm{arcsec}(cx)x^6}{6}+\frac{bc\sqrt{c^2x^2-1}\,d^3\arctan\left(\frac{}{\sqrt{c}}\right.}{6e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}$ / c^2
default	$\frac{a\left(c^2e\,x^2+c^2d\right)^3}{6c^4e}+\frac{b\,c^2\,\mathrm{arcsec}(cx)d^3}{6e}+\frac{b\,\mathrm{arcsec}(cx)d^2c^2x^2}{2}+\frac{b\,c^2e\,\mathrm{arcsec}(cx)d\,x^4}{2}+\frac{b\,c^2e^2\,\mathrm{arcsec}(cx)x^6}{6}+\frac{bc\sqrt{c^2x^2-1}\,d^3\arctan\left(\frac{}{\sqrt{c}}\right.}{6e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}$ / c^2

input

```
int(x*(e*x^2+d)^2*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

output

```
1/6*a*(e*x^2+d)^3/e+1/6*b*arcsec(c*x)*e^2*x^6+1/2*b*arcsec(c*x)*d*e*x^4+1/
2*b*arcsec(c*x)*d^2*x^2+1/6*b/e*arcsec(c*x)*d^3-1/30*b/c^3*(c^2*x^2-1)/((c
^2*x^2-1)/c^2/x^2)^(1/2)*x^3*e^2-1/6*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^
2)^(1/2)*x*d*e+1/6*b/c/e*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d
^3*arctan(1/(c^2*x^2-1)^(1/2))-2/45*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2
)^(1/2)*x*e^2-1/2*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^2-1/3*
b/c^5*e*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d-4/45*b/c^7*e^2*(c^2*x^
2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.78

$$\int x\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{15\,ac^6e^2x^6+45\,ac^6dex^4+45\,ac^6d^2x^2+15\left(bc^6e^2x^6+3\,bc^6dex^4+3\,bc^6d^2x^2\right)\operatorname{arcsec}(cx)-\left(3\,bc^4e^2x^4+45\right.}{90\,c^6}$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
1/90*(15*a*c^6*e^2*x^6 + 45*a*c^6*d*e*x^4 + 45*a*c^6*d^2*x^2 + 15*(b*c^6*e
^2*x^6 + 3*b*c^6*d*e*x^4 + 3*b*c^6*d^2*x^2)*arcsec(c*x) - (3*b*c^4*e^2*x^4
 + 45*b*c^4*d^2 + 30*b*c^2*d*e + 8*b*e^2 + (15*b*c^4*d*e + 4*b*c^2*e^2)*x^
2)*sqrt(c^2*x^2 - 1))/c^6
```

3.88.6 Sympy [A] (verification not implemented)

Time = 2.69 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.81

$$\int x\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{ad^2x^2}{2}+\frac{adex^4}{2}+\frac{ae^2x^6}{6}+\frac{bd^2x^2\operatorname{asec}(cx)}{2}+\frac{bdex^4\operatorname{asec}(cx)}{2}$$
$$+\frac{be^2x^6\operatorname{asec}(cx)}{6}-\frac{bd^2\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c}$$
$$-\frac{bde\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{2c}$$
$$-\frac{be^2\left(\begin{cases}\frac{x^4\sqrt{c^2x^2-1}}{5c}+\frac{4x^2\sqrt{c^2x^2-1}}{15c^3}+\frac{8\sqrt{c^2x^2-1}}{15c^5} & \text{for } |c^2x^2|>1\\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c}+\frac{4ix^2\sqrt{-c^2x^2+1}}{15c^3}+\frac{8i\sqrt{-c^2x^2+1}}{15c^5} & \text{otherwise}\end{cases}\right)}{6c}$$

input
```
integrate(x*(e*x**2+d)**2*(a+b*asec(c*x)),x)
```

output
```
a*d**2*x**2/2 + a*d*e*x**4/2 + a*e**2*x**6/6 + b*d**2*x**2*asec(c*x)/2 + b
*d*e*x**4*asec(c*x)/2 + b*e**2*x**6*asec(c*x)/6 - b*d**2*Piecewise((sqrt(c
**2*x**2 - 1)/c, Abs(c**2*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, True))/(2
*c) - b*d*e*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c) + 2*sqrt(c**2*x**2 -
 1)/(3*c**3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*x**2 + 1)/(3*c) + 2*
I*sqrt(-c**2*x**2 + 1)/(3*c**3), True))/(2*c) - b*e**2*Piecewise((x**4*sqr
t(c**2*x**2 - 1)/(5*c) + 4*x**2*sqrt(c**2*x**2 - 1)/(15*c**3) + 8*sqrt(c**
2*x**2 - 1)/(15*c**5), Abs(c**2*x**2) > 1), (I*x**4*sqrt(-c**2*x**2 + 1)/(
5*c) + 4*I*x**2*sqrt(-c**2*x**2 + 1)/(15*c**3) + 8*I*sqrt(-c**2*x**2 + 1)/
(15*c**5), True))/(6*c)
```

3.88.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.98

$$\int x\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$= \frac{1}{6}ae^2x^6 + \frac{1}{2}adex^4 + \frac{1}{2}ad^2x^2 + \frac{1}{2}\left(x^2\operatorname{arcsec}(cx) - \frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)bd^2$$
$$+ \frac{1}{6}\left(3\,x^4\operatorname{arcsec}(cx) - \frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)bde$$
$$+ \frac{1}{90}\left(15\,x^6\operatorname{arcsec}(cx) - \frac{3\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} + 10\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 15\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^5}\right)be^2$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*e^2*x^6 + 1/2*a*d*e*x^4 + 1/2*a*d^2*x^2 + 1/2*(x^2*arcsec(c*x) - x*s
qrt(-1/(c^2*x^2) + 1)/c)*b*d^2 + 1/6*(3*x^4*arcsec(c*x) - (c^2*x^3*(-1/(c^
2*x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b*d*e + 1/90*(15*x^6*
arcsec(c*x) - (3*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2) + 10*c^2*x^3*(-1/(c^2*x^
2) + 1)^(3/2) + 15*x*sqrt(-1/(c^2*x^2) + 1))/c^5)*b*e^2
```

3.88.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11858 vs. 2(169) = 338.

Time = 0.45 (sec) , antiderivative size = 11858, normalized size of antiderivative = 60.81

$$\int x\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
1/90*(45*b*c^4*d^2*arccos(1/(c*x))/(c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x)
 + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2)
 - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c
^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x)
 + 1)^12) + 45*a*c^4*d^2/(c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 +
15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1
/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2
*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)
+ 90*b*c^4*d^2*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^2*x^2
) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c
^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*
x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2)
- 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^2) - 90*b*c^4*d^2*sqrt(-1/(c^2*x^2)
 + 1)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2
) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15
*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c
*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)) + 9
0*a*c^4*d^2*(1/(c^2*x^2) - 1)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1
)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1
)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^...
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int x\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx = \int x\left(e\,x^2+d\right)^2\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x*(d + e*x^2)^2*(a + b*acos(1/(c*x))),x)
```

output
```
int(x*(d + e*x^2)^2*(a + b*acos(1/(c*x))), x)
```

3.89 $\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x}\,dx$

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3.89.1 Optimal result

Integrand size = 21, antiderivative size = 186

$$\begin{aligned}\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x}\,dx = &-\frac{be(6c^2d+e)\sqrt{1-\frac{1}{c^2x^2}}x}{6c^3}\\ &-\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x^3}{12c}-\frac{1}{2}ibd^2\csc^{-1}(cx)^2\\ &+dex^2(a+b\sec^{-1}(cx))+\frac{1}{4}e^2x^4(a+b\sec^{-1}(cx))\\ &+bd^2\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)\\ &-bd^2\csc^{-1}(cx)\log\left(\frac{1}{x}\right)-d^2(a+b\sec^{-1}(cx))\log\left(\frac{1}{x}\right)\\ &-\frac{1}{2}ibd^2\operatorname{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\end{aligned}$$

output

```
-1/2*I*b*d^2*arccsc(c*x)^2+d*e*x^2*(a+b*arcsec(c*x))+1/4*e^2*x^4*(a+b*arcs
ec(c*x))+b*d^2*arccsc(c*x)*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)-b*d^2*arccs
c(c*x)*ln(1/x)-d^2*(a+b*arcsec(c*x))*ln(1/x)-1/2*I*b*d^2*polylog(2,(I/c/x+
(1-1/c^2/x^2)^(1/2))^2)-1/6*b*e*(6*c^2*d+e)*x*(1-1/c^2/x^2)^(1/2)/c^3-1/12
*b*e^2*x^3*(1-1/c^2/x^2)^(1/2)/c
```

3.89.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x}\,dx = adex^2 + \frac{1}{4}ae^2x^4 - \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x(2+c^2x^2)}{12c^3} + \frac{1}{4}be^2x^4\sec^{-1}(cx) + \frac{bdex\left(-\sqrt{1-\frac{1}{c^2x^2}}+cx\sec^{-1}(cx)\right)}{c} + ad^2\log(x) + \frac{1}{2}ibd^2\left(\sec^{-1}(cx)\left(\sec^{-1}(cx)+2i\log\left(1+e^{2i\sec^{-1}(cx)}\right)\right) + \text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)\right)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x,x]
```

output
```
a*d*e*x^2 + (a*e^2*x^4)/4 - (b*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*(2 + c^2*x^2))/
(12*c^3) + (b*e^2*x^4*ArcSec[c*x])/4 + (b*d*e*x*(-Sqrt[1 - 1/(c^2*x^2)] +
c*x*ArcSec[c*x]))/c + a*d^2*Log[x] + (I/2)*b*d^2*(ArcSec[c*x]*(ArcSec[c*x]
 + (2*I)*Log[1 + E^((2*I)*ArcSec[c*x])]) + PolyLog[2, -E^((2*I)*ArcSec[c*x
])])
```

3.89.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5763, 5231, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x}\,dx$$

$$\downarrow 5763$$

$$-\int \left(\frac{d}{x^2}+e\right)^2 x^5\left(a+b\arccos\left(\frac{1}{cx}\right)\right)d\frac{1}{x}$$

$$\downarrow 5231$$

$$-\frac{b\int -\frac{e\left(\frac{4d}{x^2}+e\right)x^4-4d^2\log\left(\frac{1}{x}\right)}{4\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{c}-d^2\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+dex^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+\frac{1}{4}e^2x^4\left(a+b\arccos\left(\frac{1}{cx}\right)\right)$$

$$\downarrow 27$$

$$\frac{b\int \frac{e\left(\frac{4d}{x^2}+e\right)x^4-4d^2\log\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{4c}-d^2\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+dex^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+\frac{1}{4}e^2x^4\left(a+b\arccos\left(\frac{1}{cx}\right)\right)$$

$$\downarrow 7293$$

$$\frac{b\int\left(\frac{e\left(\frac{4d}{x^2}+e\right)x^4}{\sqrt{1-\frac{1}{c^2x^2}}}-\frac{4d^2\log\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{c^2x^2}}}\right)d\frac{1}{x}}{4c}-d^2\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+dex^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+\frac{1}{4}e^2x^4\left(a+b\arccos\left(\frac{1}{cx}\right)\right)$$

$$\downarrow 2009$$

$$-d^2\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+dex^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+\frac{1}{4}e^2x^4\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+\frac{b\left(-2icd^2\,\text{PolyLog}\left(2,e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)-2icd^2\arcsin\left(\frac{1}{cx}\right)^2+4cd^2\arcsin\left(\frac{1}{cx}\right)\log\left(1-e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)-4cd^2\log\left(\frac{1}{x}\right)\text{ar}\right.}{4c}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x,x]
```

output
```
d*e*x^2*(a + b*ArcCos[1/(c*x)]) + (e^2*x^4*(a + b*ArcCos[1/(c*x)]))/4 - d^
2*(a + b*ArcCos[1/(c*x)])*Log[x^(-1)] + (b*((-2*e*(6*d + e/c^2)*Sqrt[1 - 1
/(c^2*x^2)]*x)/3 - (e^2*Sqrt[1 - 1/(c^2*x^2)]*x^3)/3 - (2*I)*c*d^2*ArcSin[
1/(c*x)]^2 + 4*c*d^2*ArcSin[1/(c*x)]*Log[1 - E^((2*I)*ArcSin[1/(c*x)])] -
4*c*d^2*ArcSin[1/(c*x)]*Log[x^(-1)] - (2*I)*c*d^2*PolyLog[2, E^((2*I)*ArcS
in[1/(c*x)])]))/(4*c)
```

3.89.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.89.4 Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.24

method	result
parts	$a\left(\frac{e^2x^4}{4}+de\,x^2+d^2\ln\left(x\right)\right)+b\left(\frac{id^2\operatorname{arcsec}(cx)^2}{2}+\frac{e\left(12c^4d\,\operatorname{arcsec}(cx)x^2+3\,\operatorname{arcsec}(cx)e\,c^4x^4-12\sqrt{\frac{c^2x^2}{c^2x}}\right.\right.$
derivativedivides	$ade\,x^2+\frac{a\,e^2x^4}{4}+a\,d^2\ln\left(cx\right)+\frac{b\left(\frac{ic^4d^2\operatorname{arcsec}(cx)^2}{2}+\frac{e\left(12c^4d\,\operatorname{arcsec}(cx)x^2+3\,\operatorname{arcsec}(cx)e\,c^4x^4-12\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3d\right.\right.}{12}$
default	$ade\,x^2+\frac{a\,e^2x^4}{4}+a\,d^2\ln\left(cx\right)+\frac{b\left(\frac{ic^4d^2\operatorname{arcsec}(cx)^2}{2}+\frac{e\left(12c^4d\,\operatorname{arcsec}(cx)x^2+3\,\operatorname{arcsec}(cx)e\,c^4x^4-12\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3d\right.\right.}{12}$

input
```
int((e*x^2+d)^2*(a+b*arcsec(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
a*(1/4*e^2*x^4+d*e*x^2+d^2*ln(x))+b*(1/2*I*d^2*arcsec(c*x)^2+1/12/c^4*e*(1
2*c^4*d*arcsec(c*x)*x^2+3*arcsec(c*x)*e*c^4*x^4-12*((c^2*x^2-1)/c^2/x^2)^(
1/2)*c^3*d*x-((c^2*x^2-1)/c^2/x^2)^(1/2)*e*c^3*x^3-12*I*c^2*d-2*((c^2*x^2-
1)/c^2/x^2)^(1/2)*e*c*x-2*I*e)-d^2*arcsec(c*x)*ln(1+(1/c/x+I*(1-1/c^2/x^2)
^(1/2))^2)+1/2*I*d^2*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2))
```

3.89.5 Fricas [F]

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)}{x}\,dx = \int \frac{\left(ex^2+d\right)^2\left(b\operatorname{arcsec}(cx)+a\right)}{x}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arcsec(c*x))/x, x)
```

3.89.6 Sympy [F]

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)}{x}\,dx = \int \frac{\left(a+b\operatorname{asec}(cx)\right)\left(d+ex^2\right)^2}{x}\,dx$$

input
```
integrate((e*x**2+d)**2*(a+b*asec(c*x))/x,x)
```

output
```
Integral((a + b*asec(c*x))*(d + e*x**2)**2/x, x)
```

3.89.7 Maxima [F]

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{(ex^2+d)^2(b\,\text{arcsec}\,(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x,x, algorithm="maxima")
```

output
```
1/4*a*e^2*x^4 + a*d*e*x^2 + a*d^2*log(x) - 1/8*(-2*I*b*c^4*e^2*x^4*log(c)
- 4*I*b*c^4*d^2*log(-c*x + 1)*log(x) - 4*I*b*c^4*d^2*log(x)^2 - 4*I*b*c^4*
d^2*dilog(c*x) - 4*I*b*c^4*d^2*dilog(-c*x) + I*(b*e^2*(x^2/c^2 + log(c*x +
 1)/c^4 + log(c*x - 1)/c^4) + 4*b*d*e*(log(c*x + 1)/c^2 + log(c*x - 1)/c^2
) + 32*b*d^2*integrate(1/4*log(x)/(c^2*x^3 - x), x))*c^4 + 8*c^4*integrate
(1/4*(b*e^2*x^4 + 4*b*d*e*x^2 + 4*b*d^2*log(x))*sqrt(c*x + 1)*sqrt(c*x - 1
)/(c^2*x^3 - x), x) + (-8*I*b*c^4*d*e*log(c) - I*b*c^2*e^2)*x^2 - 2*(b*c^4
*e^2*x^4 + 4*b*c^4*d*e*x^2 + 4*b*c^4*d^2*log(x))*arctan(sqrt(c*x + 1)*sqrt
(c*x - 1)) + (I*b*c^4*e^2*x^4 + 4*I*b*c^4*d*e*x^2 + 4*I*b*c^4*d^2*log(x))*
log(c^2*x^2) + (-4*I*b*c^4*d^2*log(x) - 4*I*b*c^2*d*e - I*b*e^2)*log(c*x +
 1) + (-4*I*b*c^2*d*e - I*b*e^2)*log(c*x - 1) - 2*(I*b*c^4*e^2*x^4 + 4*I*b
*c^4*d*e*x^2 + 4*I*b*c^4*d^2*log(c))*log(x))/c^4
```

3.89.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Limit: Max order reached or unable to make series expan
sion Error: Bad Argument Value
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{(e\,x^2+d)^2\,\left(a+b\,\mathrm{acos}\left(\frac{1}{cx}\right)\right)}{x}\,dx$$

input
```
int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x,x)
```

output
```
int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x, x)
```

3.90 $\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^3}\,dx$

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3.90.1 Optimal result

Integrand size = 21, antiderivative size = 189

$$\begin{aligned}\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^3}\,dx = {}& \frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} \\ & - \frac{1}{4}bc^2d^2\csc^{-1}(cx) - ibde\csc^{-1}(cx)^2 \\ & - \frac{d^2(a+b\sec^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2(a+b\sec^{-1}(cx)) \\ & + 2bde\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) \\ & - 2bde\csc^{-1}(cx)\log\left(\frac{1}{x}\right) \\ & - 2de(a+b\sec^{-1}(cx))\log\left(\frac{1}{x}\right) \\ & - ibde\,\mathrm{PolyLog}\left(2, e^{2i\csc^{-1}(cx)}\right)\end{aligned}$$

output

```
-1/4*b*c^2*d^2*arccsc(c*x)-I*b*d*e*arccsc(c*x)^2-1/2*d^2*(a+b*arcsec(c*x))
/x^2+1/2*e^2*x^2*(a+b*arcsec(c*x))+2*b*d*e*arccsc(c*x)*ln(1-(I/c/x+(1-1/c^
2/x^2)^(1/2))^2)-2*b*d*e*arccsc(c*x)*ln(1/x)-2*d*e*(a+b*arcsec(c*x))*ln(1/
x)-I*b*d*e*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)+1/4*b*c*d^2*(1-1/c^2/x
^2)^(1/2)/x-1/2*b*e^2*x*(1-1/c^2/x^2)^(1/2)/c
```

3.90.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x^3}\,dx$$

$$= \frac{1}{4}\left(-\frac{2ad^2}{x^2}+2ae^2x^2-\frac{2bd^2\sec^{-1}(cx)}{x^2}+\frac{2be^2x\left(-\sqrt{1-\frac{1}{c^2x^2}}+cx\sec^{-1}(cx)\right)}{c}\right.$$

$$+\frac{bd^2\left(-1+c^2x^2+c^2x^2\sqrt{-1+c^2x^2}\arctan\left(\sqrt{-1+c^2x^2}\right)\right)}{c\sqrt{1-\frac{1}{c^2x^2}}x^3}+8ade\log(x)$$

$$\left.+4ibde\left(\sec^{-1}(cx)\left(\sec^{-1}(cx)+2i\log\left(1+e^{2i\sec^{-1}(cx)}\right)\right)+\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)\right)\right)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x^3,x]
```

output
```
((-2*a*d^2)/x^2 + 2*a*e^2*x^2 - (2*b*d^2*ArcSec[c*x])/x^2 + (2*b*e^2*x*(-S
qrt[1 - 1/(c^2*x^2)] + c*x*ArcSec[c*x]))/c + (b*d^2*(-1 + c^2*x^2 + c^2*x^
2*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]]))/(c*Sqrt[1 - 1/(c^2*x^2)]
*x^3) + 8*a*d*e*Log[x] + (4*I)*b*d*e*(ArcSec[c*x]*(ArcSec[c*x] + (2*I)*Log
[1 + E^((2*I)*ArcSec[c*x])]) + PolyLog[2, -E^((2*I)*ArcSec[c*x])]))/4
```

3.90.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.21, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5763, 5231, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x^3}\,dx$$

$$\downarrow 5763$$

$$-\int \left(\frac{d}{x^2}+e\right)^2 x^3\left(a+b\arccos\left(\frac{1}{cx}\right)\right)d\frac{1}{x}$$

$$\downarrow 5231$$

$$-\frac{b\int -\frac{-\frac{d^2}{x^2}-4e\log\left(\frac{1}{x}\right)d+e^2x^2}{2\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{c}-\frac{d^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{2x^2}-2de\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+\frac{1}{2}e^2x^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)$$

$$\downarrow 27$$

$$\frac{b\int \frac{-\frac{d^2}{x^2}-4e\log\left(\frac{1}{x}\right)d+e^2x^2}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{2c}-\frac{d^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{2x^2}-2de\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+\frac{1}{2}e^2x^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)$$

$$\downarrow 7293$$

$$\frac{b\int\left(-\frac{d^2}{\sqrt{1-\frac{1}{c^2x^2}}x^2}-\frac{4e\log\left(\frac{1}{x}\right)d}{\sqrt{1-\frac{1}{c^2x^2}}}+\frac{e^2x^2}{\sqrt{1-\frac{1}{c^2x^2}}}\right)d\frac{1}{x}}{2c}-\frac{d^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{2x^2}-2de\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+\frac{1}{2}e^2x^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)$$

$$\downarrow 2009$$

$$-\frac{d^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{2x^2}-2de\log\left(\frac{1}{x}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+\frac{1}{2}e^2x^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)+\frac{b\left(-\frac{1}{2}c^3d^2\arcsin\left(\frac{1}{cx}\right)-2icde\,\mathrm{PolyLog}\left(2,e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)-2icde\arcsin\left(\frac{1}{cx}\right)^2+4cde\arcsin\left(\frac{1}{cx}\right)\log\left(1-e^{2i\arcsin(\right.\right.}}{2c}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x^3,x]
```

output
```
-1/2*(d^2*(a + b*ArcCos[1/(c*x)]))/x^2 + (e^2*x^2*(a + b*ArcCos[1/(c*x)]))
/2 - 2*d*e*(a + b*ArcCos[1/(c*x)])*Log[x^(-1)] + (b*((c^2*d^2*Sqrt[1 - 1/(
c^2*x^2)])/(2*x) - e^2*Sqrt[1 - 1/(c^2*x^2)]*x - (c^3*d^2*ArcSin[1/(c*x)])
/2 - (2*I)*c*d*e*ArcSin[1/(c*x)]^2 + 4*c*d*e*ArcSin[1/(c*x)]*Log[1 - E^((2
*I)*ArcSin[1/(c*x)])] - 4*c*d*e*ArcSin[1/(c*x)]*Log[x^(-1)] - (2*I)*c*d*e*
PolyLog[2, E^((2*I)*ArcSin[1/(c*x)])]))/(2*c)
```

3.90. $\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^3}\,dx$

3.90.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.90.4 Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.14

method	result
parts	$a\left(\frac{e^2x^2}{2}-\frac{d^2}{2x^2}+2de\ln\left(x\right)\right)+ibde\,\mathrm{arcsec}\left(cx\right)^2+\frac{bcd^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4x}+\frac{bc^2d^2\,\mathrm{arcsec}(cx)}{4}-\frac{b\,\mathrm{arcsec}(cx$
derivativedivides	$c^2\left(\frac{ax^2e^2}{2c^2}+\frac{2ade\ln(cx)}{c^2}-\frac{ad^2}{2c^2x^2}+\frac{ibde\,\mathrm{arcsec}(cx)^2}{c^2}+\frac{bd^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4cx}+\frac{b\,\mathrm{arcsec}(cx)d^2}{4}-\frac{b\,\mathrm{arcsec}(cx)d^2}{2c^2x^2}+\right.$
default	$c^2\left(\frac{ax^2e^2}{2c^2}+\frac{2ade\ln(cx)}{c^2}-\frac{ad^2}{2c^2x^2}+\frac{ibde\,\mathrm{arcsec}(cx)^2}{c^2}+\frac{bd^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4cx}+\frac{b\,\mathrm{arcsec}(cx)d^2}{4}-\frac{b\,\mathrm{arcsec}(cx)d^2}{2c^2x^2}+\right.$

input
```
int((e*x^2+d)^2*(a+b*arcsec(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
a*(1/2*e^2*x^2-1/2*d^2/x^2+2*d*e*ln(x))+I*b*d*e*arcsec(c*x)^2+1/4*b*c*d^2/
x*((c^2*x^2-1)/c^2/x^2)^(1/2)+1/4*b*c^2*d^2*arcsec(c*x)-1/2*b*arcsec(c*x)*
d^2/x^2+1/2*b*e^2*arcsec(c*x)*x^2-1/2*b/c*e^2*((c^2*x^2-1)/c^2/x^2)^(1/2)*
x-1/2*I*b/c^2*e^2-2*b*d*e*arcsec(c*x)*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2
)+I*b*d*e*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)
```

3.90.5 Fricas [F]

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)}{x^3}\,dx = \int \frac{\left(ex^2+d\right)^2(b\operatorname{arcsec}(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arcsec(c*x))/x^3, x)
```

3.90.6 Sympy [F]

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)}{x^3}\,dx = \int \frac{\left(a+b\operatorname{asec}(cx)\right)\left(d+ex^2\right)^2}{x^3}\,dx$$

input
```
integrate((e*x**2+d)**2*(a+b*asec(c*x))/x**3,x)
```

output
```
Integral((a + b*asec(c*x))*(d + e*x**2)**2/x**3, x)
```

3.90.7 Maxima [F]

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)}{x^3}\,dx = \int \frac{\left(ex^2+d\right)^2(b\operatorname{arcsec}(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^3,x, algorithm="maxima")
```

output
```
1/2*a*e^2*x^2 - 1/4*b*d^2*((c^4*x*sqrt(-1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*
x^2) - 1) - 1) - c^3*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)))/c + 2*arcsec(c*x)
/x^2) + 2*a*d*e*log(x) - 1/2*a*d^2/x^2 - 1/4*(-2*I*b*c^2*e^2*x^2*log(c) -
4*I*b*c^2*d*e*log(-c*x + 1)*log(x) - 4*I*b*c^2*d*e*log(x)^2 - 4*I*b*c^2*d*
e*dilog(c*x) - 4*I*b*c^2*d*e*dilog(-c*x) - I*b*e^2*log(c*x - 1) + I*(b*e^2
*(log(c*x + 1)/c^2 + log(c*x - 1)/c^2) + 16*b*d*e*integrate(1/2*log(x)/(c^
2*x^3 - x), x))*c^2 + 4*c^2*integrate(1/2*(b*e^2*x^2 + 4*b*d*e*log(x))*sqr
t(c*x + 1)*sqrt(c*x - 1)/(c^2*x^3 - x), x) - 2*(b*c^2*e^2*x^2 + 4*b*c^2*d*
e*log(x))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + (I*b*c^2*e^2*x^2 + 4*I*b*c
^2*d*e*log(x))*log(c^2*x^2) + (-4*I*b*c^2*d*e*log(x) - I*b*e^2)*log(c*x +
1) - 2*(I*b*c^2*e^2*x^2 + 4*I*b*c^2*d*e*log(c))*log(x))/c^2
```

3.90.8 Giac [F]

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)}{x^3}\,dx = \int \frac{\left(ex^2+d\right)^2(b\operatorname{arcsec}(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^3,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arcsec(c*x) + a)/x^3, x)
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)}{x^3}\,dx = \int \frac{\left(e\,x^2+d\right)^2\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)}{x^3}\,dx$$

input
```
int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^3,x)
```

output
```
int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^3, x)
```

3.91 $\int \frac{x^2(a+b\sec^{-1}(cx))}{d+ex^2}\,dx$

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3.91.1 Optimal result

Integrand size = 21, antiderivative size = 546

$$\begin{aligned}\int \frac{x^2(a+b\sec^{-1}(cx))}{d+ex^2}\,dx &= \frac{x(a+b\sec^{-1}(cx))}{e} - \frac{b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce} \\ &+ \frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\ &- \frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\ &+ \frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\ &- \frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\ &+ \frac{ib\sqrt{-d}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\ &- \frac{ib\sqrt{-d}\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\ &+ \frac{ib\sqrt{-d}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\ &- \frac{ib\sqrt{-d}\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}}\end{aligned}$$

output
```
x*(a+b*arcsec(c*x))/e-b*arctanh((1-1/c^2/x^2)^(1/2))/c/e+1/2*(a+b*arcsec(c
*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1
/2)))*(-d)^(1/2)/e^(3/2)-1/2*(a+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^
2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)+1/2*(a+
b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c
^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*(a+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(
1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/
2)+1/2*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-
(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2
/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)+1/2*
I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+
e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(
1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)
```

3.91.2 Mathematica [A] (warning: unable to verify)

Time = 1.88 (sec) , antiderivative size = 1023, normalized size of antiderivative = 1.87

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \frac{ax}{e} - \frac{a\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}}$$

$$+b\left(\frac{cx\sec^{-1}(cx)+\log\left(\cos\left(\frac{1}{2}\sec^{-1}(cx)\right)-\sin\left(\frac{1}{2}\sec^{-1}(cx)\right)\right)-\log\left(\cos\left(\frac{1}{2}\sec^{-1}(cx)\right)+\sin\left(\frac{1}{2}\sec^{-1}(cx\right.\right.\right.}{ce}\right.$$

$$-\frac{\sqrt{d}\left(8\arcsin\left(\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(ic\sqrt{d}+\sqrt{e}\right)\tan\left(\frac{1}{2}\sec^{-1}(cx)\right)}{\sqrt{c^2d+e}}\right)-2i\sec^{-1}(cx)\log\left(1+\frac{i\left(\sqrt{e}-\sqrt{c^2d+e}\right)e^{i\sec^{-1}(}\right.\right.\right.}{c\sqrt{d}}$$

$$+\frac{\sqrt{d}\left(8\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\tan\left(\frac{1}{2}\sec^{-1}(cx)\right)}{\sqrt{c^2d+e}}\right)-2i\sec^{-1}(cx)\log\left(1+\frac{i\left(-\sqrt{e}+\sqrt{c^2d+e}\right)e^{i\sec}\right.\right.\right.}{c\sqrt{d}}$$

input
```
Integrate[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2),x]
```

output

```
(a*x)/e - (a*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e^(3/2) + b*((c*x*ArcSec
[c*x] + Log[Cos[ArcSec[c*x]/2] - Sin[ArcSec[c*x]/2]] - Log[Cos[ArcSec[c*x]
/2] + Sin[ArcSec[c*x]/2]])/(c*e) - (Sqrt[d]*(8*ArcSin[Sqrt[1 + (I*Sqrt[e])
/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])
/Sqrt[c^2*d + e]] - (2*I)*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e
])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*
Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]
))/(c*Sqrt[d])] - (2*I)*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])
*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sq
rt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))
/(c*Sqrt[d])] + (2*I)*ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] - 2*PolyL
og[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 2*
PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d]
)] + PolyLog[2, -E^((2*I)*ArcSec[c*x])]))/(4*e^(3/2)) + (Sqrt[d]*(8*ArcSin
[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt
[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] - (2*I)*ArcSec[c*x]*Log[1 + (I*(
-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (4*I)*ArcSin
[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^
2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (2*I)*ArcSec[c*x]*Log[1 - (I*(
Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (4*I)*ArcS...
```

3.91.3 Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 598, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5763, 5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+b\sec^{-1}(cx)\right)}{d+ex^2}\,dx$$

$$\downarrow 5763$$

$$-\int \frac{x^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{\frac{d}{x^2}+e}d\frac{1}{x}$$

$$\downarrow 5233$$

$$-\int \left(\frac{x^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e\left(\frac{d}{x^2}+e\right)}\right)d\frac{1}{x}$$

$$\downarrow 2009$$

$$\frac{\sqrt{-d}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}}-\frac{\sqrt{-d}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}}+\frac{\sqrt{-d}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^{3/2}}-\frac{\sqrt{-d}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^{3/2}}+\frac{x\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e}+\frac{ib\sqrt{-d}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^{3/2}}-\frac{ib\sqrt{-d}\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^{3/2}}+\frac{ib\sqrt{-d}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^{3/2}}-\frac{ib\sqrt{-d}\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^{3/2}}-\frac{b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce}$$

input

```
Int[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2),x]
```

output

```
(x*(a + b*ArcCos[1/(c*x)]))/e - (b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/(c*e) +
 (Sqrt[-d]*(a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)
]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcCos[1/
(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d +
 e])])/(2*e^(3/2)) + (Sqrt[-d]*(a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]
*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^(3/2)) - (Sqrt[
-d]*(a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sq
rt[e] + Sqrt[c^2*d + e])])/(2*e^(3/2)) + ((I/2)*b*Sqrt[-d]*PolyLog[2, -((c
*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/e^(3/2) -
((I/2)*b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] -
 Sqrt[c^2*d + e])])/e^(3/2) + ((I/2)*b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E
^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e]))])/e^(3/2) - ((I/2)*b*Sq
rt[-d]*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d
 + e])])/e^(3/2)
```

3.91.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

3.91.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 34.29 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.68

method	result
parts	$\frac{ax}{e} - \frac{ad\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}} + \frac{b\,\mathrm{arcsec}(cx)x}{e} + \frac{2ib\arctan\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce} - \frac{ibcd\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\right.}\right.$
derivativedivides	$\frac{a\,c^3x}{e} - \frac{a\,c^3d\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}} + b\,c^2\left(\frac{\mathrm{arcsec}(cx)cx}{e} + \frac{2i\arctan\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)}{e} + \frac{ic^2d\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2-\right.}\right.\right.$
default	$\frac{a\,c^3x}{e} - \frac{a\,c^3d\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}} + b\,c^2\left(\frac{\mathrm{arcsec}(cx)cx}{e} + \frac{2i\arctan\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)}{e} + \frac{ic^2d\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2-\right.}\right.\right.$

input
```
int(x^2*(a+b*arcsec(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
a/e*x-a*d/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b*arcsec(c*x)/e*x+2*I*b/c/
e*arctan(1/c/x+I*(1-1/c^2/x^2)^(1/2))-1/8*I*b*c/e^2*d*sum((_R1^2*c^2*d+4*_
R1^2*e+c^2*d)/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(
1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1
=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/8*I*b*c/e^2*d*sum((_R1^2*c
^2*d+c^2*d+4*e)/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I
*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_
R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))
```

3.91.5 Fricas [F]

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x^2}{ex^2+d}\,dx$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*x^2*arcsec(c*x) + a*x^2)/(e*x^2 + d), x)
```

3.91.6 Sympy [F]

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \int \frac{x^2(a+b\operatorname{asec}(cx))}{d+ex^2}\,dx$$

input
```
integrate(x**2*(a+b*asec(c*x))/(e*x**2+d),x)
```

output
```
Integral(x**2*(a + b*asec(c*x))/(d + e*x**2), x)
```

3.91.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.91.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \int \frac{x^2\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)}{e\,x^2+d}\,dx$$

input
```
int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2),x)
```

output
```
int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2), x)
```

3.92 $\int \frac{x\left(a+b\sec^{-1}(cx)\right)}{d+ex^2}\,dx$

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3.92.1 Optimal result

Integrand size = 19, antiderivative size = 487

$$\begin{aligned}\int \frac{x(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = {}& \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} \\ &+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} \\ &+ \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} \\ &+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} \\ &- \frac{(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e} \\ &- \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} \\ &- \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} \\ &- \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} \\ &- \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} + \frac{ib\,\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2e}\end{aligned}$$

output

```
-(a+b*arcsec(c*x))*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)/e+1/2*(a+b*arcsec
(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^
(1/2)))/e+1/2*(a+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^
(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e+1/2*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(
1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e+1/2*(a+b*arcse
c(c*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)
^(1/2)))/e+1/2*I*b*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)/e-1/2*I*b*p
olylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1
/2)))/e-1/2*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1
/2)-(c^2*d+e)^(1/2)))/e-1/2*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))
*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e-1/2*I*b*polylog(2,c*(1/c/x+I*(1-1
/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e
```

3.92.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 891, normalized size of antiderivative = 1.83

$$\int \frac{x(a+b\sec^{-1}(cx))}{d+ex^2}\,dx$$

$$= \frac{4ib\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\tan\left(\frac{1}{2}\sec^{-1}(cx)\right)}{\sqrt{c^2d+e}}\right)+4ib\arcsin\left(\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(ic\sqrt{d}+\sqrt{e}\right)\tan\left(\frac{1}{2}s\right.\right.}{\sqrt{c^2d+e}}$$

input

```
Integrate[(x*(a + b*ArcSec[c*x]))/(d + e*x^2),x]
```

output

```
((4*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c
*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] + (4*I)*b*ArcSin[
Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])
*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] + b*ArcSec[c*x]*Log[1 + (I*(Sqrt[e]
- Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 2*b*ArcSin[Sqrt[1 + (
I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^
(I*ArcSec[c*x]))/(c*Sqrt[d])] + b*ArcSec[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[
c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 2*b*ArcSin[Sqrt[1 - (I*Sqrt[
e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*Arc
Sec[c*x]))/(c*Sqrt[d])] + b*ArcSec[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d +
 e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 2*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*
Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]
))/(c*Sqrt[d])] + b*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(
I*ArcSec[c*x]))/(c*Sqrt[d])] - 2*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])
]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sq
rt[d])] - 2*b*ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] + a*Log[d + e*x^2
] - I*b*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(
c*Sqrt[d])] - I*b*PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c
*x]))/(c*Sqrt[d])] - I*b*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I
*ArcSec[c*x]))/(c*Sqrt[d])] - I*b*PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d +...
```

3.92.3 Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {5763, 5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+b\sec^{-1}(cx)\right)}{d+ex^2}\,dx$$

$$\downarrow 5763$$

$$-\int \frac{x\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{\frac{d}{x^2}+e}d\frac{1}{x}$$

$$\downarrow 5233$$

$$-\int \left(\frac{x\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e\left(\frac{d}{x^2}+e\right)x}\right)d\frac{1}{x}$$

$$\downarrow \text{2009}$$

$$\frac{(a+b\arccos\left(\frac{1}{cx}\right))\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e}+\frac{(a+b\arccos\left(\frac{1}{cx}\right))\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e}+$$
$$\frac{(a+b\arccos\left(\frac{1}{cx}\right))\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e}+\frac{(a+b\arccos\left(\frac{1}{cx}\right))\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e}-$$
$$\frac{\log\left(1+e^{2i\arccos\left(\frac{1}{cx}\right)}\right)(a+b\arccos\left(\frac{1}{cx}\right))}{e}-\frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e}-$$
$$\frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e}-\frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e}-$$
$$\frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e}+\frac{ib\,\text{PolyLog}\left(2,-e^{2i\arccos\left(\frac{1}{cx}\right)}\right)}{2e}$$

input
```
Int[(x*(a + b*ArcSec[c*x]))/(d + e*x^2),x]
```

output
```
((a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[
e] - Sqrt[c^2*d + e])])/(2*e) + ((a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-
d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e) + ((a + b*Ar
cCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[
c^2*d + e])])/(2*e) + ((a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*Ar
cCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e) - ((a + b*ArcCos[1/(c*
x)])*Log[1 + E^((2*I)*ArcCos[1/(c*x)])])/e - ((I/2)*b*PolyLog[2, -((c*Sqrt
[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/e - ((I/2)*b*Po
lyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/
e - ((I/2)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sq
rt[c^2*d + e]))])/e - ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]
))/(Sqrt[e] + Sqrt[c^2*d + e])])/e + ((I/2)*b*PolyLog[2, -E^((2*I)*ArcCos[
1/(c*x)])])/e
```

3.92.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

3.92.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.71 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.92

method	result
parts	$\frac{a\ln(e\,x^2+d)}{2e} - \frac{ib\,c^2 d\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2 d\_Z^4+\left(2c^2 d+4e\right)\_Z^2+c^2 d\right)} \frac{\left(\_R1^2+1\right)\left(i\,\mathrm{arcsec}(cx)\ln\left(\frac{\_R1-\frac{1}{cx}-i\sqrt{1-\frac{1}{c^2}}}{\_R1}\right.\right.}{\_R1^2\,c^2 d+c^2}\right.}{4e}$
derivativedivides	$\frac{a\,c^2\ln\left(c^2 e\,x^2+c^2 d\right)}{2e}+b\,c^2\left(-\frac{\mathrm{arcsec}(cx)\ln\left(1+i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{e}-\frac{\mathrm{arcsec}(cx)\ln\left(1-i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{e}+\frac{i\,\mathrm{dilog}\left(1+i\left(\frac{1}{cx}+\right.\right.\right.$
default	$\frac{a\,c^2\ln\left(c^2 e\,x^2+c^2 d\right)}{2e}+b\,c^2\left(-\frac{\mathrm{arcsec}(cx)\ln\left(1+i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{e}-\frac{\mathrm{arcsec}(cx)\ln\left(1-i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{e}+\frac{i\,\mathrm{dilog}\left(1+i\left(\frac{1}{cx}+\right.\right.\right.$

input
```
int(x*(a+b*arcsec(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*a/e*ln(e*x^2+d)-1/4*I*b*c^2*d/e*sum((_R1^2+1)/(_R1^2*c^2*d+c^2*d+2*e)*
(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-
I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*
d))-b/e*arcsec(c*x)*ln(1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-b/e*arcsec(c*x)*
ln(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))+I*b/e*dilog(1+I*(1/c/x+I*(1-1/c^2/x^
2)^(1/2)))+I*b/e*dilog(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-1/4*I*b/e*sum((_
R1^2*c^2*d+c^2*d+4*e)/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x
-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1))
,_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))
```

3.92. $\int \frac{x(a+b\sec^{-1}(cx))}{d+ex^2}\,dx$

3.92.5 Fricas [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x}{ex^2+d}\,dx$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*x*arcsec(c*x) + a*x)/(e*x^2 + d), x)
```

3.92.6 Sympy [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \int \frac{x(a+b\operatorname{asec}(cx))}{d+ex^2}\,dx$$

input
```
integrate(x*(a+b*asec(c*x))/(e*x**2+d),x)
```

output
```
Integral(x*(a + b*asec(c*x))/(d + e*x**2), x)
```

3.92.7 Maxima [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x}{ex^2+d}\,dx$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
b*integrate(x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x^2 + d), x) + 1/2*a*
log(e*x^2 + d)/e
```

3.92.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \int \frac{x\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)}{e\,x^2+d}\,dx$$

input
```
int((x*(a + b*acos(1/(c*x))))/(d + e*x^2),x)
```

output
```
int((x*(a + b*acos(1/(c*x))))/(d + e*x^2), x)
```

3.93 $\int \frac{a+b\sec^{-1}(cx)}{d+ex^2}\,dx$

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3.93.9 Mupad [F(-1)] . . . 698

3.93.1 Optimal result

Integrand size = 18, antiderivative size = 509

$$\begin{aligned}\int \frac{a+b\sec^{-1}(cx)}{d+ex^2}\,dx = &\frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}\\ &-\frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}\\ &+\frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}\\ &-\frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}\\ &+\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}\\ &+\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}\end{aligned}$$

output

```
1/2*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(
1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arcsec(c*x))*ln(1+c*(1/
c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2
)/e^(1/2)+1/2*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^
(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arcsec(c*x))*
ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))
/(-d)^(1/2)/e^(1/2)+1/2*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d
)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*I*b*polylog(2,c*
(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(
1/2)/e^(1/2)+1/2*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)
/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*I*b*polylog(2,c*(1/c/x+
I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^
(1/2)
```

3.93.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 871, normalized size of antiderivative = 1.71

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex^2}\,dx$$

$$= \frac{2a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) - 4b\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\tan\left(\frac{1}{2}\sec^{-1}(cx)\right)}{\sqrt{c^2d+e}}\right) + 4b\arcsin\left(\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan$$

input

```
Integrate[(a + b*ArcSec[c*x])/(d + e*x^2),x]
```

output

```
(2*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - 4*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt
[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt
[c^2*d + e]] + 4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTa
n[((I*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] - I*b*ArcS
ec[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[
d])] - (2*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (
I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + I*b*ArcSec
[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d
])] + (2*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I
*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + I*b*ArcSec
[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d]
)] - (2*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*
(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - I*b*ArcSec[c
*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])]
 + (2*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(S
qrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + b*PolyLog[2, (
(-I)*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - b*Poly
Log[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - b
*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d
])] + b*PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c...
```

3.93.3 Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 557, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5753, 5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \sec^{-1}(cx)}{d + ex^2}\,dx$$

$$\downarrow 5753$$

$$-\int \frac{a + b \arccos\left(\frac{1}{cx}\right)}{\frac{d}{x^2} + e} d\frac{1}{x}$$

$$\downarrow 5173$$

$$-\int \left(\frac{a + b \arccos\left(\frac{1}{cx}\right)}{2\sqrt{e}\left(\sqrt{e} - \frac{\sqrt{-d}}{x}\right)} + \frac{a + b \arccos\left(\frac{1}{cx}\right)}{2\sqrt{e}\left(\frac{\sqrt{-d}}{x} + \sqrt{e}\right)} \right) d\frac{1}{x}$$

$$\downarrow 2009$$

$$\frac{(a+b\arccos\left(\frac{1}{cx}\right))\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{(a+b\arccos\left(\frac{1}{cx}\right))\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}+$$
$$\frac{(a+b\arccos\left(\frac{1}{cx}\right))\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{(a+b\arccos\left(\frac{1}{cx}\right))\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}+$$
$$\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}+$$
$$\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

input
```
Int[(a + b*ArcSec[c*x])/(d + e*x^2),x]
```

output
```
((a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[
e] - Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCos[1/(c*x)])*Lo
g[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*
Sqrt[-d]*Sqrt[e]) + ((a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcC
os[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b
*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sq
rt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) + ((I/2)*b*PolyLog[2, -((c*Sqrt[-d]*
E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/(Sqrt[-d]*Sqrt[e]) -
 ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^
2*d + e])])/(Sqrt[-d]*Sqrt[e]) + ((I/2)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*Ar
cCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e]))])/(Sqrt[-d]*Sqrt[e]) - ((I/2)
*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e
])])/(Sqrt[-d]*Sqrt[e])
```

3.93.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

rule 5753
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(2*(p + 1)))
, x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p]
```

3.93.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 16.11 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.53

method	result
parts	$\frac{a \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}} + \frac{ibc \left(\sum_{\_R1=\mathrm{RootOf}\left(c^2 d \_Z^4 + \left(2c^2 d+4e\right) \_Z^2 + c^2 d\right)} \frac{\_R1 \left(i \operatorname{arcsec}(cx) \ln\left(\frac{\_R1 - \frac{1}{cx} - i\sqrt{1-\frac{1}{c^2x^2}}}{\_R1} \right) + \mathrm{d}}{\_R1^2 c^2 d + c^2 d + 2\mathrm{d}} \right.}{2}$
derivativedivides	$\frac{ac \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}} + bc^2 \left(\frac{i \left(\sum_{\_R1=\mathrm{RootOf}\left(c^2 d \_Z^4 + \left(2c^2 d+4e\right) \_Z^2 + c^2 d\right)} \frac{\_R1 \left(i \operatorname{arcsec}(cx) \ln\left(\frac{\_R1 - \frac{1}{cx} - i\sqrt{1-\frac{1}{c^2x^2}}}{\_R1} \right) +}{\_R1^2 c^2 d + c^2 d +} \right.}{2} \right.$
default	$\frac{ac \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}} + bc^2 \left(\frac{i \left(\sum_{\_R1=\mathrm{RootOf}\left(c^2 d \_Z^4 + \left(2c^2 d+4e\right) \_Z^2 + c^2 d\right)} \frac{\_R1 \left(i \operatorname{arcsec}(cx) \ln\left(\frac{\_R1 - \frac{1}{cx} - i\sqrt{1-\frac{1}{c^2x^2}}}{\_R1} \right) +}{\_R1^2 c^2 d + c^2 d +} \right.}{2} \right.$

input
```
int((a+b*arcsec(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
a/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+1/2*I*b*c*sum(_R1/(_R1^2*c^2*d+c^2*d
+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-
1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^
2+c^2*d))-1/2*I*b*c*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_
R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2
))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))
```

3.93.5 Fricas [F]

$$\int \frac{a + b \sec^{-1}(cx)}{d + ex^2}\, dx = \int \frac{b \operatorname{arcsec}(cx) + a}{ex^2 + d}\, dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arcsec(c*x) + a)/(e*x^2 + d), x)
```

3.93.6 Sympy [F]

$$\int \frac{a + b \sec^{-1}(cx)}{d + ex^2}\, dx = \int \frac{a + b \operatorname{asec}(cx)}{d + ex^2}\, dx$$

input
```
integrate((a+b*asec(c*x))/(e*x**2+d),x)
```

output
```
Integral((a + b*asec(c*x))/(d + e*x**2), x)
```

3.93.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b \sec^{-1}(cx)}{d + ex^2}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.93.8 Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \sec^{-1}(cx)}{d + ex^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \sec^{-1}(cx)}{d + ex^2}\,dx = \int \frac{a + b \operatorname{acos}\left(\frac{1}{c\,x}\right)}{e\,x^2 + d}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(d + e*x^2),x)
```

output
```
int((a + b*acos(1/(c*x)))/(d + e*x^2), x)
```

3.94 $\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)}\,dx$

3.94.1 Optimal result 699
3.94.2 Mathematica [A] (verified) 700
3.94.3 Rubi [A] (verified) 701
3.94.4 Maple [C] (warning: unable to verify) 703
3.94.5 Fricas [F] 704
3.94.6 Sympy [F] 704
3.94.7 Maxima [F] 704
3.94.8 Giac [F(-2)] 705
3.94.9 Mupad [F(-1)] 705

3.94.1 Optimal result

Integrand size = 21, antiderivative size = 459

$$\begin{aligned}\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)}\,dx = {}& \frac{i\left(a+b\sec^{-1}(cx)\right)^2}{2bd} - \frac{\left(a+b\sec^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} \\ & - \frac{\left(a+b\sec^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} \\ & - \frac{\left(a+b\sec^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d} \\ & - \frac{\left(a+b\sec^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d} \\ & + \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} + \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} \\ & + \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d} + \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}\end{aligned}$$

output
```
1/2*I*(a+b*arcsec(c*x))^2/b/d-1/2*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c
^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d-1/2*(a+b*arcsec(c*x
))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2
)))/d-1/2*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2
)/(e^(1/2)+(c^2*d+e)^(1/2)))/d-1/2*(a+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1/
c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d+1/2*I*b*polylog(2,
-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d+1
/2*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*
d+e)^(1/2)))/d+1/2*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/
2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d+1/2*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)
^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d
```

3.94.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 876, normalized size of antiderivative = 1.91

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)}\,dx$$

$$= \frac{i\left(b\sec^{-1}(cx)^2-4b\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\tan\left(\frac{1}{2}\sec^{-1}(cx)\right)}{\sqrt{c^2d+e}}\right)-4b\arcsin\left(\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\right.}{}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x*(d + e*x^2)),x]
```

output

```
((I/2)*(b*ArcSec[c*x]^2 - 4*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqr
t[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d +
e]] - 4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*S
qrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] + I*b*ArcSec[c*x]*L
og[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (2
*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e
] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + I*b*ArcSec[c*x]*Log
[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (2*
I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e
] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + I*b*ArcSec[c*x]*Log
[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (2*I
)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e]
+ Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + I*b*ArcSec[c*x]*Log[1
 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (2*I)*
b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] +
Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (2*I)*a*Log[x] + I*a*Lo
g[d + e*x^2] + b*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec
[c*x]))/(c*Sqrt[d])] + b*PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*A
rcSec[c*x]))/(c*Sqrt[d])] + b*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e])
*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + b*PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*...
```

3.94.3 Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5763, 5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \sec^{-1}(cx)}{x\left(d + ex^2\right)}\,dx$$

$$\downarrow 5763$$

$$-\int \frac{a + b \arccos\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2} + e\right)x} d\frac{1}{x}$$

$$\downarrow 5233$$

$$-\int \left(\frac{\sqrt{-d}\left(a + b \arccos\left(\frac{1}{cx}\right)\right)}{2d\left(\frac{\sqrt{-d}}{x} + \sqrt{e}\right)} - \frac{\sqrt{-d}\left(a + b \arccos\left(\frac{1}{cx}\right)\right)}{2d\left(\sqrt{e} - \frac{\sqrt{-d}}{x}\right)}\right) d\frac{1}{x}$$

$$\downarrow 2009$$

$$-\frac{(a+b\arccos\left(\frac{1}{cx}\right))\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}-\frac{(a+b\arccos\left(\frac{1}{cx}\right))\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}-$$
$$\frac{(a+b\arccos\left(\frac{1}{cx}\right))\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d}-\frac{(a+b\arccos\left(\frac{1}{cx}\right))\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d}+$$
$$\frac{i\left(a+b\arccos\left(\frac{1}{cx}\right)\right)^2}{2bd}+\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d}+$$
$$\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d}$$

input
```
Int[(a + b*ArcSec[c*x])/(x*(d + e*x^2)),x]
```

output
```
((I/2)*(a + b*ArcCos[1/(c*x)])^2)/(b*d) - ((a + b*ArcCos[1/(c*x)])*Log[1 -
 (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d) -
((a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[
e] - Sqrt[c^2*d + e])])/(2*d) - ((a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-
d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d) - ((a + b*Ar
cCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[
c^2*d + e])])/(2*d) + ((I/2)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcCos[1/(c*x
)]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/d + ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^
(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/d + ((I/2)*b*PolyLog[2,
 -((c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e]))])/d + (
(I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*
d + e])])/d
```

3.94.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

3.94.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.33 (sec) , antiderivative size = 1970, normalized size of antiderivative = 4.29

method	result	size
parts	Expression too large to display	1970
derivativedivides	Expression too large to display	1997
default	Expression too large to display	1997

input
```
int((a+b*arcsec(c*x))/x/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
a/d*ln(x)-1/2*a/d*ln(e*x^2+d)+b*(-1/2*I*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*
d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*e*polylog(2,d*c^2*(1/c/x+I*(1-1/c^2/x^2
)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))/d^3/(c^2*d+e)/c^4+1/2*I/d*s
um((_R1^2*c^2*d+2*c^2*d+4*e)/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R
1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2)
)/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-1/4*I*(e*(c^2*d+e
))^(1/2)/e/(c^2*d+e)*arcsec(c*x)^2*c^2+1/2*I*(c^2*d-2*(e*(c^2*d+e))^(1/2)+
2*e)*polylog(2,d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e
))^(1/2)-2*e))*e/d^3/c^4-1/4*I*(e*(c^2*d+e))^(1/2)/d/(c^2*d+e)*polylog(2,d
*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))+1
/4*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)/e/
(c^2*d+e)/d*ln(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d-2*(e*(c^2*d
+e))^(1/2)-2*e))*arcsec(c*x)+1/4*(e*(c^2*d+e))^(1/2)/e/(c^2*d+e)*c^2*arcse
c(c*x)*ln(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^
(1/2)-2*e))+1/2*I*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*arcsec(c*x)^2/c^2/d^2+
1/4*I*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*polylog(2,d*c^2*(1/c/x+I*(1-1/c^2/
x^2)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))/c^2/d^2+(-(e*(c^2*d+e))^
(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)/d^3*e/(c^2*d+e)/c^4*l
n(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*
e))*arcsec(c*x)-1/2*I*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d...
```

3.94.5 Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)}\,dx = \int \frac{b\operatorname{arcsec}(cx)+a}{(ex^2+d)x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arcsec(c*x) + a)/(e*x^3 + d*x), x)
```

3.94.6 Sympy [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)}\,dx = \int \frac{a+b\operatorname{asec}(cx)}{x\left(d+ex^2\right)}\,dx$$

input
```
integrate((a+b*asec(c*x))/x/(e*x**2+d),x)
```

output
```
Integral((a + b*asec(c*x))/(x*(d + e*x**2)), x)
```

3.94.7 Maxima [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)}\,dx = \int \frac{b\operatorname{arcsec}(cx)+a}{(ex^2+d)x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d),x, algorithm="maxima")
```

output
```
-1/2*a*(log(e*x^2 + d)/d - 2*log(x)/d) + b*integrate(arctan(sqrt(c*x + 1)*
sqrt(c*x - 1))/(e*x^3 + d*x), x)
```

3.94.8 Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \sec^{-1}(cx)}{x\left(d + ex^2\right)}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \sec^{-1}(cx)}{x\left(d + ex^2\right)}\, dx = \int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{x\,\left(e\,x^2 + d\right)}\, dx$$

input
```
int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)), x)
```

3.95 $\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)}\,dx$

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3.95.2 Mathematica [A] (verified) . . . 707
3.95.3 Rubi [A] (verified) . . . 708
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3.95.7 Maxima [F(-2)] . . . 712
3.95.8 Giac [F(-2)] . . . 713
3.95.9 Mupad [F(-1)] . . . 713

3.95.1 Optimal result

Integrand size = 21, antiderivative size = 551

$$\begin{aligned}\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)}\,dx = {}& \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d}-\frac{a}{dx}-\frac{b\sec^{-1}(cx)}{dx}\\ &+\frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}\\ &-\frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}\\ &+\frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}\\ &-\frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}\\ &+\frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}\\ &-\frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}\\ &+\frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}\\ &-\frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}\end{aligned}$$

output

```
-a/d/x-b*arcsec(c*x)/d/x+1/2*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^
2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*(a+
b*arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c
^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(
1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/
2)-1/2*(a+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(
e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*I*b*polylog(2,-c*(1/c/x+I
*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(
3/2)-1/2*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)
-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*I*b*polylog(2,-c*(1/c/x+I*(1-1/c
^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/
2*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d
+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+b*c*(1-1/c^2/x^2)^(1/2)/d
```

3.95.2 Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 997, normalized size of antiderivative = 1.81

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)}\,dx = -\frac{a}{dx} - \frac{a\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}} + b\left(\frac{c\sqrt{1-\frac{1}{c^2x^2}}-\frac{\sec^{-1}(cx)}{x}}{d}\right.$$

$$-\frac{\sqrt{e}\left(8\arcsin\left(\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(ic\sqrt{d}+\sqrt{e}\right)\tan\left(\frac{1}{2}\sec^{-1}(cx)\right)}{\sqrt{c^2d+e}}\right)-2i\sec^{-1}(cx)\log\left(1+\frac{i\left(\sqrt{e}-\sqrt{c^2d+e}\right)e^{i\sec^{-1}(c}}{c\sqrt{d}}\right.\right.}$$

$$+\frac{\sqrt{e}\left(8\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\tan\left(\frac{1}{2}\sec^{-1}(cx)\right)}{\sqrt{c^2d+e}}\right)-2i\sec^{-1}(cx)\log\left(1+\frac{i\left(-\sqrt{e}+\sqrt{c^2d+e}\right)e^{i\sec^{-1}}}{c\sqrt{d}}\right.\right.}$$

input

```
Integrate[(a + b*ArcSec[c*x])/(x^2*(d + e*x^2)),x]
```

output

```
-(a/(d*x)) - (a*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2) + b*((c*Sqrt[
1 - 1/(c^2*x^2)] - ArcSec[c*x]/x)/d - (Sqrt[e]*(8*ArcSin[Sqrt[1 + (I*Sqrt[
e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/
2])/Sqrt[c^2*d + e]] - (2*I)*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d
+ e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/
(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c
*x]))/(c*Sqrt[d])] - (2*I)*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d +
e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c
*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x
]))/(c*Sqrt[d])] + (2*I)*ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] - 2*Po
lyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] -
 2*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt
[d])] + PolyLog[2, -E^((2*I)*ArcSec[c*x])]))/(4*d^(3/2)) + (Sqrt[e]*(8*Arc
Sin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + S
qrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] - (2*I)*ArcSec[c*x]*Log[1 + (
I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (4*I)*Arc
Sin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt
[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (2*I)*ArcSec[c*x]*Log[1 - (
I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (4*I)*ArcS
in[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqr...
```

3.95.3 Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 603, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5763, 5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \sec^{-1}(cx)}{x^2 \left(d + ex^2\right)} \, dx$$

$$\downarrow 5763$$

$$-\int \frac{a + b \arccos\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2} + e\right) x^2} d\frac{1}{x}$$

$$\downarrow 5233$$

$$-\int \left(\frac{a + b \arccos\left(\frac{1}{cx}\right)}{d} - \frac{e\left(a + b \arccos\left(\frac{1}{cx}\right)\right)}{d\left(\frac{d}{x^2} + e\right)} \right) d\frac{1}{x}$$

$$\downarrow 2009$$

$$\frac{\sqrt{e}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}-\frac{\sqrt{e}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}+\frac{\sqrt{e}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2(-d)^{3/2}}-\frac{\sqrt{e}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2(-d)^{3/2}}-\frac{a}{dx}+\frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}}-\frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}}+\frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}}-\frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}}-\frac{b\arccos\left(\frac{1}{cx}\right)}{dx}+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d}$$

input

```
Int[(a + b*ArcSec[c*x])/(x^2*(d + e*x^2)),x]
```

output

```
(b*c*Sqrt[1 - 1/(c^2*x^2)])/d - a/(d*x) - (b*ArcCos[1/(c*x)])/(d*x) + (Sqr
t[e]*(a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(S
qrt[e] - Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcCos[1/(c*x
)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])
])/(2*(-d)^(3/2)) + (Sqrt[e]*(a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E
^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) - (Sqrt
[e]*(a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sq
rt[e] + Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) + ((I/2)*b*Sqrt[e]*PolyLog[2, -(
(c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/(-d)^(3/
2) - ((I/2)*b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[
e] - Sqrt[c^2*d + e])])/(-d)^(3/2) + ((I/2)*b*Sqrt[e]*PolyLog[2, -((c*Sqrt
[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e]))])/(-d)^(3/2) - ((
I/2)*b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sq
rt[c^2*d + e])])/(-d)^(3/2)
```

3.95.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

3.95.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 27.44 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.60

method	result
parts	$-\frac{a}{dx}-\frac{ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{d\sqrt{de}}+\frac{bc\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{d}-\frac{b\operatorname{arcsec}(cx)}{dx}-\frac{ibce\left(\sum_{\_R1=\operatorname{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\right.$
derivativedivides	$c\left(-\frac{a}{dcx}-\frac{ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{cd\sqrt{de}}+\frac{b\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{d}-\frac{b\operatorname{arcsec}(cx)}{dcx}-\frac{ibe\left(\sum_{\_R1=\operatorname{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\right.\right.$
default	$c\left(-\frac{a}{dcx}-\frac{ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{cd\sqrt{de}}+\frac{b\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{d}-\frac{b\operatorname{arcsec}(cx)}{dcx}-\frac{ibe\left(\sum_{\_R1=\operatorname{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\right.\right.$

input
```
int((a+b*arcsec(c*x))/x^2/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-a/d/x-a*e/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b*c/d*((c^2*x^2-1)/c^2/x^
2)^(1/2)-b*arcsec(c*x)/d/x-1/2*I*b*c*e/d*sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(
I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I
*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d
))+1/2*I*b*c*e/d*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-
1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/
_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))
```

3.95.5 Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)}\,dx = \int \frac{b\operatorname{arcsec}(cx)+a}{(ex^2+d)x^2}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arcsec(c*x) + a)/(e*x^4 + d*x^2), x)
```

3.95.6 Sympy [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)}\,dx = \int \frac{a+b\operatorname{asec}(cx)}{x^2\left(d+ex^2\right)}\,dx$$

input
```
integrate((a+b*asec(c*x))/x**2/(e*x**2+d),x)
```

output
```
Integral((a + b*asec(c*x))/(x**2*(d + e*x**2)), x)
```

3.95.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.95.8 Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \sec^{-1}(cx)}{x^2 \left(d + ex^2\right)} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \sec^{-1}(cx)}{x^2 \left(d + ex^2\right)} \, dx = \int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{x^2 \left(e\,x^2 + d\right)} \, dx$$

input
```
int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)), x)
```

3.96 $\int \frac{x^5\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

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3.96.1 Optimal result

Integrand size = 21, antiderivative size = 608

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = -\frac{b\sqrt{1-\frac{1}{c^2x^2}}x}{2ce^2} + \frac{d(a+b\sec^{-1}(cx))}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2(a+b\sec^{-1}(cx))}{2e^2} + \frac{bd\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}} - \frac{d(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3} - \frac{d(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3} - \frac{d(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3} - \frac{d(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3} + \frac{2d(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e^3} + \frac{ibd\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3} + \frac{ibd\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3} + \frac{ibd\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3} + \frac{ibd\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3} - \frac{ibd\,\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{e^3}$$

output

```
1/2*d*(a+b*arcsec(c*x))/e^2/(e+d/x^2)+1/2*x^2*(a+b*arcsec(c*x))/e^2+2*d*(a
+b*arcsec(c*x))*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)/e^3-d*(a+b*arcsec(c*
x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/
2)))/e^3-d*(a+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/
2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3-d*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1
/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3-d*(a+b*arcsec(c
*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1
/2)))/e^3-I*b*d*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)/e^3+I*b*d*poly
log(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)
))/e^3+I*b*d*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)
-(c^2*d+e)^(1/2)))/e^3+I*b*d*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-
d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3+I*b*d*polylog(2,c*(1/c/x+I*(1-1/c^
2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3+1/2*b*d*arctan((c^
2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2)^(1/2))/e^(5/2)/(c^2*d+e)^(1/2)-1/2*
b*x*(1-1/c^2/x^2)^(1/2)/c/e^2
```

3.96.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1255 vs. $2(608) = 1216$.

Time $= 3.18$ (sec) , antiderivative size $= 1255$, normalized size of antiderivative $= 2.06$

$$\int \frac{x^5(a + b\sec^{-1}(cx))}{(d + ex^2)^2}\,dx = \text{Too large to display}$$

input

```
Integrate[(x^5*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

output

```
-1/4*(-2*a*e*x^2 + (2*a*d^2)/(d + e*x^2) + 4*a*d*Log[d + e*x^2] + b*((2*e*
Sqrt[1 - 1/(c^2*x^2)]*x)/c - 2*e*x^2*ArcSec[c*x] + (d^(3/2)*ArcSec[c*x])/(
Sqrt[d] - I*Sqrt[e]*x) + (d^(3/2)*ArcSec[c*x])/(Sqrt[d] + I*Sqrt[e]*x) + 2
*d*ArcSin[1/(c*x)] + (16*I)*d*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqr
t[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d +
e]] + (16*I)*d*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((
I*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] + 4*d*ArcSec[c
*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])]
 + 8*d*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[
e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 4*d*ArcSec[c*x]*Lo
g[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 8*
d*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] +
 Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 4*d*ArcSec[c*x]*Log[1
- (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 8*d*Arc
Sin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[
c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 4*d*ArcSec[c*x]*Log[1 + (I*(
Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 8*d*ArcSin[Sq
rt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d
+ e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 8*d*ArcSec[c*x]*Log[1 + E^((2*I)*A
rcSec[c*x])] - (d*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt...
```

3.96.3 Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 676, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5763, 5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

$$\downarrow 5763$$

$$-\int \frac{x^3\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{\left(\frac{d}{x^2}+e\right)^2}d\frac{1}{x}$$

$$\downarrow 5233$$

$$-\int\left(\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)x^3}{e^2}-\frac{2d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)x}{e^3}+\frac{2d^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e^3\left(\frac{d}{x^2}+e\right)x}+\frac{d^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e^2\left(\frac{d}{x^2}+e\right)^2x}\right)d\frac{1}{x}$$

$$\downarrow 2009$$

$$-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{e^3}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{e^3}+\frac{2d\log\left(1+e^{2i\arccos\left(\frac{1}{cx}\right)}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e^3}+\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{2e^2\left(\frac{d}{x^2}+e\right)}+\frac{x^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{2e^2}+\frac{ibd\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{e^3}+\frac{ibd\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{e^3}+\frac{ibd\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{e^3}+\frac{ibd\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{e^3}-\frac{ibd\,\mathrm{PolyLog}\left(2,-e^{2i\arccos\left(\frac{1}{cx}\right)}\right)}{e^3}+\frac{bd\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2e^{5/2}\sqrt{c^2d+e}}-\frac{bx\sqrt{1-\frac{1}{c^2x^2}}}{2ce^2}$$

input
```
Int[(x^5*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/2*(b*Sqrt[1 - 1/(c^2*x^2)]*x)/(c*e^2) + (d*(a + b*ArcCos[1/(c*x)]))/(2*
e^2*(e + d/x^2)) + (x^2*(a + b*ArcCos[1/(c*x)]))/(2*e^2) + (b*d*ArcTan[Sqr
t[c^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)])/(2*e^(5/2)*Sqrt[c^2*d +
 e]) - (d*(a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]
))/(Sqrt[e] - Sqrt[c^2*d + e])])/e^3 - (d*(a + b*ArcCos[1/(c*x)])*Log[1 +
(c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/e^3 - (d*
(a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e
] + Sqrt[c^2*d + e])])/e^3 - (d*(a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d
]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 + (2*d*(a + b*A
rcCos[1/(c*x)])*Log[1 + E^((2*I)*ArcCos[1/(c*x)])])/e^3 + (I*b*d*PolyLog[2
, -((c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/e^3
+ (I*b*d*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2
*d + e])])/e^3 + (I*b*d*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(S
qrt[e] + Sqrt[c^2*d + e]))])/e^3 + (I*b*d*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcC
os[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 - (I*b*d*PolyLog[2, -E^((2
*I)*ArcCos[1/(c*x)])])/e^3
```

3.96. $\int \frac{x^5\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

3.96.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

3.96.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 9.59 (sec) , antiderivative size = 701, normalized size of antiderivative = 1.15

method	result
parts	$\frac{a\,x^2}{2e^2}-\frac{a\,d^2}{2e^3\left(e\,x^2+d\right)}-\frac{ad\ln\left(e\,x^2+d\right)}{e^3}+b\left(\frac{c^4\left(2c^4d\,\operatorname{arcsec}(cx)x^2+\operatorname{arcsec}(cx)e\,c^4x^4-\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3dx-\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,e\,c^3x^3-\right.}{2\left(c^2e\,x^2+c^2d\right)e^2}\right.$
derivativedivides	$\frac{a\,c^6x^2}{2e^2}-\frac{a\,c^8d^2}{2e^3\left(c^2e\,x^2+c^2d\right)}-\frac{a\,c^6d\ln\left(c^2e\,x^2+c^2d\right)}{e^3}+bc^4\left(\frac{2c^4d\,\operatorname{arcsec}(cx)x^2+\operatorname{arcsec}(cx)e\,c^4x^4-\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3dx-\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,e\,c^3x\right.}{2\left(c^2e\,x^2+c^2d\right)e^2}\right.$
default	$\frac{a\,c^6x^2}{2e^2}-\frac{a\,c^8d^2}{2e^3\left(c^2e\,x^2+c^2d\right)}-\frac{a\,c^6d\ln\left(c^2e\,x^2+c^2d\right)}{e^3}+bc^4\left(\frac{2c^4d\,\operatorname{arcsec}(cx)x^2+\operatorname{arcsec}(cx)e\,c^4x^4-\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3dx-\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,e\,c^3x\right.}{2\left(c^2e\,x^2+c^2d\right)e^2}\right.$

input
```
int(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2/e^2-1/2*a*d^2/e^3/(e*x^2+d)-a*d/e^3*ln(e*x^2+d)+b/c^6*(1/2*c^4*(
2*c^4*d*arcsec(c*x)*x^2+arcsec(c*x)*e*c^4*x^4-((c^2*x^2-1)/c^2/x^2)^(1/2)*
c^3*d*x-((c^2*x^2-1)/c^2/x^2)^(1/2)*e*c^3*x^3-I*c^2*d-I*e*c^2*x^2)/(c^2*e*
x^2+c^2*d)/e^2-1/2*I*(e*(c^2*d+e))^(1/2)/(c^2*d+e)/e^3*arctanh(1/4*(2*c^2*
d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2+2*c^2*d+4*e)/(c^2*d*e+e^2)^(1/2))*d*c^6-
2*I/e^3*d*c^6*dilog(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))+1/2*I/e^3*d^2*c^8*s
um((_R1^2+1)/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c
^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=Root
Of(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/2*I/e^3*d*c^6*sum((_R1^2*c^2*d+
c^2*d+4*e)/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2
/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf
(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-2*I/e^3*d*c^6*dilog(1+I*(1/c/x+I*(1
-1/c^2/x^2)^(1/2)))+2/e^3*d*c^6*arcsec(c*x)*ln(1+I*(1/c/x+I*(1-1/c^2/x^2)^
(1/2)))+2/e^3*d*c^6*arcsec(c*x)*ln(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2))))
```

3.96.5 Fricas [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x^5}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^5*arcsec(c*x) + a*x^5)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.96.6 Sympy [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^5(a+b\operatorname{asec}(cx))}{(d+ex^2)^2}\,dx$$

input
```
integrate(x**5*(a+b*asec(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral(x**5*(a + b*asec(c*x))/(d + e*x**2)**2, x)
```

3.96.7 Maxima [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\,\text{arcsec}\,(cx)+a)x^5}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/2*a*(d^2/(e^4*x^2 + d*e^3) - x^2/e^2 + 2*d*log(e*x^2 + d)/e^3) + b*inte
grate(x^5*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^4 + 2*d*e*x^2 + d^2),
 x)
```

3.96.8 Giac [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Timed out
```

3.96.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^5\left(a+b\,\text{acos}\left(\frac{1}{cx}\right)\right)}{(e\,x^2+d)^2}\,dx$$

input
```
int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^2,x)
```

output
```
int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^2, x)
```

3.97 $\int \frac{x^3\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

3.97.1 Optimal result 724
3.97.2 Mathematica [B] (warning: unable to verify) 725
3.97.3 Rubi [A] (verified) 726
3.97.4 Maple [C] (warning: unable to verify) 728
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3.97.8 Giac [F(-1)] 731
3.97.9 Mupad [F(-1)] 731

3.97.1 Optimal result

Integrand size = 21, antiderivative size = 570

$$\begin{aligned}\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx &= -\frac{a+b\sec^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} - \frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{3/2}\sqrt{c^2d+e}} \\ &+ \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2} \\ &+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2} \\ &+ \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2} \\ &+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2} \\ &- \frac{(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e^2} \\ &- \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2} \\ &- \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2} \\ &- \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2} \\ &- \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2} + \frac{ib\,\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2e^2}\end{aligned}$$

output
```
1/2*(-a-b*arcsec(c*x))/e/(e+d/x^2)-(a+b*arcsec(c*x))*ln(1+(1/c/x+I*(1-1/c^
2/x^2)^(1/2))^2)/e^2+1/2*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(
1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2+1/2*(a+b*arcsec(c*x))*ln(1
+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2
+1/2*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^
(1/2)+(c^2*d+e)^(1/2)))/e^2+1/2*(a+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1/c^2
/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^2+1/2*I*b*polylog(2,-
(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)/e^2-1/2*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^
2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2-1/2*I*b*polylog(2,
c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2-
1/2*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^
2*d+e)^(1/2)))/e^2-1/2*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^
(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^2-1/2*b*arctan((c^2*d+e)^(1/2)/c/x/e^(1
/2)/(1-1/c^2/x^2)^(1/2))/e^(3/2)/(c^2*d+e)^(1/2)
```

3.97.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1213 vs. $2(570) = 1140$.

Time $= 1.17$ (sec) , antiderivative size $= 1213$, normalized size of antiderivative $= 2.13$

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx$$

$$= \frac{\frac{2ad}{d+ex^2} + \frac{b\sqrt{d}\sec^{-1}(cx)}{\sqrt{d}-i\sqrt{e}x} + \frac{b\sqrt{d}\sec^{-1}(cx)}{\sqrt{d}+i\sqrt{e}x} + 2b\arcsin\left(\frac{1}{cx}\right) + 8ib\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\tan\left(\frac{1}{2}\sec^{-1}(cx\right.}{\sqrt{c^2d+e}}\right.}$$

input
```
Integrate[(x^3*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

output

```
((2*a*d)/(d + e*x^2) + (b*Sqrt[d]*ArcSec[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (
b*Sqrt[d]*ArcSec[c*x])/(Sqrt[d] + I*Sqrt[e]*x) + 2*b*ArcSin[1/(c*x)] + (8*
I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqr
t[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] + (8*I)*b*ArcSin[Sqrt
[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Tan
[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] + 2*b*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] -
Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 4*b*ArcSin[Sqrt[1 + (I*
Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I
*ArcSec[c*x]))/(c*Sqrt[d])] + 2*b*ArcSec[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[
c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 4*b*ArcSin[Sqrt[1 - (I*Sqrt[
e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*Arc
Sec[c*x]))/(c*Sqrt[d])] + 2*b*ArcSec[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d
 + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 4*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(
c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*
x]))/(c*Sqrt[d])] + 2*b*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])
*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt
[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(
c*Sqrt[d])] - 4*b*ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] - (b*Sqrt[e]*
Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt
[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))])/S...
```

3.97.3 Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 634, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5763, 5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

$$\downarrow 5763$$

$$-\int \frac{x\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{\left(\frac{d}{x^2}+e\right)^2}d\frac{1}{x}$$

$$\downarrow 5233$$

$$-\int \left(\frac{x\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e^2}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e^2\left(\frac{d}{x^2}+e\right)x}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e\left(\frac{d}{x^2}+e\right)^2x}\right)d\frac{1}{x}$$

$$\downarrow 2009$$

$$\frac{(a+b\arccos\left(\frac{1}{cx}\right))\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}+\frac{(a+b\arccos\left(\frac{1}{cx}\right))\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}+$$
$$\frac{(a+b\arccos\left(\frac{1}{cx}\right))\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^2}+\frac{(a+b\arccos\left(\frac{1}{cx}\right))\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^2}-$$
$$\frac{a+b\arccos\left(\frac{1}{cx}\right)}{2e\left(\frac{d}{x^2}+e\right)}-\frac{\log\left(1+e^{2i\arccos\left(\frac{1}{cx}\right)}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e^2}-\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^2}-$$
$$\frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^2}-\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^2}-$$
$$\frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^2}+\frac{ib\,\mathrm{PolyLog}\left(2,-e^{2i\arccos\left(\frac{1}{cx}\right)}\right)}{2e^2}-\frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2e^{3/2}\sqrt{c^2d+e}}$$

input
```
Int[(x^3*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/2*(a + b*ArcCos[1/(c*x)])/(e*(e + d/x^2)) - (b*ArcTan[Sqrt[c^2*d + e]/(
c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)])/(2*e^(3/2)*Sqrt[c^2*d + e]) + ((a + b
*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sq
rt[c^2*d + e])])/(2*e^2) + ((a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^
(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^2) + ((a + b*ArcCo
s[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2
*d + e])])/(2*e^2) + ((a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*Arc
Cos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^2) - ((a + b*ArcCos[1/(c
*x)])*Log[1 + E^((2*I)*ArcCos[1/(c*x)])])/e^2 - ((I/2)*b*PolyLog[2, -((c*S
qrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/e^2 - ((I/2)
*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e
])])/e^2 - ((I/2)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[
e] + Sqrt[c^2*d + e]))])/e^2 - ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos
[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^2 + ((I/2)*b*PolyLog[2, -E^((2
*I)*ArcCos[1/(c*x)])])/e^2
```

3.97.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

3.97.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.81 (sec) , antiderivative size = 577, normalized size of antiderivative = 1.01

method	result
parts	$\frac{ad}{2e^2(ex^2+d)}+\frac{a\ln(ex^2+d)}{2e^2}-\frac{bc^2x^2\operatorname{arcsec}(cx)}{2(c^2ex^2+c^2d)e}+\frac{ib\sqrt{e(c^2d+e)}\,\operatorname{arctanh}\left(\frac{2c^2d\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2+2c^2d+4e}{4\sqrt{c^2de+e^2}}\right)}{2(c^2d+e)e^2}+$
derivativedivides	$\frac{ac^6d}{2e^2\left(c^2ex^2+c^2d\right)}+\frac{ac^4\ln\left(c^2ex^2+c^2d\right)}{2e^2}+bc^4\left(-\frac{c^2x^2\operatorname{arcsec}(cx)}{2\left(c^2ex^2+c^2d\right)e}-\frac{ic^2d\left(\sum_{\_R1=\operatorname{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\right.\right.$
default	$\frac{ac^6d}{2e^2\left(c^2ex^2+c^2d\right)}+\frac{ac^4\ln\left(c^2ex^2+c^2d\right)}{2e^2}+bc^4\left(-\frac{c^2x^2\operatorname{arcsec}(cx)}{2\left(c^2ex^2+c^2d\right)e}-\frac{ic^2d\left(\sum_{\_R1=\operatorname{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\right.\right.$

input
```
int(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*d/e^2/(e*x^2+d)+1/2*a/e^2*ln(e*x^2+d)-1/2*b*c^2*x^2*arcsec(c*x)/(c^2
*e*x^2+c^2*d)/e+1/2*I*b*(e*(c^2*d+e))^(1/2)/(c^2*d+e)/e^2*arctanh(1/4*(2*c
^2*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2+2*c^2*d+4*e)/(c^2*d*e+e^2)^(1/2))+I*b
/e^2*dilog(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-1/4*I*b*c^2/e^2*d*sum((_R1^2
+1)/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(
1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*
_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-1/4*I*b/e^2*sum((_R1^2*c^2*d+c^2*d+4*e)/(_
R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/
_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(
2*c^2*d+4*e)*_Z^2+c^2*d))+I*b/e^2*dilog(1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))
-b/e^2*arcsec(c*x)*ln(1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-b/e^2*arcsec(c*x)
*ln(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))
```

3.97. $\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx$

3.97.5 Fricas [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\,\text{arcsec}\,(cx)+a)x^3}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^3*arcsec(c*x) + a*x^3)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.97.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*asec(c*x))/(e*x**2+d)**2,x)
```

output
```
Timed out
```

3.97.7 Maxima [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\,\text{arcsec}\,(cx)+a)x^3}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a*(d/(e^3*x^2 + d*e^2) + log(e*x^2 + d)/e^2) + b*integrate(x^3*arctan(
sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.97.8 Giac [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Timed out
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^3\left(a+b\,\text{acos}\left(\frac{1}{cx}\right)\right)}{(e\,x^2+d)^2}\,dx$$

input
```
int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^2,x)
```

output
```
int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^2, x)
```

3.98 $\int \frac{x\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

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3.98.1 Optimal result

Integrand size = 19, antiderivative size = 131

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = -\frac{a+b\sec^{-1}(cx)}{2e\,(d+ex^2)} + \frac{bcx\arctan\left(\sqrt{-1+c^2x^2}\right)}{2de\sqrt{c^2x^2}} - \frac{bcx\arctan\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{\sqrt{c^2d+e}}\right)}{2d\sqrt{e}\sqrt{c^2d+e}\sqrt{c^2x^2}}$$

output

```
1/2*(-a-b*arcsec(c*x))/e/(e*x^2+d)+1/2*b*c*x*arctan((c^2*x^2-1)^(1/2))/d/e
/(c^2*x^2)^(1/2)-1/2*b*c*x*arctan(e^(1/2)*(c^2*x^2-1)^(1/2)/(c^2*d+e)^(1/2
))/d/e^(1/2)/(c^2*d+e)^(1/2)/(c^2*x^2)^(1/2)
```

3.98.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.18

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \frac{-\frac{2a}{d+ex^2} - \frac{2b\sec^{-1}(cx)}{d+ex^2} - \frac{2b\arcsin\left(\frac{1}{cx}\right)}{d} + \frac{b\sqrt{e}\log\left(\frac{4ide+4cd\sqrt{e}\left(c\sqrt{d}-i\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x}{b\sqrt{-c^2d-e}\left(\sqrt{d}+i\sqrt{e}x\right)}\right)}{d\sqrt{-c^2d-e}} + \frac{b\sqrt{e}\log\left(\frac{-4ide+4cd\sqrt{e}\left(c\sqrt{d}+i\sqrt{-c^2d-e}\right)}{b\sqrt{-c^2d-e}\left(\sqrt{d}-i\sqrt{e}\right)}\right)}{d\sqrt{-c^2d-e}}}{4e}$$

input
```
Integrate[(x*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

output
```
((-2*a)/(d + e*x^2) - (2*b*ArcSec[c*x])/(d + e*x^2) - (2*b*ArcSin[1/(c*x)]
)/d + (b*Sqrt[e]*Log[((4*I)*d*e + 4*c*d*Sqrt[e]*(c*Sqrt[d] - I*Sqrt[-(c^2*
d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x)/(b*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[
e]*x))])/(d*Sqrt[-(c^2*d) - e]) + (b*Sqrt[e]*Log[((-4*I)*d*e + 4*c*d*Sqrt[
e]*(c*Sqrt[d] + I*Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x)/(b*Sqrt[-(c
^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))])/(d*Sqrt[-(c^2*d) - e]))/(4*e)
```

3.98.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5759, 354, 97, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

$$\downarrow 5759$$

$$\frac{bcx\int \frac{1}{x\sqrt{c^2x^2-1}(ex^2+d)}dx}{2e\sqrt{c^2x^2}} - \frac{a+b\sec^{-1}(cx)}{2e\left(d+ex^2\right)}$$

$$\downarrow 354$$

$$\frac{bcx\int \frac{1}{x^2\sqrt{c^2x^2-1}(ex^2+d)}dx^2}{4e\sqrt{c^2x^2}} - \frac{a+b\sec^{-1}(cx)}{2e\left(d+ex^2\right)}$$

$$\downarrow 97$$

$$\frac{bcx\left(\frac{\int \frac{1}{x^2\sqrt{c^2x^2-1}}dx^2}{d} - \frac{e\int \frac{1}{\sqrt{c^2x^2-1}\left(ex^2+d\right)}dx^2}{d}\right)}{4e\sqrt{c^2x^2}} - \frac{a+b\sec^{-1}(cx)}{2e\left(d+ex^2\right)}$$

$$\downarrow 73$$

$$\frac{bcx\left(\frac{2\int \frac{1}{\frac{ex^4}{c^2}+\frac{1}{c^2}}d\sqrt{c^2x^2-1}}{c^2d} - \frac{2e\int \frac{1}{\frac{ex^4}{c^2}+d+\frac{e}{c^2}}d\sqrt{c^2x^2-1}}{c^2d}\right)}{4e\sqrt{c^2x^2}} - \frac{a+b\sec^{-1}(cx)}{2e\left(d+ex^2\right)}$$

$$\downarrow 218$$

$$\frac{bcx\left(\frac{2\arctan\left(\sqrt{c^2x^2-1}\right)}{d}-\frac{2\sqrt{e}\arctan\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{d\sqrt{c^2d+e}}\right)}{4e\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{2e\left(d+ex^2\right)}$$

input
```
Int[(x*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/2*(a + b*ArcSec[c*x])/(e*(d + e*x^2)) + (b*c*x*((2*ArcTan[Sqrt[-1 + c^2
*x^2]])/d - (2*Sqrt[e]*ArcTan[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/Sqrt[c^2*d + e]
])/(d*Sqrt[c^2*d + e])))/(4*e*Sqrt[c^2*x^2])
```

3.98.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5759
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSec[c*x])/(2*e*(p + 1))), x
] - Simp[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2]))   Int[(d + e*x^2)^(p + 1)/(x*S
qrt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

3.98.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(112) = 224$.

Time = 6.50 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.02

method	result
parts	$-\frac{a}{2e(ex^2+d)}+\frac{b\left(-\frac{c^4\operatorname{arcsec}(cx)}{2e\left(c^2ex^2+c^2d\right)}-\frac{c\sqrt{c^2x^2-1}\left(2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d+e}{e}}-\ln\left(\frac{2\sqrt{c^2x^2-1}\sqrt{-\frac{c^2d+e}{e}}e-2\sqrt{-c^2de}}{cex+\sqrt{-c^2de}}\right.\right.\right.}{4e\sqrt{\frac{c^2x^2-1}{c^2x^2}}xd\sqrt{-\frac{c^2d+}{e}}}\right)}{c^2}$
derivativedivides	$-\frac{ac^4}{2e\left(c^2ex^2+c^2d\right)}+bc^4\left(-\frac{\operatorname{arcsec}(cx)}{2e\left(c^2ex^2+c^2d\right)}-\frac{\sqrt{c^2x^2-1}\left(2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d+e}{e}}-\ln\left(-\frac{2\left(\sqrt{c^2x^2-1}\sqrt{-\frac{c^2d+e}{e}}e+\sqrt{-}\right.\right.\right.}{4e\sqrt{\frac{c^2x^2-1}{c^2x^2}}c^3xd\sqrt{-}}\right)}{c^2}$
default	$-\frac{ac^4}{2e\left(c^2ex^2+c^2d\right)}+bc^4\left(-\frac{\operatorname{arcsec}(cx)}{2e\left(c^2ex^2+c^2d\right)}-\frac{\sqrt{c^2x^2-1}\left(2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d+e}{e}}-\ln\left(-\frac{2\left(\sqrt{c^2x^2-1}\sqrt{-\frac{c^2d+e}{e}}e+\sqrt{-}\right.\right.\right.}{4e\sqrt{\frac{c^2x^2-1}{c^2x^2}}c^3xd\sqrt{-}}\right)}{c^2}$

input
```
int(x*(a+b*arcsec(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/e/(e*x^2+d)+b/c^2*(-1/2*c^4/e/(c^2*e*x^2+c^2*d)*arcsec(c*x)-1/4*c/e
*(c^2*x^2-1)^(1/2)*(2*arctan(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)-ln(
2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x
+(-c^2*d*e)^(1/2)))-ln(-2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*
d*e)^(1/2)*c*x-e)/(-c*e*x+(-c^2*d*e)^(1/2))))/((c^2*x^2-1)/c^2/x^2)^(1/2)/
x/d/(-(c^2*d+e)/e)^(1/2))
```

3.98.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.93

$$\int \frac{x(a+b\sec^{-1}(cx))}{\left(d+ex^2\right)^2}\,dx$$

$$= \left[-\frac{2\,ac^2d^2+2\,ade+\sqrt{-c^2de-e^2}(bex^2+bd)\log\left(\frac{c^2ex^2-c^2d+2\sqrt{-c^2de-e^2}\sqrt{c^2x^2-1}-2\,e}{ex^2+d}\right)+2\left(bc^2d^2+bde\right)\arcsin}{4\left(c^2d^3e+d^2e^2+(c^2d^2e^2+de^3)x\right.}\right.$$

$$\left. -\frac{ac^2d^2+ade+\sqrt{c^2de+e^2}(bex^2+bd)\arctan\left(\frac{\sqrt{c^2de+e^2}\sqrt{c^2x^2-1}}{c^2d+e}\right)+\left(bc^2d^2+bde\right)\operatorname{arcsec}(cx)-2\left(bc^2d^2+\right.}{2\left(c^2d^3e+d^2e^2+(c^2d^2e^2+de^3)x^2\right)}\right]$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*a*c^2*d^2 + 2*a*d*e + sqrt(-c^2*d*e - e^2)*(b*e*x^2 + b*d)*log((c
^2*e*x^2 - c^2*d + 2*sqrt(-c^2*d*e - e^2)*sqrt(c^2*x^2 - 1) - 2*e)/(e*x^2
+ d)) + 2*(b*c^2*d^2 + b*d*e)*arcsec(c*x) - 4*(b*c^2*d^2 + b*d*e + (b*c^2*
d*e + b*e^2)*x^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^3*e + d^2*e^2 +
 (c^2*d^2*e^2 + d*e^3)*x^2), -1/2*(a*c^2*d^2 + a*d*e + sqrt(c^2*d*e + e^2)
*(b*e*x^2 + b*d)*arctan(sqrt(c^2*d*e + e^2)*sqrt(c^2*x^2 - 1)/(c^2*d + e))
 + (b*c^2*d^2 + b*d*e)*arcsec(c*x) - 2*(b*c^2*d^2 + b*d*e + (b*c^2*d*e + b
*e^2)*x^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^3*e + d^2*e^2 + (c^2*d
^2*e^2 + d*e^3)*x^2)]
```

3.98.6 Sympy [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{\left(d+ex^2\right)^2}\,dx = \int \frac{x(a+b\operatorname{asec}(cx))}{\left(d+ex^2\right)^2}\,dx$$

input
```
integrate(x*(a+b*asec(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral(x*(a + b*asec(c*x))/(d + e*x**2)**2, x)
```

3.98.7 Maxima [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x}{(ex^2+d)^2}\,dx$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*(2*(c^2*e^2*x^2 + c^2*d*e)*integrate(1/2*x*e^(1/2*log(c*x + 1) + 1/2*l
og(c*x - 1))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e + (c^2*e^2*x^4 + (c^
2*d*e - e^2)*x^2 - d*e)*e^(log(c*x + 1) + log(c*x - 1))), x) - arctan(sqrt
(c*x + 1)*sqrt(c*x - 1)))*b/(e^2*x^2 + d*e) - 1/2*a/(e^2*x^2 + d*e)
```

3.98.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)}{(e\,x^2+d)^2}\,dx$$

input
```
int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^2,x)
```

output
```
int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^2, x)
```

3.99 $\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx$

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3.99.1 Optimal result

Integrand size = 21, antiderivative size = 546

$$\begin{aligned}\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx &= -\frac{e(a+b\sec^{-1}(cx))}{2d^2\left(e+\frac{d}{x^2}\right)}+\frac{i(a+b\sec^{-1}(cx))^2}{2bd^2}\\ &\quad-\frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2d^2\sqrt{c^2d+e}}\\ &\quad-\frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}\\ &\quad-\frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}\\ &\quad-\frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}\\ &\quad-\frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}\\ &\quad+\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}\\ &\quad+\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}\end{aligned}$$

output
```
-1/2*e*(a+b*arcsec(c*x))/d^2/(e+d/x^2)+1/2*I*(a+b*arcsec(c*x))^2/b/d^2-1/2
*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2
)-(c^2*d+e)^(1/2)))/d^2-1/2*(a+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2
)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^2-1/2*(a+b*arcsec(c*x))*l
n(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/
d^2-1/2*(a+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/
(e^(1/2)+(c^2*d+e)^(1/2)))/d^2+1/2*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)
^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^2+1/2*I*b*polylog(2,c*(1/c
/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^2+1/2*I*
b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)
^(1/2)))/d^2+1/2*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/
(e^(1/2)+(c^2*d+e)^(1/2)))/d^2-1/2*b*arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1
-1/c^2/x^2)^(1/2))*e^(1/2)/d^2/(c^2*d+e)^(1/2)
```

3.99.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1190 vs. $2(546) = 1092$.

Time $= 0.98$ (sec) , antiderivative size $= 1190$, normalized size of antiderivative $= 2.18$

$$\int \frac{a + b\sec^{-1}(cx)}{x\left(d + ex^2\right)^2}\,dx$$

$$= \frac{\frac{2ad}{d+ex^2} + \frac{b\sqrt{d}\sec^{-1}(cx)}{\sqrt{d}-i\sqrt{e}x} + \frac{b\sqrt{d}\sec^{-1}(cx)}{\sqrt{d}+i\sqrt{e}x} + 2ib\sec^{-1}(cx)^2 + 2b\arcsin\left(\frac{1}{cx}\right) - 8ib\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(-ic\sqrt{}\right.}{}\right.$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x*(d + e*x^2)^2),x]
```

output

```
((2*a*d)/(d + e*x^2) + (b*Sqrt[d]*ArcSec[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (
b*Sqrt[d]*ArcSec[c*x])/(Sqrt[d] + I*Sqrt[e]*x) + (2*I)*b*ArcSec[c*x]^2 + 2
*b*ArcSin[1/(c*x)] - (8*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt
[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e
]] - (8*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*
c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] - 2*b*ArcSec[c*x
]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] -
 4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e]
 - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 2*b*ArcSec[c*x]*Log[
1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 4*b*
ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + S
qrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 2*b*ArcSec[c*x]*Log[1 -
(I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 4*b*ArcSi
n[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^
2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 2*b*ArcSec[c*x]*Log[1 + (I*(Sq
rt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 4*b*ArcSin[Sqrt
[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d +
e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 4*a*Log[x] - (b*Sqrt[e]*Log[(2*Sqrt[
d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*
x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))])/Sqrt[-(c^2*d)...
```

3.99.3 Rubi [A] (verified)

Time = 1.57 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5763, 5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \sec^{-1}(cx)}{x \left(d + ex^2\right)^2} \, dx$$

$$\downarrow 5763$$

$$-\int \frac{a + b \arccos\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2} + e\right)^2 x^3} d\frac{1}{x}$$

$$\downarrow 5233$$

$$-\int \left(\frac{a + b \arccos\left(\frac{1}{cx}\right)}{d\left(\frac{d}{x^2} + e\right) x} - \frac{e\left(a + b \arccos\left(\frac{1}{cx}\right)\right)}{d\left(\frac{d}{x^2} + e\right)^2 x} \right) d\frac{1}{x}$$

$$\downarrow 2009$$

$$-\frac{(a+b\arccos\left(\frac{1}{cx}\right))\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}-\frac{(a+b\arccos\left(\frac{1}{cx}\right))\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}-$$
$$\frac{(a+b\arccos\left(\frac{1}{cx}\right))\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^2}-\frac{(a+b\arccos\left(\frac{1}{cx}\right))\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^2}-$$
$$\frac{e\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{2d^2\left(\frac{d}{x^2}+e\right)}+\frac{i\left(a+b\arccos\left(\frac{1}{cx}\right)\right)^2}{2bd^2}+\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^2}+$$
$$\frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^2}+\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^2}+$$
$$\frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^2}-\frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2d^2\sqrt{c^2d+e}}$$

input

```
Int[(a + b*ArcSec[c*x])/(x*(d + e*x^2)^2),x]
```

output

```
-1/2*(e*(a + b*ArcCos[1/(c*x)]))/(d^2*(e + d/x^2)) + ((I/2)*(a + b*ArcCos[
1/(c*x)])^2)/(b*d^2) - (b*Sqrt[e]*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[1
 - 1/(c^2*x^2)]*x)])/(2*d^2*Sqrt[c^2*d + e]) - ((a + b*ArcCos[1/(c*x)])*Lo
g[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*
d^2) - ((a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))
/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^2) - ((a + b*ArcCos[1/(c*x)])*Log[1 -
(c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^2) -
 ((a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt
[e] + Sqrt[c^2*d + e])])/(2*d^2) + ((I/2)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*
ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/d^2 + ((I/2)*b*PolyLog[2,
 (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/d^2 + ((
I/2)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2
*d + e]))])/d^2 + ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(
Sqrt[e] + Sqrt[c^2*d + e])])/d^2
```

3.99.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

3.99.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 9.06 (sec) , antiderivative size = 2108, normalized size of antiderivative = 3.86

method	result	size
parts	Expression too large to display	2108
derivativedivides	Expression too large to display	2157
default	Expression too large to display	2157

input
```
int((a+b*arcsec(c*x))/x/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
a/d^2*ln(x)+1/2*a/d/(e*x^2+d)-1/2*a/d^2*ln(e*x^2+d)+b*(-1/2*I*(-(e*(c^2*d+
e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*e*polylog(2,d*c^2
*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))/d^4/(
c^2*d+e)/c^4-I*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)
*e+2*e^2)*e*arcsec(c*x)^2/d^4/(c^2*d+e)/c^4+(-(e*(c^2*d+e))^(1/2)*c^2*d+2*
c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)/d^4*e/(c^2*d+e)/c^4*ln(1-d*c^2*(1/c
/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsec(c*x
)-1/4*I*(e*(c^2*d+e))^(1/2)/d/e/(c^2*d+e)*arcsec(c*x)^2*c^2+1/4*(e*(c^2*d+
e))^(1/2)/d/e/(c^2*d+e)*c^2*arcsec(c*x)*ln(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2)^
(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))-1/8*I*(e*(c^2*d+e))^(1/2)/d/e
/(c^2*d+e)*polylog(2,d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d+2*(e*(c
^2*d+e))^(1/2)-2*e))*c^2+1/2*I*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*arcsec(c*
x)^2/c^2/d^3+1/2*I*(e*(c^2*d+e))^(1/2)/d^2/(c^2*d+e)*arctanh(1/4*(2*c^2*d*
(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2+2*c^2*d+4*e)/(c^2*d*e+e^2)^(1/2))-1/2*(c^2
*d-2*(e*(c^2*d+e))^(1/2)+2*e)/d^3/c^2*ln(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1
/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsec(c*x)-1/4*I*(e*(c^2*d+e))
^(1/2)/d^2/(c^2*d+e)*polylog(2,d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2
*d+2*(e*(c^2*d+e))^(1/2)-2*e))-1/2*I*(e*(c^2*d+e))^(1/2)/d^2/(c^2*d+e)*arc
sec(c*x)^2+1/2*(e*(c^2*d+e))^(1/2)/d^2/(c^2*d+e)*arcsec(c*x)*ln(1-d*c^2*(1
/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))+1/4*I...
```

3.99.5 Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = \int \frac{b\operatorname{arcsec}(cx)+a}{\left(ex^2+d\right)^2 x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arcsec(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

3.99.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*asec(c*x))/x/(e*x**2+d)**2,x)
```

output
```
Timed out
```

3.99.7 Maxima [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = \int \frac{b\operatorname{arcsec}(cx)+a}{\left(ex^2+d\right)^2 x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a*(1/(d*e*x^2 + d^2) - log(e*x^2 + d)/d^2 + 2*log(x)/d^2) + b*integrat
e(arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

3.99.8 Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \sec^{-1}(cx)}{x \left(d + ex^2\right)^2} \, dx = \int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{x \left(e\, x^2 + d\right)^2} \, dx$$

input
```
int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^2),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^2), x)
```

3.100 $\int \frac{x^4\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

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3.100.1 Optimal result

Integrand size = 21, antiderivative size = 784

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = -\frac{d(a+b\sec^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d(a+b\sec^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{x(a+b\sec^{-1}(cx))}{e^2}$$
$$- \frac{b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce^2} - \frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{c^2d+e}}$$
$$- \frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{c^2d+e}}$$
$$+ \frac{3\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$- \frac{3\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$+ \frac{3\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$- \frac{3\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$+ \frac{3ib\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$- \frac{3ib\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$+ \frac{3ib\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$- \frac{3ib\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$

output
```
x*(a+b*arcsec(c*x))/e^2-b*arctanh((1-1/c^2/x^2)^(1/2))/c/e^2+3/4*(a+b*arcs
ec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e
)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*(a+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1/c^
2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4
*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2
)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*(a+b*arcsec(c*x))*ln(1+c*(1/c/x
+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e
^(5/2)+3/4*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1
/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*I*b*polylog(2,c*(1/c/x+I*(1-1
/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)+
3/4*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^
2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^
2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-1/4*d*(
a+b*arcsec(c*x))/e^2/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*d*(a+b*arcsec(c*x))/e^2
/(d/x+(-d)^(1/2)*e^(1/2))-1/4*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(
1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))*d^(1/2)/e^2/(c^2*d+e)^(1/2)-1/4*
b*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/
x^2)^(1/2))*d^(1/2)/e^2/(c^2*d+e)^(1/2)
```

3.100.2 Mathematica [A] (warning: unable to verify)

Time = 1.80 (sec) , antiderivative size = 1331, normalized size of antiderivative = 1.70

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

output

```
(4*a*Sqrt[e]*x + (2*a*d*Sqrt[e]*x)/(d + e*x^2) - 6*a*Sqrt[d]*ArcTan[(Sqrt[
e]*x)/Sqrt[d]] + b*(4*Sqrt[e]*x*ArcSec[c*x] + (d*ArcSec[c*x])/((-I)*Sqrt[d
] + Sqrt[e]*x) + (d*ArcSec[c*x])/(I*Sqrt[d] + Sqrt[e]*x) + 12*Sqrt[d]*ArcS
in[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sq
rt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] - 12*Sqrt[d]*ArcSin[Sqrt[1 + (
I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Tan[ArcSe
c[c*x]/2])/Sqrt[c^2*d + e]] + (3*I)*Sqrt[d]*ArcSec[c*x]*Log[1 + (I*(Sqrt[e
] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (6*I)*Sqrt[d]*ArcSi
n[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^
2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (3*I)*Sqrt[d]*ArcSec[c*x]*Log[
1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (6*I
)*Sqrt[d]*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-S
qrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (3*I)*Sqrt[d]*
ArcSec[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*S
qrt[d])] + (6*I)*Sqrt[d]*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]
*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] +
(3*I)*Sqrt[d]*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcS
ec[c*x]))/(c*Sqrt[d])] - (6*I)*Sqrt[d]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt
[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(
c*Sqrt[d])] + (I*Sqrt[d]*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I...
```

3.100.3 Rubi [A] (verified)

Time = 2.94 (sec) , antiderivative size = 844, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5763, 5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

$$\downarrow 5763$$

$$-\int \frac{x^2\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{\left(\frac{d}{x^2}+e\right)^2}d\frac{1}{x}$$

$$\downarrow 5233$$

$$-\int \left(\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)x^2}{e^2}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e^2\left(\frac{d}{x^2}+e\right)}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e\left(\frac{d}{x^2}+e\right)^2}\right)d\frac{1}{x}$$

$$\downarrow 2009$$

$$\frac{x\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e^2}+\frac{3\sqrt{-d}\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4e^{5/2}}-$$
$$\frac{3\sqrt{-d}\log\left(\frac{\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4e^{5/2}}+$$
$$\frac{3\sqrt{-d}\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4e^{5/2}}-$$
$$\frac{3\sqrt{-d}\log\left(\frac{\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4e^{5/2}}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4e^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}-$$
$$\frac{b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce^2}-\frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{dc^2+e}}-\frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{dc^2+e}}+$$
$$\frac{3ib\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4e^{5/2}}-\frac{3ib\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4e^{5/2}}+$$
$$\frac{3ib\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4e^{5/2}}-\frac{3ib\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4e^{5/2}}$$

input
```
Int[(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/4*(d*(a + b*ArcCos[1/(c*x)]))/(e^2*(Sqrt[-d]*Sqrt[e] - d/x)) + (d*(a +
b*ArcCos[1/(c*x)]))/(4*e^2*(Sqrt[-d]*Sqrt[e] + d/x)) + (x*(a + b*ArcCos[1/
(c*x)]))/e^2 - (b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/(c*e^2) - (b*Sqrt[d]*Arc
Tanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/
(c^2*x^2)])])/(4*e^2*Sqrt[c^2*d + e]) - (b*Sqrt[d]*ArcTanh[(c^2*d + (Sqrt[
-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*e^2
*Sqrt[c^2*d + e]) + (3*Sqrt[-d]*(a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d
]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*e^(5/2)) - (3*Sq
rt[-d]*(a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/
(Sqrt[e] - Sqrt[c^2*d + e])])/(4*e^(5/2)) + (3*Sqrt[-d]*(a + b*ArcCos[1/(c
*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e
])])/(4*e^(5/2)) - (3*Sqrt[-d]*(a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]
*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*e^(5/2)) + (((3*I
)/4)*b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] -
 Sqrt[c^2*d + e]))])/e^(5/2) - (((3*I)/4)*b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d
]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/e^(5/2) + (((3*I)/4
)*b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sq
rt[c^2*d + e]))])/e^(5/2) - (((3*I)/4)*b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E
^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^(5/2)
```

3.100.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

3.100.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 88.11 (sec) , antiderivative size = 941, normalized size of antiderivative = 1.20

method	result	size
parts	Expression too large to display	941
derivativedivides	Expression too large to display	966
default	Expression too large to display	966

input
```
int(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
a*(1/e^2*x-1/e^2*d*(-1/2*x/(e*x^2+d)+3/2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2
))))+b/c^5*(1/2*x*c^5*arcsec(c*x)*(2*c^2*e*x^2+3*c^2*d)/(c^2*e*x^2+c^2*d)/
e^2+2*I/e^2*c^4*arctan(1/c/x+I*(1-1/c^2/x^2)^(1/2))+3/16*I/e^3*c^6*d*sum((
_R1^2*c^2*d+c^2*d+4*e)/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-
1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/
_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-3/16*I/e^3*c^6*d*su
m((_R1^2*c^2*d+4*_R1^2*e+c^2*d)/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)
*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2
)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/2*I*((c^
2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*
c*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2
*e)*d)^(1/2))/d^2/e^2+1/2*I*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(
c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*c*arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2
))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^2/e^2-1/2*I*((c^2*d+2*(
e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(
e*(c^2*d+e))^(1/2)*e+2*e^2)*c*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((c
^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e^2/(c^2*d+e)/d^2-1/2*I*(-(c^2*d
-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+
2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*c*arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))
/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e^2/(c^2*d+e)/d^2)
```

3.100.5 Fricas [F]

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\,\text{arcsec}\,(cx)+a)x^4}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^4*arcsec(c*x) + a*x^4)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.100.6 Sympy [F]

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^4(a+b\,\text{asec}\,(cx))}{(d+ex^2)^2}\,dx$$

input
```
integrate(x**4*(a+b*asec(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral(x**4*(a + b*asec(c*x))/(d + e*x**2)**2, x)
```

3.100.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.100.8 Giac [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Timed out
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^4\left(a+b\,\text{acos}\left(\frac{1}{cx}\right)\right)}{(e\,x^2+d)^2}\,dx$$

input
```
int((x^4*(a + b*acos(1/(c*x))))/(d + e*x^2)^2,x)
```

output
```
int((x^4*(a + b*acos(1/(c*x))))/(d + e*x^2)^2, x)
```

3.101 $\int \frac{x^2\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

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3.101.1 Optimal result

Integrand size = 21, antiderivative size = 745

$$\begin{aligned}\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx &= \frac{a+b\sec^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\sec^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}\\&+\frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}}+\frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}}\\&+\frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}\\&-\frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}\\&+\frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}\\&-\frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}\\&+\frac{ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}\\&-\frac{ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}\\&+\frac{ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}\\&-\frac{ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}\end{aligned}$$

output
```
1/4*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(
1/2)-(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*(a+b*arcsec(c*x))*ln(1+c*(1/
c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(3/2)/(
-d)^(1/2)+1/4*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^
(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*(a+b*arcsec(c*x))*
ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))
/e^(3/2)/(-d)^(1/2)+1/4*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d
)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*I*b*polylog(2,c*
(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(3/2
)/(-d)^(1/2)+1/4*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)
/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*I*b*polylog(2,c*(1/c/x+
I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^
(1/2)+1/4*(a+b*arcsec(c*x))/e/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*(-a-b*arcsec(c
*x))/e/(d/x+(-d)^(1/2)*e^(1/2))+1/4*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)
/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/e/d^(1/2)/(c^2*d+e)^(1/2)+
1/4*b*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/
c^2/x^2)^(1/2))/e/d^(1/2)/(c^2*d+e)^(1/2)
```

3.101.2 Mathematica [A] (warning: unable to verify)

Time = 1.27 (sec) , antiderivative size = 1245, normalized size of antiderivative = 1.67

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

output

```
((-2*a*Sqrt[e]*x)/(d + e*x^2) + (2*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d]
+ b*(ArcSec[c*x]/(I*Sqrt[d] - Sqrt[e]*x) - ArcSec[c*x]/(I*Sqrt[d] + Sqrt[e
]*x) - (4*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*
c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]])/Sqrt[d] + (4*Ar
cSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqr
t[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]])/Sqrt[d] - (I*ArcSec[c*x]*Log[1
 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])])/Sqrt[d]
 - ((2*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sq
rt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])])/Sqrt[d] + (I*Arc
Sec[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqr
t[d])])/Sqrt[d] + ((2*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]
*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])])/
Sqrt[d] + (I*ArcSec[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSe
c[c*x]))/(c*Sqrt[d])])/Sqrt[d] - ((2*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqr
t[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/
(c*Sqrt[d])])/Sqrt[d] - (I*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d +
e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])])/Sqrt[d] + ((2*I)*ArcSin[Sqrt[1 + (I*S
qrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*
ArcSec[c*x]))/(c*Sqrt[d])])/Sqrt[d] - (I*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(S
qrt[e] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))...
```

3.101.3 Rubi [A] (verified)

Time = 1.76 (sec) , antiderivative size = 801, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5763, 5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

$$\downarrow 5763$$

$$-\int \frac{a+b\arccos\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^2}d\frac{1}{x}$$

$$\downarrow 5173$$

$$-\int \left(-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{2e\left(-\frac{d^2}{x^2}-ed\right)}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4e\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2}\right)d\frac{1}{x}$$

$$\downarrow \text{2009}$$

$$\frac{\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4\sqrt{-d}e^{3/2}}-\frac{\log\left(\frac{\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4\sqrt{-d}e^{3/2}}+$$
$$\frac{\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4\sqrt{-d}e^{3/2}}-\frac{\log\left(\frac{\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4\sqrt{-d}e^{3/2}}+$$
$$\frac{a+b\arccos\left(\frac{1}{cx}\right)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\arccos\left(\frac{1}{cx}\right)}{4e\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{dc^2+e}}+$$
$$\frac{b\operatorname{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{dc^2+e}}+\frac{ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}}-$$
$$\frac{ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}}-$$
$$\frac{ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}}$$

input

```
Int[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

output

```
(a + b*ArcCos[1/(c*x)])/(4*e*(Sqrt[-d]*Sqrt[e] - d/x)) - (a + b*ArcCos[1/(
c*x)])/(4*e*(Sqrt[-d]*Sqrt[e] + d/x)) + (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt
[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*Sqrt[d]*e*S
qrt[c^2*d + e]) + (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqr
t[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*Sqrt[d]*e*Sqrt[c^2*d + e]) + ((a
+ b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] -
 Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcCos[1/(c*x)])*Log[1
+ (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*Sqrt
[-d]*e^(3/2)) + ((a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1
/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) - ((a + b*Arc
Cos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c
^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) + ((I/4)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I
*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/(Sqrt[-d]*e^(3/2)) - ((I
/4)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d
+ e])])/(Sqrt[-d]*e^(3/2)) + ((I/4)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcCos
[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e]))])/(Sqrt[-d]*e^(3/2)) - ((I/4)*b*P
olyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])
/(Sqrt[-d]*e^(3/2))
```

3.101. $\int \frac{x^2\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

3.101.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

3.101.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 30.53 (sec) , antiderivative size = 852, normalized size of antiderivative = 1.14

method	result
parts	$-\frac{ax}{2e(ex^2+d)}+\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}}+b\left(-\frac{c^5\,\mathrm{arcsec}(cx)x}{2e\left(c^2ex^2+c^2d\right)}-\frac{i\sqrt{\left(c^2d+2\sqrt{e\left(c^2d+e\right)}+2e\right)}d\left(c^2d-2\sqrt{e\left(c^2d+e\right)}+2e\right)\arctan\left($
derivativedivides	$-\frac{ac^5x}{2e\left(c^2ex^2+c^2d\right)}+\frac{ac^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}}+bc^4\left(-\frac{\mathrm{arcsec}(cx)cx}{2e\left(c^2ex^2+c^2d\right)}-\frac{i\sqrt{\left(c^2d+2\sqrt{e\left(c^2d+e\right)}+2e\right)}d\left(c^2d-2\sqrt{e\left(c^2d+e\right)}+2e\right)\arctan\right.$
default	$-\frac{ac^5x}{2e\left(c^2ex^2+c^2d\right)}+\frac{ac^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}}+bc^4\left(-\frac{\mathrm{arcsec}(cx)cx}{2e\left(c^2ex^2+c^2d\right)}-\frac{i\sqrt{\left(c^2d+2\sqrt{e\left(c^2d+e\right)}+2e\right)}d\left(c^2d-2\sqrt{e\left(c^2d+e\right)}+2e\right)\arctan\right.$

input
```
int(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/e*x/(e*x^2+d)+1/2*a/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b/c^3*(-1
/2*c^5*arcsec(c*x)/e*x/(c^2*e*x^2+c^2*d)-1/2*I*((c^2*d+2*(e*(c^2*d+e))^(1/
2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*arctan(c*d*(1/c/x+I*(1-
1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/c/d^3/e+1/2
*I*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-(e*(c^2*d+e))^(1/2)*c^2*d
+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2
)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e/(c^2*d+e)/d^3/c-1/
2*I*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1
/2)+2*e)*arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e)
)^(1/2)-2*e)*d)^(1/2))/c/d^3/e+1/2*I*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d
)^(1/2)*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2
)*arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)
-2*e)*d)^(1/2))/e/(c^2*d+e)/d^3/c+1/4*I/e*c^4*sum(_R1/(_R1^2*c^2*d+c^2*d+2
*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/
c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+
c^2*d))-1/4*I/e*c^4*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_
R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2
))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d)))
```

3.101.5 Fricas [F]

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x^2}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^2*arcsec(c*x) + a*x^2)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.101.6 Sympy [F]

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^2(a+b\operatorname{asec}(cx))}{(d+ex^2)^2}\,dx$$

input
```
integrate(x**2*(a+b*asec(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral(x**2*(a + b*asec(c*x))/(d + e*x**2)**2, x)
```

3.101.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.101.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^2\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)}{(e\,x^2+d)^2}\,dx$$

input
```
int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^2,x)
```

output
```
int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^2, x)
```

3.102 $\int \frac{a+b\sec^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx$

3.102.1 Optimal result . . . 764
3.102.2 Mathematica [A] (warning: unable to verify) . . . 765
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3.102.8 Giac [F(-2)] . . . 771
3.102.9 Mupad [F(-1)] . . . 771

3.102.1 Optimal result

Integrand size = 18, antiderivative size = 739

$$\begin{aligned}\int \frac{a+b\sec^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx &= -\frac{a+b\sec^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\sec^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}\\
&-\frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}}-\frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}}\\
&-\frac{\left(a+b\sec^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&+\frac{\left(a+b\sec^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&-\frac{\left(a+b\sec^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&+\frac{\left(a+b\sec^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&-\frac{ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&-\frac{ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\end{aligned}$$

output

```
-1/4*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^
(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(a+b*arcsec(c*x))*ln(1+c*(1
/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/
2)/e^(1/2)-1/4*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)
^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(a+b*arcsec(c*x))
*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2))
)/(-d)^(3/2)/e^(1/2)-1/4*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-
d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*I*b*polylog(2,c
*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^
(3/2)/e^(1/2)-1/4*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2
)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*I*b*polylog(2,c*(1/c/x
+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e
^(1/2)+1/4*(-a-b*arcsec(c*x))/d/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*(a+b*arcsec(
c*x))/d/(d/x+(-d)^(1/2)*e^(1/2))-1/4*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x
)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(3/2)/(c^2*d+e)^(1/2)-1
/4*b*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c
^2/x^2)^(1/2))/d^(3/2)/(c^2*d+e)^(1/2)
```

3.102.2 Mathematica [A] (warning: unable to verify)

Time = 1.60 (sec) , antiderivative size = 1239, normalized size of antiderivative = 1.68

$$\int \frac{a + b \sec^{-1}(cx)}{\left(d + ex^2\right)^2}\, dx = \text{Too large to display}$$

input

```
Integrate[(a + b*ArcSec[c*x])/(d + e*x^2)^2,x]
```

output

```
((a*x)/(d^2 + d*e*x^2) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*Sqrt[e])
 + (b*((Sqrt[d]*ArcSec[c*x])/((-I)*Sqrt[d]*Sqrt[e] + e*x) + (Sqrt[d]*ArcSe
c[c*x])/(I*Sqrt[d]*Sqrt[e] + e*x) - (4*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt
[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt
[c^2*d + e]])/Sqrt[e] + (4*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2
]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]])/S
qrt[e] - (I*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec
[c*x]))/(c*Sqrt[d])])/Sqrt[e] - ((2*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt
[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(
c*Sqrt[d])])/Sqrt[e] + (I*ArcSec[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d +
e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])])/Sqrt[e] + ((2*I)*ArcSin[Sqrt[1 - (I*S
qrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I
*ArcSec[c*x]))/(c*Sqrt[d])])/Sqrt[e] + (I*ArcSec[c*x]*Log[1 - (I*(Sqrt[e]
+ Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])])/Sqrt[e] - ((2*I)*ArcSi
n[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^
2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])])/Sqrt[e] - (I*ArcSec[c*x]*Log[1
+ (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])])/Sqrt[e]
+ ((2*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqr
t[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])])/Sqrt[e] + (I*Log[
(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[...
```

3.102.3 Rubi [A] (verified)

Time = 2.63 (sec) , antiderivative size = 795, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5753, 5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b\sec^{-1}(cx)}{(d + ex^2)^2}\,dx$$

$$\downarrow 5753$$

$$-\int \frac{a + b\arccos\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2} + e\right)^2 x^2} d\frac{1}{x}$$

$$\downarrow 5233$$

$$-\int \left(\frac{a + b\arccos\left(\frac{1}{cx}\right)}{d\left(\frac{d}{x^2} + e\right)} - \frac{e\left(a + b\arccos\left(\frac{1}{cx}\right)\right)}{d\left(\frac{d}{x^2} + e\right)^2}\right) d\frac{1}{x}$$

$$\downarrow 2009$$

$$-\frac{\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{\log\left(\frac{\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4(-d)^{3/2}\sqrt{e}}-$$
$$\frac{\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{\log\left(\frac{\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4(-d)^{3/2}\sqrt{e}}-$$
$$\frac{a+b\arccos\left(\frac{1}{cx}\right)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\arccos\left(\frac{1}{cx}\right)}{4d\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}-\frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{dc^2+e}}-$$
$$\frac{b\operatorname{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{dc^2+e}}-\frac{ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+$$
$$\frac{ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+$$
$$\frac{ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

input

```
Int[(a + b*ArcSec[c*x])/(d + e*x^2)^2,x]
```

output

```
-1/4*(a + b*ArcCos[1/(c*x)])/(d*(Sqrt[-d]*Sqrt[e] - d/x)) + (a + b*ArcCos[
1/(c*x)])/(4*d*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*ArcTanh[(c^2*d - (Sqrt[-d]*S
qrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*d^(3/2)*
Sqrt[c^2*d + e]) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sq
rt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*d^(3/2)*Sqrt[c^2*d + e]) - ((a +
 b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] -
Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcCos[1/(c*x)])*Log[1
 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d
)^(3/2)*Sqrt[e]) - ((a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCo
s[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) + ((a +
b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + S
qrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) - ((I/4)*b*PolyLog[2, -((c*Sqrt[-
d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/((-d)^(3/2)*Sqrt[
e]) + ((I/4)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sq
rt[c^2*d + e])])/((-d)^(3/2)*Sqrt[e]) - ((I/4)*b*PolyLog[2, -((c*Sqrt[-d]*
E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e]))])/((-d)^(3/2)*Sqrt[e])
 + ((I/4)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[
c^2*d + e])])/((-d)^(3/2)*Sqrt[e])
```

3.102. $\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^2}\,dx$

3.102.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5753
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(2*(p + 1)))
, x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p]
```

3.102.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 45.23 (sec) , antiderivative size = 840, normalized size of antiderivative = 1.14

method	result
parts	$\frac{ax}{2d(ex^2+d)}+\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+b\left(\frac{c^3\,\mathrm{arcsec}(cx)x}{2d\left(c^2ex^2+c^2d\right)}+\frac{ic^2\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\frac{\_R1\left(i\,\mathrm{arcse}\right.}{\right.}\right.$
derivativedivides	$\frac{a\,c^3x}{2d\left(c^2ex^2+c^2d\right)}+\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+bc^4\left(\frac{\mathrm{arcsec}(cx)x}{2cd\left(c^2ex^2+c^2d\right)}+\frac{i\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\frac{\_R1\left(i\,\mathrm{arcs}\right.}{\right.}\right.$
default	$\frac{a\,c^3x}{2d\left(c^2ex^2+c^2d\right)}+\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+bc^4\left(\frac{\mathrm{arcsec}(cx)x}{2cd\left(c^2ex^2+c^2d\right)}+\frac{i\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\frac{\_R1\left(i\,\mathrm{arcs}\right.}{\right.}\right.$

input
```
int((a+b*arcsec(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x/d/(e*x^2+d)+1/2*a/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b/c*(1/2*c
^3*arcsec(c*x)*x/d/(c^2*e*x^2+c^2*d)+1/4*I/d*c^2*sum(_R1/(_R1^2*c^2*d+c^2*
d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1
-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z
^2+c^2*d))+1/2*I*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(
c^2*d+e))^(1/2)+2*e)*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e
*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^4/c^3-1/2*I*((c^2*d+2*(e*(c^2*d+e))^(1/
2)+2*e)*d)^(1/2)*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/
2)*e+2*e^2)*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e
))^(1/2)+2*e)*d)^(1/2))/d^4/(c^2*d+e)/c^3+1/2*I*(-(c^2*d-2*(e*(c^2*d+e))^(
1/2)+2*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*arctanh(c*d*(1/c/x+I*
(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4/c^3
-1/2*I*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2*d+e))^(1/2)*c
^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*arctanh(c*d*(1/c/x+I*(1-1/c^
2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4/(c^2*d+e)/
c^3-1/4*I/d*c^2*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1
/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_
R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d)))
```

3.102.5 Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx = \int \frac{b\operatorname{arcsec}(cx)+a}{\left(ex^2+d\right)^2}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arcsec(c*x) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.102.6 Sympy [F]

$$\int \frac{a+b\sec^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx = \int \frac{a+b\operatorname{asec}(cx)}{\left(d+ex^2\right)^2}\,dx$$

input
```
integrate((a+b*asec(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral((a + b*asec(c*x))/(d + e*x**2)**2, x)
```

3.102.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.102.8 Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx = \int \frac{a+b\operatorname{acos}\left(\frac{1}{cx}\right)}{\left(e\,x^2+d\right)^2}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(d + e*x^2)^2,x)
```

output
```
int((a + b*acos(1/(c*x)))/(d + e*x^2)^2, x)
```

3.103 $\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx$

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3.103.1 Optimal result

Integrand size = 21, antiderivative size = 785

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx = \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\sec^{-1}(cx)}{d^2x} + \frac{e(a+b\sec^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$
$$- \frac{e(a+b\sec^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{be\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d+e}}$$
$$+ \frac{be\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d+e}}$$
$$- \frac{3\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$
$$+ \frac{3\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$
$$- \frac{3\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$
$$+ \frac{3\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$
$$- \frac{3ib\sqrt{e}\,\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$
$$+ \frac{3ib\sqrt{e}\,\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$
$$- \frac{3ib\sqrt{e}\,\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$
$$+ \frac{3ib\sqrt{e}\,\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$

output
```
-a/d^2/x-b*arcsec(c*x)/d^2/x-3/4*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^
2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4
*(a+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2
)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)-3/4*(a+b*arcsec(c*x))*ln(1-c*(1/c/x
+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)
^(5/2)+3/4*(a+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/
2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)-3/4*I*b*polylog(2,-c*(1/c
/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-
d)^(5/2)+3/4*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(
1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)-3/4*I*b*polylog(2,-c*(1/c/x+I*(1
-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2
)+3/4*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c
^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)+1/4*e*(a+b*arcsec(c*x))/d^2/(-d/x+(-d)^
(1/2)*e^(1/2))-1/4*e*(a+b*arcsec(c*x))/d^2/(d/x+(-d)^(1/2)*e^(1/2))+1/4*b*
e*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/
x^2)^(1/2))/d^(5/2)/(c^2*d+e)^(1/2)+1/4*b*e*arctanh((c^2*d+(-d)^(1/2)*e^(1
/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d+e)^(1
/2)+b*c*(1-1/c^2/x^2)^(1/2)/d^2
```

3.103.2 Mathematica [A] (warning: unable to verify)

Time = 1.65 (sec) , antiderivative size = 1291, normalized size of antiderivative = 1.64

$$\int \frac{a + b \sec^{-1}(cx)}{x^2 \left(d + ex^2\right)^2} \, dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x^2*(d + e*x^2)^2),x]
```

output

```
((-4*a*Sqrt[d])/x - (2*a*Sqrt[d]*e*x)/(d + e*x^2) - 6*a*Sqrt[e]*ArcTan[(Sq
rt[e]*x)/Sqrt[d]] + b*(4*c*Sqrt[d]*Sqrt[1 - 1/(c^2*x^2)] - (4*Sqrt[d]*ArcS
ec[c*x])/x - (Sqrt[d]*e*ArcSec[c*x])/((-I)*Sqrt[d]*Sqrt[e] + e*x) - (Sqrt[
d]*e*ArcSec[c*x])/(I*Sqrt[d]*Sqrt[e] + e*x) + 12*Sqrt[e]*ArcSin[Sqrt[1 - (
I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Tan[Ar
cSec[c*x]/2])/Sqrt[c^2*d + e]] - 12*Sqrt[e]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c
*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sq
rt[c^2*d + e]] + (3*I)*Sqrt[e]*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*
d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (6*I)*Sqrt[e]*ArcSin[Sqrt[1 + (I
*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(
I*ArcSec[c*x]))/(c*Sqrt[d])] - (3*I)*Sqrt[e]*ArcSec[c*x]*Log[1 + (I*(-Sqrt
[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (6*I)*Sqrt[e]*Arc
Sin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt
[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (3*I)*Sqrt[e]*ArcSec[c*x]*L
og[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (6
*I)*Sqrt[e]*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(
Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (3*I)*Sqrt[e]
*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*
Sqrt[d])] - (6*I)*Sqrt[e]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]
]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])...
```

3.103.3 Rubi [A] (verified)

Time = 2.72 (sec) , antiderivative size = 845, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5763, 5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \sec^{-1}(cx)}{x^2 \left(d + ex^2\right)^2} \, dx$$

$$\downarrow 5763$$

$$-\int \frac{a + b \arccos\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2} + e\right)^2 x^4} d\frac{1}{x}$$

$$\downarrow 5233$$

$$-\int \left(\frac{\left(a + b \arccos\left(\frac{1}{cx}\right)\right) e^2}{d^2 \left(\frac{d}{x^2} + e\right)^2} - \frac{2\left(a + b \arccos\left(\frac{1}{cx}\right)\right) e}{d^2 \left(\frac{d}{x^2} + e\right)} + \frac{a + b \arccos\left(\frac{1}{cx}\right)}{d^2} \right) d\frac{1}{x}$$

$$\downarrow 2009$$

$$-\frac{a}{d^2x}-\frac{b\arccos\left(\frac{1}{cx}\right)}{d^2x}+\frac{e\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{e\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{4d^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+$$
$$\frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{dc^2+e}}+\frac{b\operatorname{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{dc^2+e}}-$$
$$\frac{3\sqrt{e}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}+$$
$$\frac{3\sqrt{e}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)}{4(-d)^{5/2}}-$$
$$\frac{3\sqrt{e}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}+$$
$$\frac{3\sqrt{e}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)}{4(-d)^{5/2}}-\frac{3ib\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}+$$
$$\frac{3ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}-\frac{3ib\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}+$$
$$\frac{3ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2}$$

input
```
Int[(a + b*ArcSec[c*x])/(x^2*(d + e*x^2)^2),x]
```

output
```
(b*c*Sqrt[1 - 1/(c^2*x^2)])/d^2 - a/(d^2*x) - (b*ArcCos[1/(c*x)])/(d^2*x)
+ (e*(a + b*ArcCos[1/(c*x)]))/(4*d^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (e*(a + b
*ArcCos[1/(c*x)]))/(4*d^2*(Sqrt[-d]*Sqrt[e] + d/x)) + (b*e*ArcTanh[(c^2*d
- (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])]
)/(4*d^(5/2)*Sqrt[c^2*d + e]) + (b*e*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x
)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*d^(5/2)*Sqrt[c^2*
d + e]) - (3*Sqrt[e]*(a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcC
os[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) + (3*Sqrt[e]*(a
 + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e]
- Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) - (3*Sqrt[e]*(a + b*ArcCos[1/(c*x)])*L
og[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4
*(-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I
*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) - (((3*I)/
4)*b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sq
rt[c^2*d + e]))])/(-d)^(5/2) + (((3*I)/4)*b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]
*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(-d)^(5/2) - (((3*I)
/4)*b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + S
qrt[c^2*d + e]))])/(-d)^(5/2) + (((3*I)/4)*b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d
]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(-d)^(5/2)
```

3.103.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

3.103.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 81.58 (sec) , antiderivative size = 933, normalized size of antiderivative = 1.19

method	result	size
parts	Expression too large to display	933
derivativedivides	Expression too large to display	960
default	Expression too large to display	960

input
```
int((a+b*arcsec(c*x))/x^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/d^2/x-e/d^2*(1/2*x/(e*x^2+d)+3/2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
))+b*c*(-1/2*(arcsec(c*x)+I)/d^2*(I*((c^2*x^2-1)/c^2/x^2)^(1/2)*c*x+1)/x/c
+1/2*(I*((c^2*x^2-1)/c^2/x^2)^(1/2)*c*x-1)*(arcsec(c*x)-I)/d^2/x/c-1/2*x*c
*arcsec(c*x)*e/(c^2*e*x^2+c^2*d)/d^2-1/2*I*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2
*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*arctan(c*d*(1/c/x+I*(1-1/c^
2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))*e/d^5/c^5+1/2*I
*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-(e*(c^2*d+e))^(1/2)*c^2*d+2
*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*e*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2
)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^5/c^5/(c^2*d+e)-1/
2*I*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1
/2)+2*e)*arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e)
)^(1/2)-2*e)*d)^(1/2))*e/d^5/c^5+1/2*I*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)
*d)^(1/2)*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e
^2)*e*arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(
1/2)-2*e)*d)^(1/2))/d^5/c^5/(c^2*d+e)+3/4*I*e/d^2*sum(1/_R1/(_R1^2*c^2*d+c
^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((
_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)
*_Z^2+c^2*d))-3/4*I*e/d^2*sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*l
n((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^
(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))
```

3.103.5 Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx = \int \frac{b\operatorname{arcsec}(cx)+a}{\left(ex^2+d\right)^2x^2}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arcsec(c*x) + a)/(e^2*x^6 + 2*d*e*x^4 + d^2*x^2), x)
```

3.103.6 Sympy [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx = \int \frac{a+b\operatorname{asec}(cx)}{x^2\left(d+ex^2\right)^2}\,dx$$

input
```
integrate((a+b*asec(c*x))/x**2/(e*x**2+d)**2,x)
```

output
```
Integral((a + b*asec(c*x))/(x**2*(d + e*x**2)**2), x)
```

3.103.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.103.8 Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \sec^{-1}(cx)}{x^2 \left(d + ex^2\right)^2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \sec^{-1}(cx)}{x^2 \left(d + ex^2\right)^2} \, dx = \int \frac{a + b \operatorname{acos}\left(\frac{1}{c\,x}\right)}{x^2 \left(e\,x^2 + d\right)^2} \, dx$$

input
```
int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)^2),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)^2), x)
```

3.104 $\int \frac{x^5\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

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3.104.1 Optimal result

Integrand size = 21, antiderivative size = 707

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = -\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\sec^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\sec^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)}$$
$$- \frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}} - \frac{b(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{8e^{5/2}\left(c^2d+e\right)^{3/2}}$$
$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}$$
$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}$$
$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3}$$
$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3}$$
$$- \frac{(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e^3}$$
$$- \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}$$
$$- \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}$$
$$- \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3}$$
$$- \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3} + \frac{ib\,\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2e^3}$$

output
```
1/4*(-a-b*arcsec(c*x))/e/(e+d/x^2)^2+1/2*(-a-b*arcsec(c*x))/e^2/(e+d/x^2)-
1/8*b*(c^2*d+2*e)*arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2)^(1/2))/
e^(5/2)/(c^2*d+e)^(3/2)-(a+b*arcsec(c*x))*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2
))^2)/e^3+1/2*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^
(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3+1/2*(a+b*arcsec(c*x))*ln(1+c*(1/c/x+I
*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3+1/2*(a+b*a
rcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*
d+e)^(1/2)))/e^3+1/2*(a+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2)
)*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3+1/2*I*b*polylog(2,-(1/c/x+I*(1
-1/c^2/x^2)^(1/2))^2)/e^3-1/2*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2
))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3-1/2*I*b*polylog(2,c*(1/c/x+I*
(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3-1/2*I*b*pol
ylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2
)))/e^3-1/2*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1
/2)+(c^2*d+e)^(1/2)))/e^3-1/2*b*arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^
2/x^2)^(1/2))/e^(5/2)/(c^2*d+e)^(1/2)-1/8*b*c*d*(1-1/c^2/x^2)^(1/2)/e^2/(c
^2*d+e)/(e+d/x^2)/x
```

3.104.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1805 vs. $2(707) = 1414$.

Time = 7.51 (sec) , antiderivative size = 1805, normalized size of antiderivative = 2.55

$$\int \frac{x^5(a + b\sec^{-1}(cx))}{(d + ex^2)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^5*(a + b*ArcSec[c*x]))/(d + e*x^2)^3,x]
```

output

```
-1/4*(a*d^2)/(e^3*(d + e*x^2)^2) + (a*d)/(e^3*(d + e*x^2)) + (a*Log[d + e*
x^2])/(2*e^3) + b*((((-7*I)/16)*Sqrt[d]*(-(ArcSec[c*x]/(I*Sqrt[d]*Sqrt[e]
+ e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] +
c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2
*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/e^(5/2)
+ (((7*I)/16)*Sqrt[d]*(-(ArcSec[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x)) - (I*(A
rcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*(I*c*Sqrt[d]
 + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt
[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/e^(5/2) - (d*(-(ArcSec
[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2)) + (ArcSin[1/(c*x)]/Sqrt[e] -
 I*((c*Sqrt[d]*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/((c^2*d + e)*((-I)*Sqrt[d]
 + Sqrt[e]*x)) + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(I*Sqrt[
e] + c*(c*Sqrt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d +
 e)*((-I)*Sqrt[d] + Sqrt[e]*x))])/(c^2*d + e)^(3/2)))/d))/(16*e^(5/2)) - (
d*((I*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] +
 Sqrt[e]*x)) - ArcSec[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) + ArcSin[1/
(c*x)]/(d*Sqrt[e]) - (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d + e]*((-
I)*Sqrt[e] + c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2
*c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/(16*e^(5/2)
) + ((I/4)*(8*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[...
```

3.104.3 Rubi [A] (verified)

Time = 1.90 (sec) , antiderivative size = 775, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5763, 5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$$

$$\downarrow 5763$$

$$-\int \frac{x\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{\left(\frac{d}{x^2}+e\right)^3}d\frac{1}{x}$$

$$\downarrow 5233$$

$$-\int\left(\frac{x\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e^3}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e^3\left(\frac{d}{x^2}+e\right)x}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e^2\left(\frac{d}{x^2}+e\right)^2x}-\frac{d\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{e\left(\frac{d}{x^2}+e\right)^3x}\right)d\frac{1}{x}$$

$$\downarrow \text{2009}$$

$$\frac{(a+b\arccos\left(\frac{1}{cx}\right))\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}+\frac{(a+b\arccos\left(\frac{1}{cx}\right))\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}+$$
$$\frac{(a+b\arccos\left(\frac{1}{cx}\right))\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^3}+\frac{(a+b\arccos\left(\frac{1}{cx}\right))\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^3}-$$
$$\frac{a+b\arccos\left(\frac{1}{cx}\right)}{2e^2\left(\frac{d}{x^2}+e\right)}-\frac{a+b\arccos\left(\frac{1}{cx}\right)}{4e\left(\frac{d}{x^2}+e\right)^2}-\frac{\log\left(1+e^{2i\arccos\left(\frac{1}{cx}\right)}\right)(a+b\arccos\left(\frac{1}{cx}\right))}{e^3}-$$
$$\frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^3}-\frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^3}-$$
$$\frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^3}-\frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^3}+$$
$$\frac{ib\,\text{PolyLog}\left(2,-e^{2i\arccos\left(\frac{1}{cx}\right)}\right)}{2e^3}-\frac{b\left(c^2d+2e\right)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{8e^{5/2}\left(c^2d+e\right)^{3/2}}-\frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2e^{5/2}\sqrt{c^2d+e}}-$$
$$\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2x\left(c^2d+e\right)\left(\frac{d}{x^2}+e\right)}$$

input
```
Int[(x^5*(a + b*ArcSec[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/8*(b*c*d*Sqrt[1 - 1/(c^2*x^2)])/(e^2*(c^2*d + e)*(e + d/x^2)*x) - (a +
b*ArcCos[1/(c*x)])/(4*e*(e + d/x^2)^2) - (a + b*ArcCos[1/(c*x)])/(2*e^2*(e
 + d/x^2)) - (b*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)
])/(2*e^(5/2)*Sqrt[c^2*d + e]) - (b*(c^2*d + 2*e)*ArcTan[Sqrt[c^2*d + e]/(
c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)])/(8*e^(5/2)*(c^2*d + e)^(3/2)) + ((a +
 b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] -
Sqrt[c^2*d + e])])/(2*e^3) + ((a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*
E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^3) + ((a + b*Arc
Cos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c
^2*d + e])])/(2*e^3) + ((a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*A
rcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^3) - ((a + b*ArcCos[1/
(c*x)])*Log[1 + E^((2*I)*ArcCos[1/(c*x)])])/e^3 - ((I/2)*b*PolyLog[2, -((c
*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/e^3 - ((I/
2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d +
 e])])/e^3 - ((I/2)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqr
t[e] + Sqrt[c^2*d + e]))])/e^3 - ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcC
os[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 + ((I/2)*b*PolyLog[2, -E^(
(2*I)*ArcCos[1/(c*x)])])/e^3
```

3.104.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

3.104.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.69 (sec) , antiderivative size = 1396, normalized size of antiderivative = 1.97

method	result	size
parts	Expression too large to display	1396
derivativedivides	Expression too large to display	1409
default	Expression too large to display	1409

input
```
int(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/4*d^2/e^3/(e*x^2+d)^2+1/2/e^3*ln(e*x^2+d)+d/e^3/(e*x^2+d))+b/c^6*(-1
/8*c^6*(4*c^6*d^2*arcsec(c*x)*x^2+6*c^6*d*e*arcsec(c*x)*x^4+((c^2*x^2-1)/c
^2/x^2)^(1/2)*c^5*d^2*x+((c^2*x^2-1)/c^2/x^2)^(1/2)*c^5*d*e*x^3+4*c^4*d*e*
arcsec(c*x)*x^2+6*arcsec(c*x)*e^2*c^4*x^4+I*c^4*d^2+2*I*c^4*d*e*x^2+I*e^2*
c^4*x^4)/e^2/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2+I/(c^2*d+e)/e^2*c^6*dilog(1+I*(
1/c/x+I*(1-1/c^2/x^2)^(1/2)))+I/(c^2*d+e)/e^2*c^6*dilog(1-I*(1/c/x+I*(1-1/
c^2/x^2)^(1/2)))-1/(c^2*d+e)/e^2*c^6*arcsec(c*x)*ln(1-I*(1/c/x+I*(1-1/c^2/
x^2)^(1/2)))-1/(c^2*d+e)/e^2*c^6*arcsec(c*x)*ln(1+I*(1/c/x+I*(1-1/c^2/x^2)
^(1/2)))-1/4*I/(c^2*d+e)/e^2*c^6*sum((_R1^2*c^2*d+c^2*d+4*e)/(_R1^2*c^2*d+
c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog(
(_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e
)*_Z^2+c^2*d))-1/4*I/(c^2*d+e)/e^2*c^8*d*sum((_R1^2+1)/(_R1^2*c^2*d+c^2*d+
2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1
/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2
+c^2*d))-1/(c^2*d+e)/e^3*c^8*d*arcsec(c*x)*ln(1+I*(1/c/x+I*(1-1/c^2/x^2)^(
1/2)))-1/(c^2*d+e)/e^3*c^8*d*arcsec(c*x)*ln(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/
2)))-1/4*I/(c^2*d+e)/e^3*c^8*d*sum((_R1^2*c^2*d+c^2*d+4*e)/(_R1^2*c^2*d+c^
2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_
R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*
_Z^2+c^2*d))-1/4*I/(c^2*d+e)/e^3*c^10*d^2*sum((_R1^2+1)/(_R1^2*c^2*d+c^...
```

3.104.5 Fricas [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\,\text{arcsec}\,(cx)+a)x^5}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^5*arcsec(c*x) + a*x^5)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

3.104.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(a+b*asec(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.104.7 Maxima [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\,\text{arcsec}\,(cx)+a)x^5}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/4*a*((4*d*e*x^2 + 3*d^2)/(e^5*x^4 + 2*d*e^4*x^2 + d^2*e^3) + 2*log(e*x^2
 + d)/e^3) + b*integrate(x^5*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(e^3*x^6
+ 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

3.104.8 Giac [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
Timed out
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{x^5\left(a+b\,\text{acos}\left(\frac{1}{cx}\right)\right)}{(e\,x^2+d)^3}\,dx$$

input
```
int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^3,x)
```

output
```
int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^3, x)
```

3.105 $\int \frac{x^3\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

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3.105.1 Optimal result

Integrand size = 21, antiderivative size = 157

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \frac{bcx\sqrt{-1+c^2x^2}}{8e\left(c^2d+e\right)\sqrt{c^2x^2}\left(d+ex^2\right)} + \frac{x^4(a+b\sec^{-1}(cx))}{4d\left(d+ex^2\right)^2} - \frac{bc(c^2d+2e)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{\sqrt{c^2d+e}}\right)}{8de^{3/2}\left(c^2d+e\right)^{3/2}\sqrt{c^2x^2}}$$

output
```
1/4*x^4*(a+b*arcsec(c*x))/d/(e*x^2+d)^2-1/8*b*c*(c^2*d+2*e)*x*arctan(e^(1/
2)*(c^2*x^2-1)^(1/2)/(c^2*d+e)^(1/2))/d/e^(3/2)/(c^2*d+e)^(3/2)/(c^2*x^2)^
(1/2)+1/8*b*c*x*(c^2*x^2-1)^(1/2)/e/(c^2*d+e)/(e*x^2+d)/(c^2*x^2)^(1/2)
```

3.105.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.48

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx =$$

$$-\frac{-\frac{4ad}{(d+ex^2)^2}+\frac{8a}{d+ex^2}-\frac{2bce\sqrt{1-\frac{1}{c^2x^2}}x}{(c^2d+e)(d+ex^2)}+\frac{4b\left(d+2ex^2\right)\sec^{-1}(cx)}{(d+ex^2)^2}+\frac{4b\arcsin\left(\frac{1}{cx}\right)}{d}+\frac{b\sqrt{e}(c^2d+2e)\log\left(-\frac{16d\sqrt{-c^2d-e}e^{3/2}\left(\sqrt{e}+c\left(\right.\right.\right.}{b\left(c^2d+2e\right.}}{d(-c^2d-e)^{3/2}}}{16e^2}$$

input
```
Integrate[(x^3*(a + b*ArcSec[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/16*((-4*a*d)/(d + e*x^2)^2 + (8*a)/(d + e*x^2) - (2*b*c*e*Sqrt[1 - 1/(c
^2*x^2)]*x)/((c^2*d + e)*(d + e*x^2)) + (4*b*(d + 2*e*x^2)*ArcSec[c*x])/(d
 + e*x^2)^2 + (4*b*ArcSin[1/(c*x)])/d + (b*Sqrt[e]*(c^2*d + 2*e)*Log[(-16*
d*Sqrt[-(c^2*d) - e]*e^(3/2)*(Sqrt[e] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e
]*Sqrt[1 - 1/(c^2*x^2)])*x))/(b*(c^2*d + 2*e)*(I*Sqrt[d] + Sqrt[e]*x))])/(
d*(-(c^2*d) - e)^(3/2)) + (b*Sqrt[e]*(c^2*d + 2*e)*Log[((16*I)*d*Sqrt[-(c^
2*d) - e]*e^(3/2)*(-Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 -
 1/(c^2*x^2)])*x))/(b*(c^2*d + 2*e)*(Sqrt[d] + I*Sqrt[e]*x))])/(d*(-(c^2*d
) - e)^(3/2)))/e^2
```

3.105.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5761, 27, 354, 87, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$$

$$\downarrow 5761$$

$$\frac{x^4\left(a+b\sec^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} - \frac{bcx\int \frac{x^3}{4d\sqrt{c^2x^2-1}\left(ex^2+d\right)^2}\,dx}{\sqrt{c^2x^2}}$$

$$\downarrow 27$$

$$\frac{x^4\left(a+b\sec^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} - \frac{bcx\int \frac{x^3}{\sqrt{c^2x^2-1}\left(ex^2+d\right)^2}\,dx}{4d\sqrt{c^2x^2}}$$

$$\downarrow 354$$

$$\frac{x^4\left(a+b\sec^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} - \frac{bcx\int \frac{x^2}{\sqrt{c^2x^2-1}\left(ex^2+d\right)^2}\,dx^2}{8d\sqrt{c^2x^2}}$$

$$\downarrow 87$$

$$\frac{x^4\left(a+b\sec^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}-\frac{bcx\left(\frac{\left(c^2d+2e\right)\int\frac{1}{\sqrt{c^2x^2-1}\left(ex^2+d\right)}dx^2}{2e\left(c^2d+e\right)}-\frac{d\sqrt{c^2x^2-1}}{e\left(c^2d+e\right)\left(d+ex^2\right)}\right)}{8d\sqrt{c^2x^2}}$$

$$\downarrow 73$$

$$\frac{x^4\left(a+b\sec^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}-\frac{bcx\left(\frac{\left(c^2d+2e\right)\int\frac{1}{\frac{ex^4}{c^2}+d+\frac{e}{c^2}}d\sqrt{c^2x^2-1}}{c^2e\left(c^2d+e\right)}-\frac{d\sqrt{c^2x^2-1}}{e\left(c^2d+e\right)\left(d+ex^2\right)}\right)}{8d\sqrt{c^2x^2}}$$

$$\downarrow 218$$

$$\frac{x^4\left(a+b\sec^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}-\frac{bcx\left(\frac{\left(c^2d+2e\right)\arctan\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{e^{3/2}\left(c^2d+e\right)^{3/2}}-\frac{d\sqrt{c^2x^2-1}}{e\left(c^2d+e\right)\left(d+ex^2\right)}\right)}{8d\sqrt{c^2x^2}}$$

input
```
Int[(x^3*(a + b*ArcSec[c*x]))/(d + e*x^2)^3,x]
```

output
```
(x^4*(a + b*ArcSec[c*x]))/(4*d*(d + e*x^2)^2) - (b*c*x*(-((d*Sqrt[-1 + c^2
*x^2])/(e*(c^2*d + e)*(d + e*x^2))) + ((c^2*d + 2*e)*ArcTan[(Sqrt[e]*Sqrt[
-1 + c^2*x^2])/Sqrt[c^2*d + e]])/(e^(3/2)*(c^2*d + e)^(3/2))))/(8*d*Sqrt[c
^2*x^2])
```

3.105.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.105.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 940 vs. $2(135) = 270$.

Time $= 5.55$ (sec) , antiderivative size $= 941$, normalized size of antiderivative $= 5.99$

method	result
parts	$a\left(-\frac{1}{2e^2(ex^2+d)}+\frac{d}{4e^2(ex^2+d)^2}\right)+b\left(\frac{c^8\operatorname{arcsec}(cx)d}{4e^2\left(c^2ex^2+c^2d\right)^2}-\frac{c^6\operatorname{arcsec}(cx)}{2e^2\left(c^2ex^2+c^2d\right)}-\frac{c^3\sqrt{c^2x^2-1}\left(-4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\right.}{}\right.$
derivativedivides	$\frac{ac^6\left(-\frac{1}{2e^2\left(c^2ex^2+c^2d\right)}+\frac{dc^2}{4e^2\left(c^2ex^2+c^2d\right)^2}\right)+bc^6\left(-\frac{\operatorname{arcsec}(cx)}{2e^2\left(c^2ex^2+c^2d\right)}+\frac{\operatorname{arcsec}(cx)dc^2}{4e^2\left(c^2ex^2+c^2d\right)^2}-\frac{\sqrt{c^2x^2-1}\left(-4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\right.}{}\right.}{}$
default	$\frac{ac^6\left(-\frac{1}{2e^2\left(c^2ex^2+c^2d\right)}+\frac{dc^2}{4e^2\left(c^2ex^2+c^2d\right)^2}\right)+bc^6\left(-\frac{\operatorname{arcsec}(cx)}{2e^2\left(c^2ex^2+c^2d\right)}+\frac{\operatorname{arcsec}(cx)dc^2}{4e^2\left(c^2ex^2+c^2d\right)^2}-\frac{\sqrt{c^2x^2-1}\left(-4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\right.}{}\right.}{}$

input
```
int(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output

```
a*(-1/2/e^2/(e*x^2+d)+1/4*d/e^2/(e*x^2+d)^2)+b/c^4*(1/4*c^8*arcsec(c*x)*d/
e^2/(c^2*e*x^2+c^2*d)^2-1/2*c^6*arcsec(c*x)/e^2/(c^2*e*x^2+c^2*d)-1/16*c^3
*(c^2*x^2-1)^(1/2)/e*(-4*arctan(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)*
c^4*d^2-4*arctan(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)*c^4*d*e*x^2+ln(
2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x
+(-c^2*d*e)^(1/2)))*c^4*d^2+ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e
-(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-c^2*d*e)^(1/2)))*c^4*d*e*x^2+ln(-2*((c^2
*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-c*e*x+(-c^2
*d*e)^(1/2)))*c^4*d^2+ln(-2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^
2*d*e)^(1/2)*c*x-e)/(-c*e*x+(-c^2*d*e)^(1/2)))*c^4*d*e*x^2+2*(c^2*x^2-1)^(
1/2)*(-(c^2*d+e)/e)^(1/2)*c^2*d*e-4*arctan(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e
)/e)^(1/2)*c^2*d*e-4*arctan(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)*e^2*
c^2*x^2+2*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*
c*x-e)/(c*e*x+(-c^2*d*e)^(1/2)))*c^2*d*e+2*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*
d+e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-c^2*d*e)^(1/2)))*e^2*c^2*
x^2+2*ln(-2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x
-e)/(-c*e*x+(-c^2*d*e)^(1/2)))*c^2*d*e+2*ln(-2*((c^2*x^2-1)^(1/2)*(-(c^2*d
+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-c*e*x+(-c^2*d*e)^(1/2)))*e^2*c^2*
x^2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/d/(-(c^2*d+e)/e)^(1/2)/(c^2*d+e)/(c*e*x
+(-c^2*d*e)^(1/2))/(-c*e*x+(-c^2*d*e)^(1/2)))
```

3.105.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(135) = 270$.

Time $= 0.51$ (sec) , antiderivative size $= 1015$, normalized size of antiderivative $= 6.46$

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx$$

$$= \left[-\frac{4\,ac^4d^4+8\,ac^2d^3e+4\,ad^2e^2+8\,(ac^4d^3e+2\,ac^2d^2e^2+ade^3)x^2+(bc^2d^3+(bc^2de^2+2\,be^3)x^4+2\,bd^2e}{} \right.$$

$$\left. -\frac{2\,ac^4d^4+4\,ac^2d^3e+2\,ad^2e^2+4\,(ac^4d^3e+2\,ac^2d^2e^2+ade^3)x^2+(bc^2d^3+(bc^2de^2+2\,be^3)x^4+2\,bd^2e-}{} \right]$$

input

```
integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output

```
[-1/16*(4*a*c^4*d^4 + 8*a*c^2*d^3*e + 4*a*d^2*e^2 + 8*(a*c^4*d^3*e + 2*a*c
^2*d^2*e^2 + a*d*e^3)*x^2 + (b*c^2*d^3 + (b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b
*d^2*e + 2*(b*c^2*d^2*e + 2*b*d*e^2)*x^2)*sqrt(-c^2*d*e - e^2)*log((c^2*e*
x^2 - c^2*d + 2*sqrt(-c^2*d*e - e^2)*sqrt(c^2*x^2 - 1) - 2*e)/(e*x^2 + d))
 + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2
*e^2 + b*d*e^3)*x^2)*arcsec(c*x) - 8*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^
2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2
*d^2*e^2 + b*d*e^3)*x^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)) - 2*(b*c^2*d^3*e
 + b*d^2*e^2 + (b*c^2*d^2*e^2 + b*d*e^3)*x^2)*sqrt(c^2*x^2 - 1))/(c^4*d^5*
e^2 + 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4 + 2*c^2*d^2*e^5 + d*e^6)*x^4
+ 2*(c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^2), -1/8*(2*a*c^4*d^4 + 4*a*
c^2*d^3*e + 2*a*d^2*e^2 + 4*(a*c^4*d^3*e + 2*a*c^2*d^2*e^2 + a*d*e^3)*x^2
+ (b*c^2*d^3 + (b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(b*c^2*d^2*e +
2*b*d*e^2)*x^2)*sqrt(c^2*d*e + e^2)*arctan(sqrt(c^2*d*e + e^2)*sqrt(c^2*x^
2 - 1)/(c^2*d + e)) + 2*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b*c^4*
d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*arcsec(c*x) - 4*(b*c^4*d^4 + 2*b*c
^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*
c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*arctan(-c*x + sqrt(c^2*x^2 - 1
)) - (b*c^2*d^3*e + b*d^2*e^2 + (b*c^2*d^2*e^2 + b*d*e^3)*x^2)*sqrt(c^2*x^
2 - 1))/(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4 + 2*c^2*d...
```

3.105.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

input

```
integrate(x**3*(a+b*asec(c*x))/(e*x**2+d)**3,x)
```

output

```
Timed out
```

3.105.7 Maxima [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\,\text{arcsec}\,(cx)+a)x^3}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/4*(2*e*x^2 + d)*a/(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2) - 1/4*((2*e*x^2 + d
)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 4*(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2
)*integrate(1/4*(2*c^2*e*x^3 + c^2*d*x)*e^(1/2*log(c*x + 1) + 1/2*log(c*x
- 1))/(c^2*e^4*x^6 + (2*c^2*d*e^3 - e^4)*x^4 - d^2*e^2 + (c^2*d^2*e^2 - 2*
d*e^3)*x^2 + (c^2*e^4*x^6 + (2*c^2*d*e^3 - e^4)*x^4 - d^2*e^2 + (c^2*d^2*e
^2 - 2*d*e^3)*x^2)*e^(log(c*x + 1) + log(c*x - 1))), x))*b/(e^4*x^4 + 2*d*
e^3*x^2 + d^2*e^2)
```

3.105.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{x^3\left(a+b\,\text{acos}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^3}\,dx$$

input
```
int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^3,x)
```

output
```
int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^3, x)
```

3.106 $\int \frac{x\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

3.106.1 Optimal result 798
3.106.2 Mathematica [C] (verified) 798
3.106.3 Rubi [A] (verified) 799
3.106.4 Maple [B] (verified) 802
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3.106.7 Maxima [F] 804
3.106.8 Giac [F(-2)] 805
3.106.9 Mupad [F(-1)] 805

3.106.1 Optimal result

Integrand size = 19, antiderivative size = 193

$$\begin{aligned}\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = &-\frac{bcx\sqrt{-1+c^2x^2}}{8d\left(c^2d+e\right)\sqrt{c^2x^2}\left(d+ex^2\right)}\\ &-\frac{a+b\sec^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{bcx\arctan\left(\sqrt{-1+c^2x^2}\right)}{4d^2e\sqrt{c^2x^2}}\\ &-\frac{bc(3c^2d+2e)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{\sqrt{c^2d+e}}\right)}{8d^2\sqrt{e}\left(c^2d+e\right)^{3/2}\sqrt{c^2x^2}}\end{aligned}$$

output

```
1/4*(-a-b*arcsec(c*x))/e/(e*x^2+d)^2+1/4*b*c*x*arctan((c^2*x^2-1)^(1/2))/d
^2/e/(c^2*x^2)^(1/2)-1/8*b*c*(3*c^2*d+2*e)*x*arctan(e^(1/2)*(c^2*x^2-1)^(1
/2)/(c^2*d+e)^(1/2))/d^2/(c^2*d+e)^(3/2)/e^(1/2)/(c^2*x^2)^(1/2)-1/8*b*c*x
*(c^2*x^2-1)^(1/2)/d/(c^2*d+e)/(e*x^2+d)/(c^2*x^2)^(1/2)
```

3.106.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.00

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx$$

$$= \frac{1}{16}\left(-\frac{4a}{e\left(d+ex^2\right)^2} - \frac{2bc\sqrt{1-\frac{1}{c^2x^2}}x}{d\left(c^2d+e\right)\left(d+ex^2\right)} - \frac{4b\sec^{-1}(cx)}{e\left(d+ex^2\right)^2} - \frac{4b\arcsin\left(\frac{1}{cx}\right)}{d^2e}\right.$$

$$- \frac{b(3c^2d+2e)\log\left(-\frac{16d^2\sqrt{-c^2d-e}\sqrt{e}\left(\sqrt{e}+c\left(ic\sqrt{d}-\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{b(3c^2d+2e)\left(i\sqrt{d}+\sqrt{e}x\right)}\right)}{d^2\left(-c^2d-e\right)^{3/2}\sqrt{e}}$$

$$\left. - \frac{b(3c^2d+2e)\log\left(\frac{16id^2\sqrt{-c^2d-e}\sqrt{e}\left(-\sqrt{e}+c\left(ic\sqrt{d}+\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{b(3c^2d+2e)\left(\sqrt{d}+i\sqrt{e}x\right)}\right)}{d^2\left(-c^2d-e\right)^{3/2}\sqrt{e}}\right)$$

input
```
Integrate[(x*(a + b*ArcSec[c*x]))/(d + e*x^2)^3,x]
```

output
```
((-4*a)/(e*(d + e*x^2)^2) - (2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x)/(d*(c^2*d + e)
*(d + e*x^2)) - (4*b*ArcSec[c*x])/(e*(d + e*x^2)^2) - (4*b*ArcSin[1/(c*x)]
)/(d^2*e) - (b*(3*c^2*d + 2*e)*Log[(-16*d^2*Sqrt[-(c^2*d) - e]*Sqrt[e]*(Sq
rt[e] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(b*
(3*c^2*d + 2*e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d^2*(-(c^2*d) - e)^(3/2)*Sqrt[
e]) - (b*(3*c^2*d + 2*e)*Log[((16*I)*d^2*Sqrt[-(c^2*d) - e]*Sqrt[e]*(-Sqrt
[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(b*(3
*c^2*d + 2*e)*(Sqrt[d] + I*Sqrt[e]*x))])/(d^2*(-(c^2*d) - e)^(3/2)*Sqrt[e]
))/16
```

3.106.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5759, 354, 114, 27, 174, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$$

$$\downarrow 5759$$

$$\frac{bcx\int \frac{1}{x\sqrt{c^2x^2-1}\left(ex^2+d\right)^2}dx}{4e\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\downarrow 354$$

$$\frac{bcx\int \frac{1}{x^2\sqrt{c^2x^2-1}\left(ex^2+d\right)^2}dx^2}{8e\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\downarrow 114$$

$$\frac{bcx\left(-\frac{\int -\frac{-ex^2c^2+2dc^2+2e}{2x^2\sqrt{c^2x^2-1}\left(ex^2+d\right)}dx^2}{d(c^2d+e)}-\frac{e\sqrt{c^2x^2-1}}{d(c^2d+e)(d+ex^2)}\right)}{8e\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\downarrow 27$$

$$\frac{bcx\left(\frac{\int \frac{2\left(dc^2+e\right)-c^2ex^2}{x^2\sqrt{c^2x^2-1}\left(ex^2+d\right)}dx^2}{2d(c^2d+e)}-\frac{e\sqrt{c^2x^2-1}}{d(c^2d+e)(d+ex^2)}\right)}{8e\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\downarrow 174$$

$$\frac{bcx\left(\frac{\frac{2\left(c^2d+e\right)\int \frac{1}{x^2\sqrt{c^2x^2-1}}dx^2}{d}-\frac{e\left(3c^2d+2e\right)\int \frac{1}{\sqrt{c^2x^2-1}\left(ex^2+d\right)}dx^2}{d}}{2d(c^2d+e)}-\frac{e\sqrt{c^2x^2-1}}{d(c^2d+e)(d+ex^2)}\right)}{8e\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\downarrow 73$$

$$\frac{bcx\left(\frac{\frac{4\left(c^2d+e\right)\int \frac{1}{\frac{x^4}{c^2}+\frac{1}{c^2}}d\sqrt{c^2x^2-1}}{c^2d}-\frac{2e\left(3c^2d+2e\right)\int \frac{1}{\frac{ex^4}{c^2}+d+\frac{e}{c^2}}d\sqrt{c^2x^2-1}}{c^2d}}{2d(c^2d+e)}-\frac{e\sqrt{c^2x^2-1}}{d(c^2d+e)(d+ex^2)}\right)}{8e\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\downarrow 218$$

$$\frac{bcx\left(\frac{\frac{4\arctan\left(\sqrt{c^2x^2-1}\right)\left(c^2d+e\right)}{d}-\frac{2\sqrt{e}\left(3c^2d+2e\right)\arctan\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{d\sqrt{c^2d+e}}}{2d(c^2d+e)}-\frac{e\sqrt{c^2x^2-1}}{d(c^2d+e)(d+ex^2)}\right)}{8e\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

input
```
Int[(x*(a + b*ArcSec[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/4*(a + b*ArcSec[c*x])/(e*(d + e*x^2)^2) + (b*c*x*(-((e*Sqrt[-1 + c^2*x^
2])/(d*(c^2*d + e)*(d + e*x^2))) + ((4*(c^2*d + e)*ArcTan[Sqrt[-1 + c^2*x^
2]])/d - (2*Sqrt[e]*(3*c^2*d + 2*e)*ArcTan[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/Sq
rt[c^2*d + e]])/(d*Sqrt[c^2*d + e]))/(2*d*(c^2*d + e))))/(8*e*Sqrt[c^2*x^2
])
```

3.106.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5759
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSec[c*x])/(2*e*(p + 1))), x
] - Simp[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2]))   Int[(d + e*x^2)^(p + 1)/(x*S
qrt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

3.106.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 893 vs. $2(168) = 336$.

Time = 5.55 (sec) , antiderivative size = 894, normalized size of antiderivative = 4.63

method	result
parts	$-\frac{a}{4e(ex^2+d)^2}+\frac{b\left(-\frac{c^6\operatorname{arcsec}(cx)}{4e\left(c^2ex^2+c^2d\right)^2}+\frac{c\sqrt{c^2x^2-1}\left(4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d+e}{e}}\,c^4dex^2+4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d}{e}}\right.\right.}{}$
derivativedivides	$-\frac{a\,c^6}{4e\left(c^2ex^2+c^2d\right)^2}+bc^6\left(-\frac{\operatorname{arcsec}(cx)}{4e\left(c^2ex^2+c^2d\right)^2}+\frac{\sqrt{c^2x^2-1}\left(4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d+e}{e}}\,c^4d^2+4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d}{e}}\right.\right.}{}$
default	$-\frac{a\,c^6}{4e\left(c^2ex^2+c^2d\right)^2}+bc^6\left(-\frac{\operatorname{arcsec}(cx)}{4e\left(c^2ex^2+c^2d\right)^2}+\frac{\sqrt{c^2x^2-1}\left(4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d+e}{e}}\,c^4d^2+4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d}{e}}\right.\right.}{}$

input
```
int(x*(a+b*arcsec(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/e/(e*x^2+d)^2+b/c^2*(-1/4*c^6/e/(c^2*e*x^2+c^2*d)^2*arcsec(c*x)+1/1
6*c*(c^2*x^2-1)^(1/2)*(4*arctan(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)*
c^4*d*e*x^2+4*arctan(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)*c^4*d^2-3*l
n(-2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-c
*e*x+(-c^2*d*e)^(1/2)))*c^4*d*e*x^2-3*ln(-2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)
/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-c*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2-3*l
n(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(c*e
*x+(-c^2*d*e)^(1/2)))*c^4*d*e*x^2-3*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)
^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2+4*arcta
n(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)*e^2*c^2*x^2+4*arctan(1/(c^2*x^
2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)*c^2*d*e+2*(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e
)^(1/2)*c^2*d*e-2*ln(-2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*
e)^(1/2)*c*x-e)/(-c*e*x+(-c^2*d*e)^(1/2)))*e^2*c^2*x^2-2*ln(-2*((c^2*x^2-1
)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-c*e*x+(-c^2*d*e)^
(1/2)))*c^2*d*e-2*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e-(-c^2*d*e
)^(1/2)*c*x-e)/(c*e*x+(-c^2*d*e)^(1/2)))*e^2*c^2*x^2-2*ln(2*((c^2*x^2-1)^(
1/2)*(-(c^2*d+e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-c^2*d*e)^(1/2
)))*c^2*d*e)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/d^2/(-(c^2*d+e)/e)^(1/2)/(c^2*d
+e)/(-c*e*x+(-c^2*d*e)^(1/2))/(c*e*x+(-c^2*d*e)^(1/2)))
```

3.106.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(165) = 330$.

Time $= 0.49$ (sec) , antiderivative size $= 888$, normalized size of antiderivative $= 4.60$

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx$$

$$= \left[-\frac{4\,ac^4d^4+8\,ac^2d^3e+4\,ad^2e^2+(3\,bc^2d^3+(3\,bc^2de^2+2\,be^3)x^4+2\,bd^2e+2\,(3\,bc^2d^2e+2\,bde^2)x^2)\sqrt{-c^2}}{} \right.$$

$$\left. -\frac{2\,ac^4d^4+4\,ac^2d^3e+2\,ad^2e^2+(3\,bc^2d^3+(3\,bc^2de^2+2\,be^3)x^4+2\,bd^2e+2\,(3\,bc^2d^2e+2\,bde^2)x^2)\sqrt{c^2de}}{} \right.$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
[-1/16*(4*a*c^4*d^4 + 8*a*c^2*d^3*e + 4*a*d^2*e^2 + (3*b*c^2*d^3 + (3*b*c^
2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(3*b*c^2*d^2*e + 2*b*d*e^2)*x^2)*sq
rt(-c^2*d*e - e^2)*log((c^2*e*x^2 - c^2*d + 2*sqrt(-c^2*d*e - e^2)*sqrt(c^
2*x^2 - 1) - 2*e)/(e*x^2 + d)) + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2)
*arcsec(c*x) - 8*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 +
 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*
x^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)) + 2*(b*c^2*d^3*e + b*d^2*e^2 + (b*c^
2*d^2*e^2 + b*d*e^3)*x^2)*sqrt(c^2*x^2 - 1))/(c^4*d^6*e + 2*c^2*d^5*e^2 +
d^4*e^3 + (c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 + 2
*c^2*d^4*e^3 + d^3*e^4)*x^2), -1/8*(2*a*c^4*d^4 + 4*a*c^2*d^3*e + 2*a*d^2*
e^2 + (3*b*c^2*d^3 + (3*b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(3*b*c^
2*d^2*e + 2*b*d*e^2)*x^2)*sqrt(c^2*d*e + e^2)*arctan(sqrt(c^2*d*e + e^2)*s
qrt(c^2*x^2 - 1)/(c^2*d + e)) + 2*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2)*
arcsec(c*x) - 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 +
2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x
^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)) + (b*c^2*d^3*e + b*d^2*e^2 + (b*c^2*d
^2*e^2 + b*d*e^3)*x^2)*sqrt(c^2*x^2 - 1))/(c^4*d^6*e + 2*c^2*d^5*e^2 + d^4
*e^3 + (c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 + 2*c^
2*d^4*e^3 + d^3*e^4)*x^2)]
```

3.106.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x*(a+b*asec(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.106.7 Maxima [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x}{(ex^2+d)^3}\,dx$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/4*(4*(c^2*e^3*x^4 + 2*c^2*d*e^2*x^2 + c^2*d^2*e)*integrate(1/4*x*e^(1/2*
log(c*x + 1) + 1/2*log(c*x - 1))/(c^2*e^3*x^6 + (2*c^2*d*e^2 - e^3)*x^4 -
d^2*e + (c^2*d^2*e - 2*d*e^2)*x^2 + (c^2*e^3*x^6 + (2*c^2*d*e^2 - e^3)*x^4
 - d^2*e + (c^2*d^2*e - 2*d*e^2)*x^2)*e^(log(c*x + 1) + log(c*x - 1))), x)
 - arctan(sqrt(c*x + 1)*sqrt(c*x - 1)))*b/(e^3*x^4 + 2*d*e^2*x^2 + d^2*e)
- 1/4*a/(e^3*x^4 + 2*d*e^2*x^2 + d^2*e)
```

3.106.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{x\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)}{(e\,x^2+d)^3}\,dx$$

input
```
int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^3,x)
```

output
```
int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^3, x)
```

3.107 $\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx$

3.107.1 Optimal result 806
3.107.2 Mathematica [B] (warning: unable to verify) 807
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3.107.1 Optimal result

Integrand size = 21, antiderivative size = 685

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx = \frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2(a+b\sec^{-1}(cx))}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e(a+b\sec^{-1}(cx))}{d^3\left(e+\frac{d}{x^2}\right)}$$
$$+ \frac{i(a+b\sec^{-1}(cx))^2}{2bd^3} - \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{d^3\sqrt{c^2d+e}}$$
$$+ \frac{b\sqrt{e}(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{8d^3\left(c^2d+e\right)^{3/2}}$$
$$- \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}$$
$$- \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}$$
$$- \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}$$
$$- \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}$$
$$+ \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} + \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}$$
$$+ \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3} + \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}$$

output
```
1/4*e^2*(a+b*arcsec(c*x))/d^3/(e+d/x^2)^2-e*(a+b*arcsec(c*x))/d^3/(e+d/x^2
)+1/2*I*(a+b*arcsec(c*x))^2/b/d^3-1/2*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1
-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^3-1/2*(a+b*arcs
ec(c*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e
)^(1/2)))/d^3-1/2*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(
-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3-1/2*(a+b*arcsec(c*x))*ln(1+c*(1/c
/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3+1/2*I*
b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)
^(1/2)))/d^3+1/2*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/
(e^(1/2)-(c^2*d+e)^(1/2)))/d^3+1/2*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)
^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3+1/2*I*b*polylog(2,c*(1/c
/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3+1/8*b*
(c^2*d+2*e)*arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2)^(1/2))*e^(1/2
)/d^3/(c^2*d+e)^(3/2)-b*arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2)^(
1/2))*e^(1/2)/d^3/(c^2*d+e)^(1/2)+1/8*b*c*e*(1-1/c^2/x^2)^(1/2)/d^2/(c^2*d
+e)/(e+d/x^2)/x
```

3.107.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1871 vs. $2(685) = 1370$.

Time = 6.06 (sec) , antiderivative size = 1871, normalized size of antiderivative = 2.73

$$\int \frac{a + b \sec^{-1}(cx)}{x \left(d + ex^2\right)^3} \, dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x*(d + e*x^2)^3),x]
```

output

```
a/(4*d*(d + e*x^2)^2) + a/(2*d^2*(d + e*x^2)) + (a*Log[x])/d^3 - (a*Log[d
+ e*x^2])/(2*d^3) + b*((((-5*I)/16)*Sqrt[e]*(-(ArcSec[c*x]/(I*Sqrt[d]*Sqrt
[e] + e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e
] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-
(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/d^(5
/2) + (((5*I)/16)*Sqrt[e]*(-(ArcSec[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x)) - (
I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*(I*c*Sqr
t[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(
Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/d^(5/2) + (Sqrt[e]
*(-(ArcSec[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2)) + (ArcSin[1/(c*x)]
/Sqrt[e] - I*((c*Sqrt[d]*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/((c^2*d + e)*((-
I)*Sqrt[d] + Sqrt[e]*x)) + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e
]*(I*Sqrt[e] + c*(c*Sqrt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(
(2*c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x))])/(c^2*d + e)^(3/2)))/d))/(16*d^
2) + (Sqrt[e]*((I*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*
(I*Sqrt[d] + Sqrt[e]*x)) - ArcSec[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2)
 + ArcSin[1/(c*x)]/(d*Sqrt[e]) - (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^
2*d + e]*((-I)*Sqrt[e] + c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^
2)])*x))/((2*c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))
/(16*d^2) + ((I/2)*ArcSec[c*x]^2 - ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[...
```

3.107.3 Rubi [A] (verified)

Time = 1.82 (sec) , antiderivative size = 745, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5763, 5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \sec^{-1}(cx)}{x \left(d + ex^2\right)^3} \, dx$$

$$\downarrow 5763$$

$$-\int \frac{a + b \arccos\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2} + e\right)^3 x^5} d\frac{1}{x}$$

$$\downarrow 5233$$

$$-\int \left(\frac{\left(a + b \arccos\left(\frac{1}{cx}\right)\right) e^2}{d^2 \left(\frac{d}{x^2} + e\right)^3 x} - \frac{2\left(a + b \arccos\left(\frac{1}{cx}\right)\right) e}{d^2 \left(\frac{d}{x^2} + e\right)^2 x} + \frac{a + b \arccos\left(\frac{1}{cx}\right)}{d^2 \left(\frac{d}{x^2} + e\right) x} \right) d\frac{1}{x}$$

$$\downarrow \text{2009}$$

$$-\frac{(a+b\arccos\left(\frac{1}{cx}\right))\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}-\frac{(a+b\arccos\left(\frac{1}{cx}\right))\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}-$$
$$\frac{(a+b\arccos\left(\frac{1}{cx}\right))\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^3}-\frac{(a+b\arccos\left(\frac{1}{cx}\right))\log\left(1+\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^3}+$$
$$\frac{e^2(a+b\arccos\left(\frac{1}{cx}\right))}{4d^3\left(\frac{d}{x^2}+e\right)^2}-\frac{e(a+b\arccos\left(\frac{1}{cx}\right))}{d^3\left(\frac{d}{x^2}+e\right)}+\frac{i(a+b\arccos\left(\frac{1}{cx}\right))^2}{2bd^3}+$$
$$\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^3}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^3}+$$
$$\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^3}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^3}+$$
$$\frac{b\sqrt{e}(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{8d^3(c^2d+e)^{3/2}}-\frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{d^3\sqrt{c^2d+e}}+\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2x(c^2d+e)\left(\frac{d}{x^2}+e\right)}$$

input

```
Int[(a + b*ArcSec[c*x])/(x*(d + e*x^2)^3),x]
```

output

```
(b*c*e*Sqrt[1 - 1/(c^2*x^2)])/(8*d^2*(c^2*d + e)*(e + d/x^2)*x) + (e^2*(a
+ b*ArcCos[1/(c*x)]))/(4*d^3*(e + d/x^2)^2) - (e*(a + b*ArcCos[1/(c*x)]))/
(d^3*(e + d/x^2)) + ((I/2)*(a + b*ArcCos[1/(c*x)])^2)/(b*d^3) - (b*Sqrt[e]
*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)])/(d^3*Sqrt[c^
2*d + e]) + (b*Sqrt[e]*(c^2*d + 2*e)*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqr
t[1 - 1/(c^2*x^2)]*x)])/(8*d^3*(c^2*d + e)^(3/2)) - ((a + b*ArcCos[1/(c*x)
])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])]
)/(2*d^3) - ((a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*
x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^3) - ((a + b*ArcCos[1/(c*x)])*Log
[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d
^3) - ((a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/
(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^3) + ((I/2)*b*PolyLog[2, -((c*Sqrt[-d]*
E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/d^3 + ((I/2)*b*PolyL
og[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/d^3
 + ((I/2)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqr
t[c^2*d + e]))])/d^3 + ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)
]))/(Sqrt[e] + Sqrt[c^2*d + e])])/d^3
```

3.107.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

3.107.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.87 (sec) , antiderivative size = 3533, normalized size of antiderivative = 5.16

method	result	size
parts	Expression too large to display	3533
derivativedivides	Expression too large to display	3607
default	Expression too large to display	3607

input
```
int((a+b*arcsec(c*x))/x/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output

```
a/d^3*ln(x)+1/2*a/d^2/(e*x^2+d)-1/2*a/d^3*ln(e*x^2+d)+1/4*a/d/(e*x^2+d)^2+
b*(1/4*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2
)/e/(c^4*d^2+2*c^2*d*e+e^2)*c^2/d^2*ln(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2
))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsec(c*x)-I*(-(e*(c^2*d+e))^(1/
2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*e*polylog(2,d*c^2*(1/c/x
+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))/d^4/(c^4*d^2
+2*c^2*d*e+e^2)/c^2+3/4*I*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*polylog(2,d*c^
2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*e/c^
2/d^4/(c^2*d+e)+2*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1
/2)*e+2*e^2)/d^4*e/(c^4*d^2+2*c^2*d*e+e^2)/c^2*ln(1-d*c^2*(1/c/x+I*(1-1/c^
2/x^2)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsec(c*x)-I*(-(e*(c^
2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*e^2*arcsec(c*
x)^2/(c^4*d^2+2*c^2*d*e+e^2)/d^5/c^4-1/8*I*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2
/d/e*polylog(2,d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e
))^(1/2)-2*e))*c^4+3/2*I*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*arcsec(c*x)^2*e
/c^2/d^4/(c^2*d+e)+I*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*arcsec(c*x)^2*e^2/c
^4/d^5/(c^2*d+e)-1/2*I*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e
))^(1/2)*e+2*e^2)*e^2*polylog(2,d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^
2*d-2*(e*(c^2*d+e))^(1/2)-2*e))/(c^4*d^2+2*c^2*d*e+e^2)/d^5/c^4-2*I*(-(e*(
c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*e*arcsec...
```

3.107.5 Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx = \int \frac{b\,\text{arcsec}\left(cx\right)+a}{\left(ex^2+d\right)^3 x}\,dx$$

input

```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^3,x, algorithm="fricas")
```

output

```
integral((b*arcsec(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x)
, x)
```

3.107.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b \sec^{-1}(cx)}{x \left(d + ex^2\right)^3} \, dx = \text{Timed out}$$

input
```
integrate((a+b*asec(c*x))/x/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.107.7 Maxima [F]

$$\int \frac{a + b \sec^{-1}(cx)}{x \left(d + ex^2\right)^3} \, dx = \int \frac{b \operatorname{arcsec}(cx) + a}{\left(ex^2 + d\right)^3 x} \, dx$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/4*a*((2*e*x^2 + 3*d)/(d^2*e^2*x^4 + 2*d^3*e*x^2 + d^4) - 2*log(e*x^2 + d
)/d^3 + 4*log(x)/d^3) + b*integrate(arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(e
^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x), x)
```

3.107.8 Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \sec^{-1}(cx)}{x \left(d + ex^2\right)^3} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \sec^{-1}(cx)}{x\left(d + ex^2\right)^3}\,dx = \int \frac{a + b\,\text{acos}\left(\frac{1}{cx}\right)}{x\left(e\,x^2 + d\right)^3}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^3),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^3), x)
```

3.108 $\int \frac{x^4\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

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3.108.1 Optimal result

Integrand size = 21, antiderivative size = 1124

$$\begin{aligned}
\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx &= \frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} \\
&+ \frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{-d}(a+b\sec^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} \\
&+ \frac{3(a+b\sec^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}(a+b\sec^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} \\
&- \frac{3(a+b\sec^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e\left(c^2d+e\right)^{3/2}} \\
&+ \frac{3b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{c^2d+e}} \\
&+ \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e\left(c^2d+e\right)^{3/2}} \\
&+ \frac{3b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{c^2d+e}} \\
&+ \frac{3(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&- \frac{3(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&+ \frac{3(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&- \frac{3(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&+ \frac{3ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&- \frac{3ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&+ \frac{3ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&- \frac{3ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}}
\end{aligned}$$

output
```
3/16*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^
(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)-3/16*(a+b*arcsec(c*x))*ln(1+c*(
1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)
/(-d)^(1/2)+3/16*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-
d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)-3/16*(a+b*arcsec(c*
x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/
2)))/e^(5/2)/(-d)^(1/2)+3/16*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2)
)*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+3/16*I*b*polylo
g(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))
/e^(5/2)/(-d)^(1/2)-3/16*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d
)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)-3/16*I*b*polylog(2,c
*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(5/
2)/(-d)^(1/2)+1/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d
+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/e/(c^2*d+e)^(3/2)/d^(1/2)+1/16*b*arctanh((c
^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/
e/(c^2*d+e)^(3/2)/d^(1/2)+1/16*(a+b*arcsec(c*x))*(-d)^(1/2)/e^(3/2)/(-d/x+
(-d)^(1/2)*e^(1/2))^2+3/16*(a+b*arcsec(c*x))/e^2/(-d/x+(-d)^(1/2)*e^(1/2))
-1/16*(a+b*arcsec(c*x))*(-d)^(1/2)/e^(3/2)/(d/x+(-d)^(1/2)*e^(1/2))^2-3/16
*(a+b*arcsec(c*x))/e^2/(d/x+(-d)^(1/2)*e^(1/2))+3/16*b*arctanh((c^2*d-(-d)
^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/e^2/d^...
```

3.108.2 Mathematica [A] (warning: unable to verify)

Time = 6.07 (sec) , antiderivative size = 1819, normalized size of antiderivative = 1.62

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^3,x]
```

output

```
(a*d*x)/(4*e^2*(d + e*x^2)^2) - (5*a*x)/(8*e^2*(d + e*x^2)) + (3*a*ArcTan[
(Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[d]*e^(5/2)) + b*((5*(-(ArcSec[c*x]/(I*Sqrt[d
]*Sqrt[e] + e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(
Sqrt[e] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(
Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d])
)/(16*e^2) + (5*(-(ArcSec[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSin[
1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*(I*c*Sqrt[d] + Sqr
t[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] +
I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/(16*e^2) + ((I/16)*Sqrt[d]*(
-(ArcSec[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2)) + (ArcSin[1/(c*x)]/S
qrt[e] - I*((c*Sqrt[d]*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/((c^2*d + e)*((-I)
*Sqrt[d] + Sqrt[e]*x)) + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*
(I*Sqrt[e] + c*(c*Sqrt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2
*c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x))])/(c^2*d + e)^(3/2)))/d))/e^2 - ((
I/16)*Sqrt[d]*((I*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*
(I*Sqrt[d] + Sqrt[e]*x)) - ArcSec[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2)
 + ArcSin[1/(c*x)]/(d*Sqrt[e]) - (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^
2*d + e]*((-I)*Sqrt[e] + c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^
2)])*x))/((2*c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))
/e^2 + (3*(8*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(...
```

3.108.3 Rubi [A] (verified)

Time = 2.25 (sec) , antiderivative size = 1188, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5763, 5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$$

$$\downarrow 5763$$

$$-\int \frac{a+b\arccos\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^3}d\frac{1}{x}$$

$$\downarrow 5173$$

$$-\int\left(-\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)d^3}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^3}-\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)d^3}{8(-d)^{3/2}e^{3/2}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^3}-\frac{3\left(a+b\arccos\left(\frac{1}{cx}\right)\right)d}{8e^2\left(-\frac{d^2}{x^2}-ed\right)}-\frac{3\left(a+b\arccos\right.}{16e^2\left(\sqrt{-d}\sqrt{e}\right.}\right.$$

$$\downarrow 2009$$

$$\frac{b\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}c}{16e^{3/2}\left(dc^2+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{b\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}c}{16e^{3/2}\left(dc^2+e\right)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{3\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-$$
$$\frac{3\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{16e^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{\sqrt{-d}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}-\frac{\sqrt{-d}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{16e^{3/2}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2}+$$
$$\frac{3b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{dc^2+e}}+\frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e\left(dc^2+e\right)^{3/2}}+$$
$$\frac{3b\operatorname{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{dc^2+e}}+\frac{b\operatorname{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e\left(dc^2+e\right)^{3/2}}+$$
$$\frac{3\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}-\frac{3\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)}{16\sqrt{-d}e^{5/2}}+$$
$$\frac{3\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}-\frac{3\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)}{16\sqrt{-d}e^{5/2}}+$$
$$\frac{3ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}-\frac{3ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}+$$
$$\frac{3ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}-\frac{3ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}$$

input
```
Int[(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^3,x]
```

output
```
(b*c*Sqrt[-d]*Sqrt[1 - 1/(c^2*x^2)])/(16*e^(3/2)*(c^2*d + e)*(Sqrt[-d]*Sqr
t[e] - d/x)) + (b*c*Sqrt[-d]*Sqrt[1 - 1/(c^2*x^2)])/(16*e^(3/2)*(c^2*d + e
)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[-d]*(a + b*ArcCos[1/(c*x)]))/(16*e^(3/
2)*(Sqrt[-d]*Sqrt[e] - d/x)^2) + (3*(a + b*ArcCos[1/(c*x)]))/(16*e^2*(Sqrt
[-d]*Sqrt[e] - d/x)) - (Sqrt[-d]*(a + b*ArcCos[1/(c*x)]))/(16*e^(3/2)*(Sqr
t[-d]*Sqrt[e] + d/x)^2) - (3*(a + b*ArcCos[1/(c*x)]))/(16*e^2*(Sqrt[-d]*Sq
rt[e] + d/x)) + (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[
c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*Sqrt[d]*e*(c^2*d + e)^(3/2)) + (3*
b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1
 - 1/(c^2*x^2)])])/(16*Sqrt[d]*e^2*Sqrt[c^2*d + e]) + (b*ArcTanh[(c^2*d +
(Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/
(16*Sqrt[d]*e*(c^2*d + e)^(3/2)) + (3*b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e]
)/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*Sqrt[d]*e^2*S
qrt[c^2*d + e]) + (3*(a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcC
os[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) - (3*(a
+ b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] -
 Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) + (3*(a + b*ArcCos[1/(c*x)])*Log
[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*
Sqrt[-d]*e^(5/2)) - (3*(a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*Ar
cCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) + (...
```

3.108.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

3.108.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 66.80 (sec) , antiderivative size = 1822, normalized size of antiderivative = 1.62

method	result	size
parts	Expression too large to display	1822
derivativedivides	Expression too large to display	1845
default	Expression too large to display	1845

input
```
int(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a*((-5/8/e*x^3-3/8*d/e^2*x)/(e*x^2+d)^2+3/8/e^2/(d*e)^(1/2)*arctan(e*x/(d*
e)^(1/2)))+b/c^5*(-1/8*x*c^7*(3*d^2*c^4*arcsec(c*x)+5*c^4*d*e*arcsec(c*x)*
x^2-((c^2*x^2-1)/c^2/x^2)^(1/2)*c^3*d*e*x-((c^2*x^2-1)/c^2/x^2)^(1/2)*e^2*
c^3*x^3+3*c^2*d*e*arcsec(c*x)+5*e^2*arcsec(c*x)*c^2*x^2)/e^2/(c^2*d+e)/(c^
2*e*x^2+c^2*d)^2-1/2*I*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d
+2*(e*(c^2*d+e))^(1/2)+2*e)*c*arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((
-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/e/d^3+3/8*I*(-(c^2*d
-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+
2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*c^3*arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2
))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2/e^2/d^2+3/16*
I/(c^2*d+e)/e*c^6*sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1
/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_
R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-1/2*I*((c^2*d+2*(e*(
c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*c*arctan(c
*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/
2))/(c^2*d+e)/e/d^3-3/16*I/(c^2*d+e)/e*c^6*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*
e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c
/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c
^2*d))+1/2*I*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-(e*(c^2*d+e))^(
1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*c*arctan(c*d*(1/c/x...
```

3.108.5 Fricas [F]

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x^4}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^4*arcsec(c*x) + a*x^4)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

3.108.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(a+b*asec(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.108.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.108.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{x^4\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)}{(e\,x^2+d)^3}\,dx$$

input
```
int((x^4*(a + b*acos(1/(c*x))))/(d + e*x^2)^3,x)
```

output
```
int((x^4*(a + b*acos(1/(c*x))))/(d + e*x^2)^3, x)
```

3.109 $\int \frac{x^2\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

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3.109.2 Mathematica [A] (warning: unable to verify) 825
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3.109.9 Mupad [F(-1)] 831

3.109.1 Optimal result

Integrand size = 21, antiderivative size = 1124

$$\begin{aligned}
\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx &= \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e}\,(c^2d+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} \\
&+ \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e}\,(c^2d+e)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&+ \frac{a+b\sec^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{a+b\sec^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} \\
&- \frac{a+b\sec^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} - \frac{a+b\sec^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&- \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\left(c^2d+e\right)^{3/2}} + \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}e\sqrt{c^2d+e}} \\
&- \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\left(c^2d+e\right)^{3/2}} + \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}e\sqrt{c^2d+e}} \\
&- \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} \\
&+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} \\
&- \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} \\
&+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} \\
&- \frac{ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} \\
&+ \frac{ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} \\
&- \frac{ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} \\
&+ \frac{ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}}
\end{aligned}$$

output

```
-1/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-
1/c^2/x^2)^(1/2))/d^(3/2)/(c^2*d+e)^(3/2)-1/16*b*arctanh((c^2*d+(-d)^(1/2)
*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(3/2)/(c^2*d+
e)^(3/2)-1/16*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^
(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*(a+b*arcsec(c*x))
*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2))
)/(-d)^(3/2)/e^(3/2)-1/16*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^
(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*(a+b*
arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2
*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^
2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*I*
b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)
^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1
/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*I*b*pol
ylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2
)))/(-d)^(3/2)/e^(3/2)+1/16*(a+b*arcsec(c*x))/(-d)^(1/2)/e^(1/2)/(-d/x+(-d
)^(1/2)*e^(1/2))^2+1/16*(a+b*arcsec(c*x))/d/e/(-d/x+(-d)^(1/2)*e^(1/2))+1/
16*(-a-b*arcsec(c*x))/(-d)^(1/2)/e^(1/2)/(d/x+(-d)^(1/2)*e^(1/2))^2+1/16*(
-a-b*arcsec(c*x))/d/e/(d/x+(-d)^(1/2)*e^(1/2))+1/16*b*arctanh((c^2*d-(-d)^
(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(3/2)...
```

3.109.2 Mathematica [A] (warning: unable to verify)

Time = 6.07 (sec) , antiderivative size = 1827, normalized size of antiderivative = 1.63

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Too large to display}$$

input

```
Integrate[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2)^3,x]
```

output

```
-1/4*(a*x)/(e*(d + e*x^2)^2) + (a*x)/(8*d*e*(d + e*x^2)) + (a*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(3/2)) + b*(-1/16*(-(ArcSec[c*x]/(I*Sqrt[d]*
Sqrt[e] + e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sq
rt[e] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sq
rt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d])/(
d*e) - (-(ArcSec[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSin[1/(c*x)]/
Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d
) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]
*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d])/(16*d*e) - ((I/16)*(-(ArcSec[c*x]/(Sqr
t[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2)) + (ArcSin[1/(c*x)]/Sqrt[e] - I*((c*Sqr
t[d]*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/((c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]
*x)) + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(I*Sqrt[e] + c*(c*
Sqrt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)*((-I)*
Sqrt[d] + Sqrt[e]*x))])/(c^2*d + e)^(3/2)))/d))/(Sqrt[d]*e) + ((I/16)*((I*
c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[
e]*x)) - ArcSec[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) + ArcSin[1/(c*x)]
/(d*Sqrt[e]) - (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d + e]*((-I)*Sqr
t[e] + c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d
 + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/(Sqrt[d]*e) + (8*
ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] ...
```

3.109.3 Rubi [A] (verified)

Time = 3.43 (sec) , antiderivative size = 1188, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5763, 5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$$

$$\downarrow 5763$$

$$-\int \frac{a+b\arccos\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^3 x^2}d\frac{1}{x}$$

$$\downarrow 5233$$

$$-\int \left(\frac{a+b\arccos\left(\frac{1}{cx}\right)}{d\left(\frac{d}{x^2}+e\right)^2}-\frac{e\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{d\left(\frac{d}{x^2}+e\right)^3}\right)d\frac{1}{x}$$

↓ 2009

$$\frac{b\sqrt{1-\frac{1}{c^2x^2}}c}{16\sqrt{-d}\sqrt{e}\left(dc^2+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}c}{16\sqrt{-d}\sqrt{e}\left(dc^2+e\right)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{a+b\arccos\left(\frac{1}{cx}\right)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-$$
$$\frac{a+b\arccos\left(\frac{1}{cx}\right)}{16de\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{a+b\arccos\left(\frac{1}{cx}\right)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}-\frac{a+b\arccos\left(\frac{1}{cx}\right)}{16\sqrt{-d}\sqrt{e}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2}+$$
$$\frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}e\sqrt{dc^2+e}}-\frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\left(dc^2+e\right)^{3/2}}+$$
$$\frac{b\operatorname{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}e\sqrt{dc^2+e}}-\frac{b\operatorname{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\left(dc^2+e\right)^{3/2}}-$$
$$\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}+\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)}{16(-d)^{3/2}e^{3/2}}-$$
$$\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}+\frac{\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)}{16(-d)^{3/2}e^{3/2}}-$$
$$\frac{ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}+\frac{ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}-$$
$$\frac{ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}+\frac{ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}$$

input
```
Int[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2)^3,x]
```

output
```
(b*c*Sqrt[1 - 1/(c^2*x^2)])/(16*Sqrt[-d]*Sqrt[e]*(c^2*d + e)*(Sqrt[-d]*Sqr
t[e] - d/x)) + (b*c*Sqrt[1 - 1/(c^2*x^2)])/(16*Sqrt[-d]*Sqrt[e]*(c^2*d + e
)*(Sqrt[-d]*Sqrt[e] + d/x)) + (a + b*ArcCos[1/(c*x)])/(16*Sqrt[-d]*Sqrt[e]
*(Sqrt[-d]*Sqrt[e] - d/x)^2) + (a + b*ArcCos[1/(c*x)])/(16*d*e*(Sqrt[-d]*S
qrt[e] - d/x)) - (a + b*ArcCos[1/(c*x)])/(16*Sqrt[-d]*Sqrt[e]*(Sqrt[-d]*Sq
rt[e] + d/x)^2) - (a + b*ArcCos[1/(c*x)])/(16*d*e*(Sqrt[-d]*Sqrt[e] + d/x)
) - (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*S
qrt[1 - 1/(c^2*x^2)])])/(16*d^(3/2)*(c^2*d + e)^(3/2)) + (b*ArcTanh[(c^2*d
 - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])
])/(16*d^(3/2)*e*Sqrt[c^2*d + e]) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])
/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(3/2)*(c^2*d
 + e)^(3/2)) + (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c
^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(3/2)*e*Sqrt[c^2*d + e]) - ((a +
b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - S
qrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) + ((a + b*ArcCos[1/(c*x)])*Log[1
 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-
d)^(3/2)*e^(3/2)) - ((a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcC
os[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) + ((a
+ b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] +
 Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) - ((I/16)*b*PolyLog[2, -((c...
```

3.109.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5763
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

3.109.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 33.26 (sec) , antiderivative size = 1288, normalized size of antiderivative = 1.15

method	result	size
parts	Expression too large to display	1288
derivativedivides	Expression too large to display	1307
default	Expression too large to display	1307

input
```
int(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a*((1/8/d*x^3-1/8/e*x)/(e*x^2+d)^2+1/8/e/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1
/2)))+b/c^3*(1/8*x*c^5*(c^4*d*e*arcsec(c*x)*x^2-d^2*c^4*arcsec(c*x)-((c^2*
x^2-1)/c^2/x^2)^(1/2)*e^2*c^3*x^3-((c^2*x^2-1)/c^2/x^2)^(1/2)*c^3*d*e*x+e^
2*arcsec(c*x)*c^2*x^2-c^2*d*e*arcsec(c*x))/d/e/(c^2*d+e)/(c^2*e*x^2+c^2*d)
^2-1/8*I*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e)
)^(1/2)+2*e)*c*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*
d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)/e/d^3+1/8*I*((c^2*d+2*(e*(c^2*d+e))^(
1/2)+2*e)*d)^(1/2)*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(
1/2)*e+2*e^2)*c*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2
*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2/e/d^3-1/8*I*(-(c^2*d-2*(e*(c^2*d+e
))^(1/2)+2*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*c*arctanh(c*d*(1/
c/x+I*(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(
c^2*d+e)/e/d^3+1/8*I*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2
*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*c*arctanh(c*d*
(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2)
)/(c^2*d+e)^2/e/d^3+1/16*I/(c^2*d+e)/d*c^4*sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)
*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x
-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2
*d))+1/16*I/(c^2*d+e)/e*c^6*sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)
*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/...
```

3.109.5 Fricas [F]

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x^2}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^2*arcsec(c*x) + a*x^2)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

3.109.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*asec(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.109.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.109.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{x^2\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)}{(e\,x^2+d)^3}\,dx$$

input
```
int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^3,x)
```

output
```
int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^3, x)
```

3.110 $\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^3}\,dx$

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3.110.2 Mathematica [A] (warning: unable to verify) 834
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3.110.1 Optimal result

Integrand size = 18, antiderivative size = 1114

$$\begin{aligned}
\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^3}\,dx &= \frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} \\
&+ \frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&+ \frac{\sqrt{e}(a+b\sec^{-1}(cx))}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} - \frac{5(a+b\sec^{-1}(cx))}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} \\
&- \frac{\sqrt{e}(a+b\sec^{-1}(cx))}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} + \frac{5(a+b\sec^{-1}(cx))}{16d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&+ \frac{be\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\left(c^2d+e\right)^{3/2}} - \frac{5b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\sqrt{c^2d+e}} \\
&+ \frac{be\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\left(c^2d+e\right)^{3/2}} - \frac{5b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\sqrt{c^2d+e}} \\
&+ \frac{3(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} \\
&- \frac{3(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} \\
&+ \frac{3(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} \\
&- \frac{3(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} \\
&+ \frac{3ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} \\
&+ \frac{3ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}}
\end{aligned}$$

output

```
1/16*b*e*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1
-1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d+e)^(3/2)+1/16*b*e*arctanh((c^2*d+(-d)^(1
/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(5/2)/(c^2
*d+e)^(3/2)+3/16*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-
d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*(a+b*arcsec(c*
x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/
2)))/(-d)^(5/2)/e^(1/2)+3/16*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^
2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*(a
+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(
c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^
2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/1
6*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d
+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2
)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*I*b
*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(
1/2)))/(-d)^(5/2)/e^(1/2)+1/16*(a+b*arcsec(c*x))*e^(1/2)/(-d)^(3/2)/(-d/x+
(-d)^(1/2)*e^(1/2))^2-5/16*(a+b*arcsec(c*x))/d^2/(-d/x+(-d)^(1/2)*e^(1/2))
-1/16*(a+b*arcsec(c*x))*e^(1/2)/(-d)^(3/2)/(d/x+(-d)^(1/2)*e^(1/2))^2+5/16
*(a+b*arcsec(c*x))/d^2/(d/x+(-d)^(1/2)*e^(1/2))-5/16*b*arctanh((c^2*d-(-d)
^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(5/2...
```

3.110.2 Mathematica [A] (warning: unable to verify)

Time = 6.04 (sec) , antiderivative size = 1812, normalized size of antiderivative = 1.63

$$\int \frac{a + b \sec^{-1}(cx)}{(d + ex^2)^3}\,dx = \text{Too large to display}$$

input

```
Integrate[(a + b*ArcSec[c*x])/(d + e*x^2)^3,x]
```

output

```
(a*x)/(4*d*(d + e*x^2)^2) + (3*a*x)/(8*d^2*(d + e*x^2)) + (3*a*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]) + b*((-3*(-(ArcSec[c*x]/(I*Sqrt[d]*S
qrt[e] + e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sqr
t[e] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqr
t[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/(
16*d^2) - (3*(-(ArcSec[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSin[1/(
c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-
(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*S
qrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/(16*d^2) + ((I/16)*(-(ArcSec[c*
x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2)) + (ArcSin[1/(c*x)]/Sqrt[e] - I*
((c*Sqrt[d]*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/((c^2*d + e)*((-I)*Sqrt[d] +
Sqrt[e]*x)) + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(I*Sqrt[e]
+ c*(c*Sqrt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)
*((-I)*Sqrt[d] + Sqrt[e]*x))])/(c^2*d + e)^(3/2)))/d))/d^(3/2) - ((I/16)*(
(I*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sq
rt[e]*x)) - ArcSec[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) + ArcSin[1/(c*
x)]/(d*Sqrt[e]) - (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d + e]*((-I)*
Sqrt[e] + c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^
2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/d^(3/2) + (3*(
8*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d...
```

3.110.3 Rubi [A] (verified)

Time = 4.27 (sec) , antiderivative size = 1178, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5753, 5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b\sec^{-1}(cx)}{(d + ex^2)^3}\,dx$$

$$\downarrow 5753$$

$$-\int \frac{a + b\arccos\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2} + e\right)^3 x^4} d\frac{1}{x}$$

$$\downarrow 5233$$

$$-\int \left(\frac{\left(a + b\arccos\left(\frac{1}{cx}\right)\right)e^2}{d^2\left(\frac{d}{x^2} + e\right)^3} - \frac{2\left(a + b\arccos\left(\frac{1}{cx}\right)\right)e}{d^2\left(\frac{d}{x^2} + e\right)^2} + \frac{a + b\arccos\left(\frac{1}{cx}\right)}{d^2\left(\frac{d}{x^2} + e\right)}\right) d\frac{1}{x}$$

↓ 2009

$$\frac{b\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}c}{16(-d)^{3/2}\left(dc^2+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{b\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}c}{16(-d)^{3/2}\left(dc^2+e\right)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}-\frac{5\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+$$
$$\frac{5\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{16d^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{\sqrt{e}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}-\frac{\sqrt{e}\left(a+b\arccos\left(\frac{1}{cx}\right)\right)}{16(-d)^{3/2}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2}-$$
$$\frac{5b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\sqrt{dc^2+e}}+\frac{be\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\left(dc^2+e\right)^{3/2}}-$$
$$\frac{5b\operatorname{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\sqrt{dc^2+e}}+\frac{be\operatorname{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\left(dc^2+e\right)^{3/2}}+$$
$$\frac{3\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}-\frac{3\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)}{16(-d)^{5/2}\sqrt{e}}+$$
$$\frac{3\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}-\frac{3\left(a+b\arccos\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)}{16(-d)^{5/2}\sqrt{e}}+$$
$$\frac{3ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}-\frac{3ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}+$$
$$\frac{3ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}-\frac{3ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\arccos\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}$$

input
```
Int[(a + b*ArcSec[c*x])/(d + e*x^2)^3,x]
```

output
```
(b*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)])/(16*(-d)^(3/2)*(c^2*d + e)*(Sqrt[-d]*S
qrt[e] - d/x)) + (b*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)])/(16*(-d)^(3/2)*(c^2*d
 + e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[e]*(a + b*ArcCos[1/(c*x)]))/(16*(-
d)^(3/2)*(Sqrt[-d]*Sqrt[e] - d/x)^2) - (5*(a + b*ArcCos[1/(c*x)]))/(16*d^2
*(Sqrt[-d]*Sqrt[e] - d/x)) - (Sqrt[e]*(a + b*ArcCos[1/(c*x)]))/(16*(-d)^(3
/2)*(Sqrt[-d]*Sqrt[e] + d/x)^2) + (5*(a + b*ArcCos[1/(c*x)]))/(16*d^2*(Sqr
t[-d]*Sqrt[e] + d/x)) + (b*e*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqr
t[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*(c^2*d + e)^(3/2
)) - (5*b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e
]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*Sqrt[c^2*d + e]) + (b*e*ArcTanh[(c^
2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2
)])])/(16*d^(5/2)*(c^2*d + e)^(3/2)) - (5*b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqr
t[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*S
qrt[c^2*d + e]) + (3*(a + b*ArcCos[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^(I*ArcC
os[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*(
a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e]
 - Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) + (3*(a + b*ArcCos[1/(c*x)])
*Log[1 - (c*Sqrt[-d]*E^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/
(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcCos[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E
^(I*ArcCos[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt...
```

3.110.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5753
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(2*(p + 1)))
, x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p]
```

3.110.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 94.04 (sec) , antiderivative size = 1812, normalized size of antiderivative = 1.63

method	result	size
parts	Expression too large to display	1812
derivativedivides	Expression too large to display	1837
default	Expression too large to display	1837

input
```
int((a+b*arcsec(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x/d/(e*x^2+d)^2+3/8*a/d^2*x/(e*x^2+d)+3/8*a/d^2/(d*e)^(1/2)*arctan(e
*x/(d*e)^(1/2))+b/c*(1/8*x*c^3*(5*d^2*c^4*arcsec(c*x)+3*c^4*d*e*arcsec(c*x
)*x^2+((c^2*x^2-1)/c^2/x^2)^(1/2)*c^3*d*e*x+((c^2*x^2-1)/c^2/x^2)^(1/2)*e^
2*c^3*x^3+5*c^2*d*e*arcsec(c*x)+3*e^2*arcsec(c*x)*c^2*x^2)/d^2/(c^2*d+e)/(
c^2*e*x^2+c^2*d)^2-5/8*I*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*
(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*arctanh(c*
d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/
2))/(c^2*d+e)^2/d^4/c-3/16*I/(c^2*d+e)/d^2*c^2*e*sum(1/_R1/(_R1^2*c^2*d+c^
2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_
R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*
_Z^2+c^2*d))+1/2*I*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d+2*(
e*(c^2*d+e))^(1/2)+2*e)*arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((-c^2*d
+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))*e/c^3/d^5/(c^2*d+e)-1/2*I*((c^2*d+2*
(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*
(e*(c^2*d+e))^(1/2)*e+2*e^2)*e*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((
c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2/d^5/c^3+5/8*I*((c^2
*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*a
rctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)
*d)^(1/2))/c/d^4/(c^2*d+e)+5/8*I*arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))
/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))*(-(c^2*d-2*(e*(c^2*d+e))...
```

3.110.5 Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^3}\,dx = \int \frac{b\,\text{arcsec}\,(cx)+a}{(ex^2+d)^3}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arcsec(c*x) + a)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3),
x)
```

3.110.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*asec(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.110.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.110.8 Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \sec^{-1}(cx)}{(d + ex^2)^3}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \sec^{-1}(cx)}{(d + ex^2)^3}\, dx = \int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{(e\,x^2 + d)^3}\, dx$$

input
```
int((a + b*acos(1/(c*x)))/(d + e*x^2)^3,x)
```

output
```
int((a + b*acos(1/(c*x)))/(d + e*x^2)^3, x)
```

3.111 $\int x^5\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx$

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3.111.1 Optimal result

Integrand size = 23, antiderivative size = 403

$$\begin{aligned}
&\int x^5\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx \\
&= \frac{b(23c^4d^2+12c^2de-75e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} \\
&\quad+\frac{b(29c^2d-25e)\,x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{840c^3e^2\sqrt{c^2x^2}}-\frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce^2\sqrt{c^2x^2}} \\
&\quad+\frac{d^2(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{3e^3}-\frac{2d(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5e^3} \\
&\quad+\frac{(d+ex^2)^{7/2}\,(a+b\sec^{-1}(cx))}{7e^3}+\frac{8bcd^{7/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{105e^3\sqrt{c^2x^2}} \\
&\quad-\frac{b(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3)\,x\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{1680c^6e^{5/2}\sqrt{c^2x^2}}
\end{aligned}$$

output

```
1/3*d^2*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/e^3-2/5*d*(e*x^2+d)^(5/2)*(a+b*a
rcsec(c*x))/e^3+1/7*(e*x^2+d)^(7/2)*(a+b*arcsec(c*x))/e^3+8/105*b*c*d^(7/2
)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e^3/(c^2*x^2)^(1/2)-
1/1680*b*(105*c^6*d^3-35*c^4*d^2*e+63*c^2*d*e^2+75*e^3)*x*arctanh(e^(1/2)*
(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^6/e^(5/2)/(c^2*x^2)^(1/2)+1/840*b*(
29*c^2*d-25*e)*x*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(1/2)/c^3/e^2/(c^2*x^2)^(1/2)
-1/42*b*x*(e*x^2+d)^(5/2)*(c^2*x^2-1)^(1/2)/c/e^2/(c^2*x^2)^(1/2)+1/1680*b
*(23*c^4*d^2+12*c^2*d*e-75*e^2)*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c^5/e^
2/(c^2*x^2)^(1/2)
```

3.111.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.62 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.85

$$\int x^5\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$=\frac{32a(d+ex^2)\left(8d^3-4d^2ex^2+3de^2x^4+15e^3x^6\right)-\frac{2be\sqrt{1-\frac{1}{c^2x^2}}x\left(d+ex^2\right)\left(75e^2+2c^2e\left(19d+25ex^2\right)+c^4\left(-41d^2+22dex^2+40e\right.\right.}{c^5}$$

input
```
Integrate[x^5*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
```

output
```
(32*a*(d + e*x^2)*(8*d^3 - 4*d^2*e*x^2 + 3*d*e^2*x^4 + 15*e^3*x^6) - (2*b*
e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2)*(75*e^2 + 2*c^2*e*(19*d + 25*e*x^2)
+ c^4*(-41*d^2 + 22*d*e*x^2 + 40*e^2*x^4)))/c^5 + (128*b*d^4*Sqrt[1 + d/(e
*x^2)]*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))])/(c*x) + (b*e*(
105*c^6*d^3 - 35*c^4*d^2*e + 63*c^2*d*e^2 + 75*e^3)*Sqrt[1 - 1/(c^2*x^2)]*
x^3*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/(
c^5*Sqrt[1 - c^2*x^2]) + 32*b*(d + e*x^2)*(8*d^3 - 4*d^2*e*x^2 + 3*d*e^2*x
^4 + 15*e^3*x^6)*ArcSec[c*x])/(3360*e^3*Sqrt[d + e*x^2])
```

3.111.3 Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.92, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {5761, 27, 7282, 2118, 27, 171, 27, 171, 27, 175, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\downarrow 5761$$

$$-\frac{bcx\int\frac{\left(ex^2+d\right)^{3/2}\left(15e^2x^4-12dex^2+8d^2\right)}{105e^3x\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}+$$
$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}$$

$$\downarrow 27$$

$$-\frac{bcx\int\frac{\left(ex^2+d\right)^{3/2}\left(15e^2x^4-12dex^2+8d^2\right)}{x\sqrt{c^2x^2-1}}dx}{105e^3\sqrt{c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}$$

$$\downarrow 7282$$

$$-\frac{bcx\int\frac{\left(ex^2+d\right)^{3/2}\left(15e^2x^4-12dex^2+8d^2\right)}{x^2\sqrt{c^2x^2-1}}dx^2}{210e^3\sqrt{c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}$$

$$\downarrow 2118$$

$$-\frac{bcx\left(\frac{\int\frac{3e\left(ex^2+d\right)^{3/2}\left(16c^2d^2-\left(29c^2d-25e\right)ex^2\right)}{2x^2\sqrt{c^2x^2-1}}dx^2}{3c^2e}+\frac{5e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{5/2}}{c^2}\right)}{210e^3\sqrt{c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}$$

$$\downarrow 27$$

$$-\frac{bcx\left(\frac{\int\frac{\left(ex^2+d\right)^{3/2}\left(16c^2d^2-\left(29c^2d-25e\right)ex^2\right)}{x^2\sqrt{c^2x^2-1}}dx^2}{2c^2}+\frac{5e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{5/2}}{c^2}\right)}{210e^3\sqrt{c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}$$

$$\downarrow 171$$

$$-\frac{bcx\left(\frac{\int\frac{\sqrt{ex^2+d}\left(64c^4d^3-e\left(23d^2c^4+12dec^2-75e^2\right)x^2\right)}{2x^2\sqrt{c^2x^2-1}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(29c^2d-25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}+\frac{5e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{5/2}}{c^2}\right)}{210e^3\sqrt{c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}$$

$$\downarrow 27$$

$$-\frac{bcx\left(\frac{\int \frac{\sqrt{ex^2+d}\left(64c^4d^3-e\left(23d^2c^4+12dec^2-75e^2\right)x^2\right)}{x^2\sqrt{c^2x^2-1}}dx^2}{4c^2}-\frac{e\sqrt{c^2x^2-1}\left(29c^2d-25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}+\frac{5e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{5/2}}{c^2}\right)}{2c^2}}{210e^3\sqrt{c^2x^2}}+$$
$$\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^3}-$$
$$\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}$$

$$\downarrow 171$$

$$-\frac{bcx\left(\frac{\int \frac{128d^4c^6+e\left(105d^3c^6-35d^2ec^4+63de^2c^2+75e^3\right)x^2}{2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{c^2}-\frac{e\sqrt{c^2x^2-1}\left(23c^4d^2+12c^2de-75e^2\right)\sqrt{d+ex^2}}{c^2}-\frac{e\sqrt{c^2x^2-1}\left(29c^2d-25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}+\frac{5e}{4c^2}\right)}{2c^2}}{210e^3\sqrt{c^2x^2}}$$
$$\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^3}-$$
$$\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}$$

$$\downarrow 27$$

$$-\frac{bcx\left(\frac{\int \frac{128d^4c^6+e\left(105d^3c^6-35d^2ec^4+63de^2c^2+75e^3\right)x^2}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(23c^4d^2+12c^2de-75e^2\right)\sqrt{d+ex^2}}{c^2}-\frac{e\sqrt{c^2x^2-1}\left(29c^2d-25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}+\frac{5e}{4c^2}\right)}{2c^2}}{210e^3\sqrt{c^2x^2}}$$
$$\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^3}-$$
$$\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}$$

$$\downarrow 175$$

$$-\frac{bcx\left(\frac{\frac{128c^6d^4\int\frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+e\left(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3\right)\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(23c^4d^2+12c^2de-75e^2\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\right.}{2c^2}$$

$$\left.\frac{210e^3\sqrt{c^2x^2}}{\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}}\right.$$

$$\downarrow 66$$

$$-\frac{bcx\left(\frac{\frac{128c^6d^4\int\frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\left(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3\right)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(23c^4d^2+12c^2de-75e^2\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\right.}{2c^2}$$

$$\left.\frac{210e^3\sqrt{c^2x^2}}{\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}}\right.$$

$$\downarrow 104$$

$$-\frac{bcx\left(\frac{\frac{256c^6d^4\int\frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+2e\left(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3\right)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(23c^4d^2+12c^2de-75e^2\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\right.}{2c^2}$$

$$\left.\frac{210e^3\sqrt{c^2x^2}}{\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}}\right.$$

$$\downarrow 217$$

$$-\frac{bcx\left(\frac{\frac{2e\left(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3\right)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-256c^6d^{7/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(23c^4d^2+12c^2de-75e^2\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{e\sqrt{c}}{2c^2}\right.}{210e^3\sqrt{c^2x^2}}$$

$$\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}$$

$$\downarrow 221$$

$$\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}-$$

$$\frac{bcx\left(\frac{\frac{2\sqrt{e}\left(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3\right)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-256c^6d^{7/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(23c^4d^2+12c^2de-75e^2\right)\sqrt{d+ex^2}}{c^2}}{4c^2}\right.}{210e^3\sqrt{c^2x^2}}$$

input
```
Int[x^5*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
```

output
```
(d^2*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/(3*e^3) - (2*d*(d + e*x^2)^(5/
2)*(a + b*ArcSec[c*x]))/(5*e^3) + ((d + e*x^2)^(7/2)*(a + b*ArcSec[c*x]))/
(7*e^3) - (b*c*x*((5*e*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(5/2))/c^2 + (-1/2*(
(29*c^2*d - 25*e)*e*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/c^2 + (-((e*(23*
c^4*d^2 + 12*c^2*d*e - 75*e^2)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/c^2) +
(-256*c^6*d^(7/2)*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])] + (
2*Sqrt[e]*(105*c^6*d^3 - 35*c^4*d^2*e + 63*c^2*d*e^2 + 75*e^3)*ArcTanh[(Sq
rt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/c)/(2*c^2))/(4*c^2))/(2*c^
2)))/(210*e^3*Sqrt[c^2*x^2])
```

3.111.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

3.111.4 Maple [F]

$$\int x^5(a+b\,\text{arcsec}\,(cx))\,\sqrt{e\,x^2+d}dx$$

input
```
int(x^5*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x)
```

output
```
int(x^5*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x)
```

3.111.5 Fricas [A] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 1701, normalized size of antiderivative = 4.22

$$\int x^5\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/6720*(128*b*c^7*sqrt(-d)*d^3*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(
c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^
2 + d)*sqrt(-d) + 8*d^2)/x^4) + (105*b*c^6*d^3 - 35*b*c^4*d^2*e + 63*b*c^2
*d*e^2 + 75*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^
4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqr
t(e*x^2 + d)*sqrt(e) + e^2) + 4*(240*a*c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^4 -
64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 + 16*(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*
x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*arcsec(c*x) - (40*b*c^5*e^3*x^4 - 4
1*b*c^5*d^2*e + 38*b*c^3*d*e^2 + 75*b*c*e^3 + 2*(11*b*c^5*d*e^2 + 25*b*c^3
*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^7*e^3), 1/6720*(256*b*c^
7*d^(7/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2
 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (105*b*c^6*d^3
- 35*b*c^4*d^2*e + 63*b*c^2*d*e^2 + 75*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 +
c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d -
 c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(240*a*c^7*e^3*
x^6 + 48*a*c^7*d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 + 16*(15*b*c
^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*arcsec(c
*x) - (40*b*c^5*e^3*x^4 - 41*b*c^5*d^2*e + 38*b*c^3*d*e^2 + 75*b*c*e^3 + 2
*(11*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/
(c^7*e^3), 1/3360*(64*b*c^7*sqrt(-d)*d^3*log(((c^4*d^2 - 6*c^2*d*e + e^...
```

3.111.6 Sympy [F]

$$\int x^5\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int x^5(a+b\,\text{asec}\,(cx))\,\sqrt{d+ex^2}\,dx$$

input
```
integrate(x**5*(a+b*asec(c*x))*(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**5*(a + b*asec(c*x))*sqrt(d + e*x**2), x)
```

3.111.7 Maxima [F(-2)]

Exception generated.

$$\int x^5\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.111.8 Giac [F]

$$\int x^5\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\operatorname{arcsec}(cx)+a)x^5\,dx$$

input
```
integrate(x^5*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)*x^5, x)
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int x^5\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int x^5\,\sqrt{e\,x^2+d}\left(a+b\operatorname{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^5*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))),x)
```

output
```
int(x^5*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))), x)
```

3.112 $\int x^3\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx$

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3.112.1 Optimal result

Integrand size = 23, antiderivative size = 294

$$\begin{aligned}\int x^3\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx = &-\frac{b(c^2d+9e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}}\\ &-\frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}}\\ &-\frac{d(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2}\\ &+\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^2}\\ &-\frac{2bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{15e^2\sqrt{c^2x^2}}\\ &+\frac{b(15c^4d^2-10c^2de-9e^2)\,x\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{120c^4e^{3/2}\sqrt{c^2x^2}}\end{aligned}$$

output

```
-1/3*d*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/e^2+1/5*(e*x^2+d)^(5/2)*(a+b*arcs
ec(c*x))/e^2-2/15*b*c*d^(5/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)
^(1/2))/e^2/(c^2*x^2)^(1/2)+1/120*b*(15*c^4*d^2-10*c^2*d*e-9*e^2)*x*arctan
h(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^4/e^(3/2)/(c^2*x^2)^(1/2)
-1/20*b*x*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(1/2)/c/e/(c^2*x^2)^(1/2)-1/120*b*(c
^2*d+9*e)*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c^3/e/(c^2*x^2)^(1/2)
```

3.112.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.70 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.89

$$\int x^3\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$=\frac{16a(d+ex^2)^2(-2d+3ex^2)-\frac{2be\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)(9e+c^2(7d+6ex^2))}{c^3}+\frac{b\left(-16c^2d^3\sqrt{1+\frac{d}{ex^2}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)\right.}{240e^2\sqrt{d+}}$$

input
```
Integrate[x^3*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
```

output
```
(16*a*(d + e*x^2)^2*(-2*d + 3*e*x^2) - (2*b*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d +
 e*x^2)*(9*e + c^2*(7*d + 6*e*x^2)))/c^3 + (b*(-16*c^2*d^3*Sqrt[1 + d/(e*x
^2)]*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))] + (e*(-15*c^4*d^2
 + 10*c^2*d*e + 9*e^2)*Sqrt[1 - 1/(c^2*x^2)]*x^4*Sqrt[1 + (e*x^2)/d]*Appel
lF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/Sqrt[1 - c^2*x^2]))/(c^3*x) +
16*b*(d + e*x^2)^2*(-2*d + 3*e*x^2)*ArcSec[c*x])/(240*e^2*Sqrt[d + e*x^2])
```

3.112.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.91, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {5761, 27, 435, 171, 27, 171, 27, 175, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\downarrow 5761$$

$$-\frac{bcx\int-\frac{(2d-3ex^2)(ex^2+d)^{3/2}}{15e^2x\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{(d+ex^2)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}-\frac{d(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}$$

$$\downarrow 27$$

$$\frac{bcx\int\frac{(2d-3ex^2)(ex^2+d)^{3/2}}{x\sqrt{c^2x^2-1}}dx}{15e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}-\frac{d(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}$$

$$\downarrow 435$$

$$\frac{bcx\int\frac{\left(2d-3ex^2\right)\left(ex^2+d\right)^{3/2}}{x^2\sqrt{c^2x^2-1}}dx^2}{30e^2\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}$$

$$\downarrow 171$$

$$\frac{bcx\left(\frac{\int\frac{\sqrt{ex^2+d}\left(8c^2d^2-e\left(dc^2+9e\right)x^2\right)}{2x^2\sqrt{c^2x^2-1}}dx^2}{2c^2}-\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^2\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}$$

$$\downarrow 27$$

$$\frac{bcx\left(\frac{\int\frac{\sqrt{ex^2+d}\left(8c^2d^2-e\left(dc^2+9e\right)x^2\right)}{x^2\sqrt{c^2x^2-1}}dx^2}{4c^2}-\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^2\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}$$

$$\downarrow 171$$

$$\frac{bcx\left(\frac{\int\frac{16d^3c^4+e\left(15d^2c^4-10dec^2-9e^2\right)x^2}{2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{4c^2}-\frac{e\sqrt{c^2x^2-1}\left(c^2d+9e\right)\sqrt{d+ex^2}}{c^2}-\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^2\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}$$

$$\downarrow 27$$

$$\frac{bcx\left(\frac{\int\frac{16d^3c^4+e\left(15d^2c^4-10dec^2-9e^2\right)x^2}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(c^2d+9e\right)\sqrt{d+ex^2}}{c^2}-\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^2\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}$$

$$\downarrow 175$$

$$bcx\left(\frac{\frac{16c^4d^3\int\frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+e\left(15c^4d^2-10c^2de-9e^2\right)\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(c^2d+9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}+\frac{30e^2\sqrt{c^2x^2}}$$

$$\downarrow 66$$

$$bcx\left(\frac{\frac{16c^4d^3\int\frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\left(15c^4d^2-10c^2de-9e^2\right)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(c^2d+9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)+$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}+\frac{30e^2\sqrt{c^2x^2}}$$

$$\downarrow 104$$

$$bcx\left(\frac{\frac{32c^4d^3\int\frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+2e\left(15c^4d^2-10c^2de-9e^2\right)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(c^2d+9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)+$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}+\frac{30e^2\sqrt{c^2x^2}}$$

$$\downarrow 217$$

$$bcx\left(\frac{\frac{2e\left(15c^4d^2-10c^2de-9e^2\right)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-32c^4d^{5/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(c^2d+9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)+$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}+\frac{30e^2\sqrt{c^2x^2}}$$

$$\downarrow 221$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}+$$

$$bcx\left(\frac{\frac{2\sqrt{e}\left(15c^4d^2-10c^2de-9e^2\right)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-32c^4d^{5/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(c^2d+9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/}}{2c^2}\right)+$$

$$30e^2\sqrt{c^2x^2}$$

3.112. $\int x^3\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx$

input
```
Int[x^3*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
```

output
```
-1/3*(d*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/e^2 + ((d + e*x^2)^(5/2)*(a
 + b*ArcSec[c*x]))/(5*e^2) + (b*c*x*((-3*e*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^
(3/2))/(2*c^2) + (-((e*(c^2*d + 9*e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/c
^2) + (-32*c^4*d^(5/2)*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])
] + (2*Sqrt[e]*(15*c^4*d^2 - 10*c^2*d*e - 9*e^2)*ArcTanh[(Sqrt[e]*Sqrt[-1
+ c^2*x^2])/(c*Sqrt[d + e*x^2])])/c)/(2*c^2))/(4*c^2)))/(30*e^2*Sqrt[c^2*x
^2])
```

3.112.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
))))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.112.4 Maple [F]

$$\int x^3(a+b\,\text{arcsec}\,(cx))\,\sqrt{e\,x^2+d}dx$$

input
```
int(x^3*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x)
```

output
```
int(x^3*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x)
```

3.112.5 Fricas [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 1383, normalized size of antiderivative = 4.70

$$\int x^3\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/480*(16*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^
2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2
+ d)*sqrt(-d) + 8*d^2)/x^4) - (15*b*c^4*d^2 - 10*b*c^2*d*e - 9*b*e^2)*sqrt
(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 -
4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) +
e^2) + 4*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 + 8*(3*b*c^5*e
^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*arcsec(c*x) - (6*b*c^3*e^2*x^2 + 7*b
*c^3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e^2), -1/48
0*(32*b*c^5*d^(5/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*
sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (15*b
*c^4*d^2 - 10*b*c^2*d*e - 9*b*e^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6
*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(
c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 4*(24*a*c^5*e^2*x^4 + 8*a*c^
5*d*e*x^2 - 16*a*c^5*d^2 + 8*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^
2)*arcsec(c*x) - (6*b*c^3*e^2*x^2 + 7*b*c^3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2
- 1))*sqrt(e*x^2 + d))/(c^5*e^2), 1/240*(8*b*c^5*sqrt(-d)*d^2*log(((c^4*d^
2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c
^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) - (15*b*c^4*d^
2 - 10*b*c^2*d*e - 9*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*
sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*...
```

3.112.6 Sympy [F]

$$\int x^3\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int x^3(a+b\,\text{asec}\,(cx))\,\sqrt{d+ex^2}\,dx$$

input
```
integrate(x**3*(a+b*asec(c*x))*(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**3*(a + b*asec(c*x))*sqrt(d + e*x**2), x)
```

3.112.7 Maxima [F(-2)]

Exception generated.

$$\int x^3\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.112.8 Giac [F]

$$\int x^3\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\operatorname{arcsec}(cx)+a)x^3\,dx$$

input
```
integrate(x^3*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)*x^3, x)
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int x^3\,\sqrt{e\,x^2+d}\left(a+b\operatorname{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^3*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))),x)
```

output
```
int(x^3*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))), x)
```

3.113 $\int x\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx$

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3.113.1 Optimal result

Integrand size = 21, antiderivative size = 195

$$\begin{aligned}\int x\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx = &-\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}}\\ &+\frac{\left(d+ex^2\right)^{3/2}(a+b\sec^{-1}(cx))}{3e}\\ &+\frac{bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e\sqrt{c^2x^2}}\\ &-\frac{b(3c^2d+e)\,x\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2\sqrt{e}\sqrt{c^2x^2}}\end{aligned}$$

output

```
1/3*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/e+1/3*b*c*d^(3/2)*x*arctan((e*x^2+d)
^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e/(c^2*x^2)^(1/2)-1/6*b*(3*c^2*d+e)*x*ar
ctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^2/e^(1/2)/(c^2*x^2)^(
1/2)-1/6*b*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c/(c^2*x^2)^(1/2)
```

3.113.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.59 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.09

$$\int x\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$=\frac{\frac{2bd^2\sqrt{1+\frac{d}{ex^2}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)}{cex}+\frac{\frac{b\left(3c^2d+e\right)\sqrt{1-\frac{1}{c^2x^2}}x^3\sqrt{1+\frac{ex^2}{d}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,c^2x^2,-\frac{ex^2}{d}\right)}{\sqrt{1-c^2x^2}}+\frac{2\left(d+ex^2\right)\left(-be\sqrt{1-\frac{1}{c^2x^2}}x+2ac\right.}{c}}{12\sqrt{d+ex^2}}$$

input
```
Integrate[x*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
```

output
```
((2*b*d^2*Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e
*x^2))])/(c*e*x) + ((b*(3*c^2*d + e)*Sqrt[1 - 1/(c^2*x^2)]*x^3*Sqrt[1 + (e
*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/Sqrt[1 - c^2*x^2
] + (2*(d + e*x^2)*(-(b*e*Sqrt[1 - 1/(c^2*x^2)]*x) + 2*a*c*(d + e*x^2) + 2
*b*c*(d + e*x^2)*ArcSec[c*x]))/e)/c)/(12*Sqrt[d + e*x^2])
```

3.113.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5759, 354, 113, 27, 175, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\downarrow 5759$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{bcx\int\frac{\left(ex^2+d\right)^{3/2}}{x\sqrt{c^2x^2-1}}dx}{3e\sqrt{c^2x^2}}$$

$$\downarrow 354$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{bcx\int\frac{\left(ex^2+d\right)^{3/2}}{x^2\sqrt{c^2x^2-1}}dx^2}{6e\sqrt{c^2x^2}}$$

$$\downarrow 113$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{bcx\left(\frac{\int\frac{2c^2d^2+e\left(3dc^2+e\right)x^2}{2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{c^2}+\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e\sqrt{c^2x^2}}$$

$$\downarrow 27$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{bcx\left(\frac{\int\frac{2c^2d^2+e\left(3dc^2+e\right)x^2}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e\sqrt{c^2x^2}}$$

$$\downarrow 175$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{bcx\left(\frac{2c^2d^2\int\frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+e\left(3c^2d+e\right)\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e\sqrt{c^2x^2}}$$

$$\downarrow 66$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{bcx\left(\frac{2c^2d^2\int\frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\left(3c^2d+e\right)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e\sqrt{c^2x^2}}$$

$$\downarrow 104$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{bcx\left(\frac{4c^2d^2\int\frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+2e\left(3c^2d+e\right)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e\sqrt{c^2x^2}}$$

$$\downarrow 217$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{bcx\left(\frac{2e\left(3c^2d+e\right)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-4c^2d^{3/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}+\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e\sqrt{c^2x^2}}$$

$$\downarrow 221$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e} - \frac{bcx\left(\frac{2\sqrt{e}\left(3c^2d+e\right)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c} - 4c^2d^{3/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2} + \frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e\sqrt{c^2x^2}}$$

input
```
Int[x*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
```

output
```
((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/(3*e) - (b*c*x*((e*Sqrt[-1 + c^2*x
^2]*Sqrt[d + e*x^2])/c^2 + (-4*c^2*d^(3/2)*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]
*Sqrt[-1 + c^2*x^2])] + (2*Sqrt[e]*(3*c^2*d + e)*ArcTanh[(Sqrt[e]*Sqrt[-1
+ c^2*x^2])/(c*Sqrt[d + e*x^2])])/c)/(2*c^2)))/(6*e*Sqrt[c^2*x^2])
```

3.113.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 113
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
))))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5759
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSec[c*x])/(2*e*(p + 1))), x
] - Simp[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2]))   Int[(d + e*x^2)^(p + 1)/(x*S
qrt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

3.113.4 Maple [F]

$$\int x(a + b \operatorname{arcsec}(cx)) \sqrt{e\,x^2 + d}\,dx$$

input
```
int(x*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x)
```

output
```
int(x*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x)
```

3.113.5 Fricas [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 1100, normalized size of antiderivative = 5.64

$$\int x\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/24*(2*b*c^3*sqrt(-d)*d*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^
2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)
*sqrt(-d) + 8*d^2)/x^4) + (3*b*c^2*d + b*e)*sqrt(e)*log(8*c^4*e^2*x^4 + c^
4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c
*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(2*a*c^3*e*x^2 +
2*a*c^3*d - sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 + b*c^3*d)*arcsec(c*x
))*sqrt(e*x^2 + d))/(c^3*e), 1/24*(4*b*c^3*d^(3/2)*arctan(-1/2*sqrt(c^2*x^
2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2
*d^2 - d*e)*x^2 - d^2)) + (3*b*c^2*d + b*e)*sqrt(e)*log(8*c^4*e^2*x^4 + c^
4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c
*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(2*a*c^3*e*x^2 +
2*a*c^3*d - sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 + b*c^3*d)*arcsec(c*x
))*sqrt(e*x^2 + d))/(c^3*e), 1/12*(b*c^3*sqrt(-d)*d*log(((c^4*d^2 - 6*c^2*
d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*
x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + (3*b*c^2*d + b*e)*sqrt
(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d
)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(2*a*c^3*e*x
^2 + 2*a*c^3*d - sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 + b*c^3*d)*arcse
c(c*x))*sqrt(e*x^2 + d))/(c^3*e), 1/12*(2*b*c^3*d^(3/2)*arctan(-1/2*sqrt(c
^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^...
```

3.113.6 Sympy [F]

$$\int x\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int x(a+b\,\text{asec}\,(cx))\,\sqrt{d+ex^2}\,dx$$

input
```
integrate(x*(a+b*asec(c*x))*(e*x**2+d)**(1/2),x)
```

output
```
Integral(x*(a + b*asec(c*x))*sqrt(d + e*x**2), x)
```

3.113.7 Maxima [F]

$$\int x\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\,\text{arcsec}\,(cx)+a)x\,dx$$

input
```
integrate(x*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(e*x^2 + d)^(3/2)*a/e + 1/3*((e*x^2 + d)^(3/2)*arctan(sqrt(c*x + 1)*sq
rt(c*x - 1)) - 3*e*integrate((3*(c^2*e*x^3 - e*x + (c^2*e*x^3 - e*x)*e^(lo
g(c*x + 1) + log(c*x - 1)))*sqrt(e*x^2 + d)*log(x) + (3*c^2*e*x^3*log(c) -
 3*e*x*log(c) + ((3*c^2*log(c) + c^2)*e*x^3 + (c^2*d - 3*e*log(c))*x)*e^(l
og(c*x + 1) + log(c*x - 1)))*sqrt(e*x^2 + d))/(c^2*e*x^2 + (c^2*e*x^2 - e)
*e^(log(c*x + 1) + log(c*x - 1)) - e), x))*b/e
```

3.113.8 Giac [F]

$$\int x\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\,\text{arcsec}\,(cx)+a)x\,dx$$

input
```
integrate(x*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)*x, x)
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int x\,\sqrt{e\,x^2+d}\left(a+b\,\text{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))),x)
```

output
```
int(x*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))), x)
```

3.114 $\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x}\,dx$

3.114.1 Optimal result . . . 866
3.114.2 Mathematica [N/A] . . . 866
3.114.3 Rubi [N/A] . . . 867
3.114.4 Maple [N/A] (verified) . . . 867
3.114.5 Fricas [N/A] . . . 868
3.114.6 Sympy [N/A] . . . 868
3.114.7 Maxima [F(-2)] . . . 868
3.114.8 Giac [N/A] . . . 869
3.114.9 Mupad [N/A] . . . 869

3.114.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x}\,dx = \text{Int}\left(\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x}, x\right)$$

output
```
Unintegrable((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x,x)
```

3.114.2 Mathematica [N/A]

Not integrable

Time = 7.66 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x}\,dx$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x,x]
```

output
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x, x]
```

3.114.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{x}\,dx$$
$$\downarrow 5771$$
$$\int \frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{x}dx$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x,x]
```

output
```
$Aborted
```

3.114.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.114.4 Maple [N/A] (verified)

Not integrable

Time = 0.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\ \text{arcsec}\,(cx))\sqrt{e\,x^2+d}}{x}dx$$

input
```
int((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x,x)
```

output
```
int((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x,x)
```

3.114.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\text{arcsec}\,(cx)+a)}{x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/x, x)
```

3.114.6 Sympy [N/A]

Not integrable

Time = 10.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{(a+b\,\text{asec}\,(cx))\sqrt{d+ex^2}}{x}\,dx$$

input
```
integrate((a+b*asec(c*x))*(e*x**2+d)**(1/2)/x,x)
```

output
```
Integral((a + b*asec(c*x))*sqrt(d + e*x**2)/x, x)
```

3.114.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.114.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\text{arcsec}\,(cx)+a)}{x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/x, x)
```

3.114.9 Mupad [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\text{acos}\left(\frac{1}{cx}\right)\right)}{x}\,dx$$

input
```
int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x,x)
```

output
```
int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x, x)
```

3.115 $\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}\,dx$

3.115.1 Optimal result . . . 870
3.115.2 Mathematica [N/A] . . . 870
3.115.3 Rubi [N/A] . . . 871
3.115.4 Maple [N/A] (verified) . . . 871
3.115.5 Fricas [N/A] . . . 872
3.115.6 Sympy [N/A] . . . 872
3.115.7 Maxima [F(-2)] . . . 872
3.115.8 Giac [N/A] . . . 873
3.115.9 Mupad [N/A] . . . 873

3.115.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}\,dx = \text{Int}\left(\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}, x\right)$$

output
```
Unintegrable((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^3,x)
```

3.115.2 Mathematica [N/A]

Not integrable

Time = 4.88 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}\,dx$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^3,x]
```

output
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^3, x]
```

3.115.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{x^3}\,dx$$
$$\downarrow 5771$$
$$\int \frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{x^3}dx$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^3,x]
```

output
```
$Aborted
```

3.115.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.115.4 Maple [N/A] (verified)

Not integrable

Time = 0.74 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\ \text{arcsec}\,(cx))\sqrt{e\,x^2+d}}{x^3}dx$$

input
```
int((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^3,x)
```

output
```
int((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^3,x)
```

3.115.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\text{arcsec}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/x^3, x)
```

3.115.6 Sympy [N/A]

Not integrable

Time = 14.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{(a+b\,\text{asec}\,(cx))\sqrt{d+ex^2}}{x^3}\,dx$$

input
```
integrate((a+b*asec(c*x))*(e*x**2+d)**(1/2)/x**3,x)
```

output
```
Integral((a + b*asec(c*x))*sqrt(d + e*x**2)/x**3, x)
```

3.115.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.115.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\text{arcsec}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/x^3, x)
```

3.115.9 Mupad [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\text{acos}\left(\frac{1}{cx}\right)\right)}{x^3}\,dx$$

input
```
int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x^3,x)
```

output
```
int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x^3, x)
```

3.116 $\int x^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx$

3.116.1 Optimal result . . . 874
3.116.2 Mathematica [N/A] . . . 874
3.116.3 Rubi [N/A] . . . 875
3.116.4 Maple [N/A] (verified) . . . 875
3.116.5 Fricas [N/A] . . . 876
3.116.6 Sympy [N/A] . . . 876
3.116.7 Maxima [F(-2)] . . . 876
3.116.8 Giac [N/A] . . . 877
3.116.9 Mupad [N/A] . . . 877

3.116.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx = \text{Int}\left(x^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx)),x\right)$$

output
```
Unintegrable(x^2*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x)
```

3.116.2 Mathematica [N/A]

Not integrable

Time = 12.60 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx = \int x^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx$$

input
```
Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
```

output
```
Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]), x]
```

3.116.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\downarrow 5771$$

$$\int x^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx$$

input
```
Int[x^2*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
```

output
```
$Aborted
```

3.116.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrable[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.116.4 Maple [N/A] (verified)

Not integrable

Time = 0.39 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^2(a+b\ \text{arcsec}\,(cx))\,\sqrt{e\,x^2+d}dx$$

input
```
int(x^2*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x)
```

output
```
int(x^2*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x)
```

3.116.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\operatorname{arcsec}(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x^2*arcsec(c*x) + a*x^2)*sqrt(e*x^2 + d), x)
```

3.116.6 Sympy [N/A]

Not integrable

Time = 116.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int x^2(a+b\operatorname{asec}(cx))\sqrt{d+ex^2}\,dx$$

input
```
integrate(x**2*(a+b*asec(c*x))*(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**2*(a + b*asec(c*x))*sqrt(d + e*x**2), x)
```

3.116.7 Maxima [F(-2)]

Exception generated.

$$\int x^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.116.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\,\mathrm{arcsec}\,(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)*x^2, x)
```

3.116.9 Mupad [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int x^2\,\sqrt{e\,x^2+d}\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^2*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))),x)
```

output
```
int(x^2*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))), x)
```

3.117 $\int \sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx$

3.117.1 Optimal result . . . 878
3.117.2 Mathematica [N/A] . . . 878
3.117.3 Rubi [N/A] . . . 879
3.117.4 Maple [N/A] (verified) . . . 879
3.117.5 Fricas [N/A] . . . 880
3.117.6 Sympy [N/A] . . . 880
3.117.7 Maxima [F(-2)] . . . 880
3.117.8 Giac [N/A] . . . 881
3.117.9 Mupad [N/A] . . . 881

3.117.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx = \text{Int}\left(\sqrt{d+ex^2}(a+b\sec^{-1}(cx)),x\right)$$

output
```
Unintegrable((a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x)
```

3.117.2 Mathematica [N/A]

Not integrable

Time = 16.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx = \int \sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx$$

input
```
Integrate[Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
```

output
```
Integrate[Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]), x]
```

3.117.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\downarrow 5771$$

$$\int \sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx$$

input
```
Int[Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
```

output
```
$Aborted
```

3.117.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrable[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.117.4 Maple [N/A] (verified)

Not integrable

Time = 0.41 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int (a+b\,\text{arcsec}\,(cx))\sqrt{e\,x^2+d}dx$$

input
```
int((a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x)
```

3.117.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\operatorname{arcsec}(cx)+a)\,dx$$

input
```
integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a), x)
```

3.117.6 Sympy [N/A]

Not integrable

Time = 47.32 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int (a+b\operatorname{asec}(cx))\sqrt{d+ex^2}\,dx$$

input
```
integrate((a+b*asec(c*x))*(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asec(c*x))*sqrt(d + e*x**2), x)
```

3.117.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.117.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\,\text{arcsec}\,(cx)+a)\,dx$$

input
```
integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a), x)
```

3.117.9 Mupad [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int \sqrt{e\,x^2+d}\left(a+b\,\text{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))),x)
```

output
```
int((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))), x)
```

3.118 $\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}\,dx$

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3.118.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}\,dx = \text{Int}\left(\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}, x\right)$$

output
```
Unintegrable((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

3.118.2 Mathematica [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}\,dx$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^2,x]
```

output
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^2, x]
```

3.118.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{x^2}\,dx$$
$$\downarrow 5771$$
$$\int \frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{x^2}dx$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^2,x]
```

output
```
$Aborted
```

3.118.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.118.4 Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\ \text{arcsec}\,(cx))\sqrt{e\,x^2+d}}{x^2}dx$$

input
```
int((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

output
```
int((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

3.118.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\text{arcsec}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/x^2, x)
```

3.118.6 Sympy [N/A]

Not integrable

Time = 7.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}\,dx = \int \frac{(a+b\,\text{asec}\,(cx))\sqrt{d+ex^2}}{x^2}\,dx$$

input
```
integrate((a+b*asec(c*x))*(e*x**2+d)**(1/2)/x**2,x)
```

output
```
Integral((a + b*asec(c*x))*sqrt(d + e*x**2)/x**2, x)
```

3.118.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.118.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\text{arcsec}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/x^2, x)
```

3.118.9 Mupad [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\text{acos}\left(\frac{1}{cx}\right)\right)}{x^2}\,dx$$

input
```
int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x^2,x)
```

output
```
int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x^2, x)
```

3.119 $\int \frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{x^4}\,dx$

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3.119.1 Optimal result

Integrand size = 23, antiderivative size = 328

$$
\begin{aligned}
&\int \frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{x^4}\,dx \\
&= \frac{2bc\left(c^2d+2e\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} \\
&\quad + \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3} \\
&\quad - \frac{2bc^2\left(c^2d+2e\right)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|\,-\frac{e}{c^2d}\right)}{9d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&\quad + \frac{b\left(c^2d+e\right)\left(2c^2d+3e\right)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{9d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}
$$

output

```
-1/3*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/d/x^3+2/9*b*c*(c^2*d+2*e)*(c^2*x^2-
1)^(1/2)*(e*x^2+d)^(1/2)/d/(c^2*x^2)^(1/2)+1/9*b*c*(c^2*x^2-1)^(1/2)*(e*x^
2+d)^(1/2)/x^2/(c^2*x^2)^(1/2)-2/9*b*c^2*(c^2*d+2*e)*x*EllipticE(c*x,(-e/c
^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2
-1)^(1/2)/(1+e*x^2/d)^(1/2)+1/9*b*(c^2*d+e)*(2*c^2*d+3*e)*x*EllipticF(c*x,
(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(c^2*x^2)^(1/2)/(
c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)
```

3.119.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.12 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^4}\,dx$$
$$= \frac{\sqrt{d+ex^2}\left(-3a(d+ex^2)+bc\sqrt{1-\frac{1}{c^2x^2}}x(d+2c^2dx^2+4ex^2)-3b(d+ex^2)\sec^{-1}(cx)\right)}{9dx^3}$$
$$-\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(2c^2d(c^2d+2e)\,E\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)|-\frac{e}{c^2d}\right)-(2c^4d^2+5c^2de+3e^2)\,\text{EllipticF}\right.}{9\sqrt{-c^2}d\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^4,x]
```

output
```
(Sqrt[d + e*x^2]*(-3*a*(d + e*x^2) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + 2*c^
2*d*x^2 + 4*e*x^2) - 3*b*(d + e*x^2)*ArcSec[c*x]))/(9*d*x^3) - ((I/9)*b*c*
Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(2*c^2*d*(c^2*d + 2*e)*Ellipti
cE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] - (2*c^4*d^2 + 5*c^2*d*e + 3*e^2
)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d*Sqrt[1
- c^2*x^2]*Sqrt[d + e*x^2])
```

3.119.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {5761, 27, 376, 25, 445, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^4}\,dx$$
$$\downarrow 5761$$
$$-\frac{bcx\int -\frac{(ex^2+d)^{3/2}}{3dx^4\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}-\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3dx^3}$$
$$\downarrow 27$$

$$\frac{bcx\int\frac{\left(ex^2+d\right)^{3/2}}{x^4\sqrt{c^2x^2-1}}dx}{3d\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

$$\downarrow 376$$

$$\frac{bcx\left(\frac{d\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{3x^3}-\frac{1}{3}\int-\frac{e\left(dc^2+3e\right)x^2+2d\left(dc^2+2e\right)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx\right)}{3d\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

$$\downarrow 25$$

$$\frac{bcx\left(\frac{1}{3}\int\frac{e\left(dc^2+3e\right)x^2+2d\left(dc^2+2e\right)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{d\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{3x^3}\right)}{3d\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

$$\downarrow 445$$

$$\frac{bcx\left(\frac{1}{3}\left(\frac{\int\frac{de\left(-2\left(dc^2+2e\right)x^2c^2+dc^2+3e\right)}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{d}+\frac{2\sqrt{c^2x^2-1}\left(c^2d+2e\right)\sqrt{d+ex^2}}{x}\right)+\frac{d\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{3x^3}\right)}{3d\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

$$\downarrow 27$$

$$\frac{bcx\left(\frac{1}{3}\left(e\int\frac{-2\left(dc^2+2e\right)x^2c^2+dc^2+3e}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{2\sqrt{c^2x^2-1}\left(c^2d+2e\right)\sqrt{d+ex^2}}{x}\right)+\frac{d\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{3x^3}\right)}{3d\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

$$\downarrow 399$$

$$\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{\left(c^2d+e\right)\left(2c^2d+3e\right)\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{e}-\frac{2c^2\left(c^2d+2e\right)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{2\sqrt{c^2x^2-1}\left(c^2d+2e\right)\sqrt{d+ex^2}}{x}\right)+\frac{d\sqrt{c^2x^2-1}\sqrt{d+}}{3x^3}\right)}{3d\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

$$\downarrow 323$$

$$\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{\left(c^2d+e\right)\left(2c^2d+3e\right)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}-\frac{2c^2\left(c^2d+2e\right)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{2\sqrt{c^2x^2-1}\left(c^2d+2e\right)\sqrt{d+ex^2}}{x}\right)+\frac{d\sqrt{}}{}\right)}{3d\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

3.119. $\int\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{x^4}dx$

$$\downarrow 323$$

$$\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}\left(c^2d+e\right)\left(2c^2d+3e\right)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{2c^2\left(c^2d+2e\right)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{2\sqrt{c^2x^2-1}\left(c^2d+2e\right)\sqrt{d+ex^2}}{x}\right)\right)}{3d\sqrt{c^2x^2}}}{\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}}$$

$$\downarrow 321$$

$$\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}\left(c^2d+e\right)\left(2c^2d+3e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{2c^2\left(c^2d+2e\right)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{2\sqrt{c^2x^2-1}\left(c^2d+2e\right)\sqrt{d+}}{x}\right)\right)}{3d\sqrt{c^2x^2}}}{\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}}$$

$$\downarrow 331$$

$$\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}\left(c^2d+e\right)\left(2c^2d+3e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{2c^2\sqrt{1-c^2x^2}\left(c^2d+2e\right)\int\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{c^2x^2-1}}\right)+\frac{2\sqrt{c^2x^2-1}\left(c^2d-\right.}{x}\right)\right)}{3d\sqrt{c^2x^2}}}{\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}}$$

$$\downarrow 330$$

$$\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}\left(c^2d+e\right)\left(2c^2d+3e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{2c^2\sqrt{1-c^2x^2}\left(c^2d+2e\right)\sqrt{d+ex^2}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\right)+\frac{2\sqrt{c^2x}}{}\right)\right)}{3d\sqrt{c^2x^2}}}{\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}}$$

$$\downarrow 327$$

$$\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}\left(c^2d+e\right)\left(2c^2d+3e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{2c\sqrt{1-c^2x^2}\left(c^2d+2e\right)\sqrt{d+ex^2}\,E\left(\arcsin(cx)\middle|-\frac{e}{c^2d}\right)}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\right)+\frac{2}{}\right)\right)}{3d\sqrt{c^2x^2}}}{\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}}$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^4,x]
```

output
```
-1/3*((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/(d*x^3) + (b*c*x*((d*Sqrt[-1
+ c^2*x^2]*Sqrt[d + e*x^2])/(3*x^3) + ((2*(c^2*d + 2*e)*Sqrt[-1 + c^2*x^2]
*Sqrt[d + e*x^2])/x + e*((-2*c*(c^2*d + 2*e)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*
x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(e*Sqrt[-1 + c^2*x^2]*Sqrt[1 +
(e*x^2)/d]) + ((c^2*d + e)*(2*c^2*d + 3*e)*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x
^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(c*e*Sqrt[-1 + c^2*x^2]*Sqrt[
d + e*x^2]))))/3))/(3*d*Sqrt[c^2*x^2])
```

3.119.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.119.4 Maple [F]

$$\int \frac{(a+b\,\text{arcsec}\,(cx))\,\sqrt{e\,x^2+d}}{x^4}dx$$

input
```
int((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^4,x)
```

output
```
int((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^4,x)
```

3.119.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^4}\,dx =$$
$$-\frac{\left(3\,acdex^2+3\,acd^2+3\,(bcdex^2+bcd^2)\,\text{arcsec}\,(cx)-(bcd^2+2\,(bc^3d^2+2\,bcde)x^2)\sqrt{c^2x^2-1}\right)\sqrt{ex^2+d}}{}$$

input
```
integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
-1/9*((3*a*c*d*e*x^2 + 3*a*c*d^2 + 3*(b*c*d*e*x^2 + b*c*d^2)*arcsec(c*x) -
 (b*c*d^2 + 2*(b*c^3*d^2 + 2*b*c*d*e)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 +
 d) - (2*(b*c^6*d^2 + 2*b*c^4*d*e)*x^3*elliptic_e(arcsin(c*x), -e/(c^2*d))
 - (2*b*c^6*d^2 + (4*b*c^4 + b*c^2)*d*e + 3*b*e^2)*x^3*elliptic_f(arcsin(c
*x), -e/(c^2*d)))*sqrt(-d))/(c*d^2*x^3)
```

3.119.6 Sympy [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^4}\,dx = \int \frac{(a+b\,\text{asec}\,(cx))\,\sqrt{d+ex^2}}{x^4}\,dx$$

input
```
integrate((a+b*asec(c*x))*(e*x**2+d)**(1/2)/x**4,x)
```

output
```
Integral((a + b*asec(c*x))*sqrt(d + e*x**2)/x**4, x)
```

3.119.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.119.8 Giac [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^4}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\text{arcsec}\,(cx)+a)}{x^4}\,dx$$

input
```
integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/x^4, x)
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^4}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\text{acos}\left(\frac{1}{cx}\right)\right)}{x^4}\,dx$$

input
```
int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x^4,x)
```

output
```
int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x^4, x)
```

3.120 $\int \frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{x^6}\,dx$

3.120.1 Optimal result 894
3.120.2 Mathematica [C] (verified) 895
3.120.3 Rubi [A] (verified) 896
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3.120.8 Giac [F] 903
3.120.9 Mupad [F(-1)] 903

3.120.1 Optimal result

Integrand size = 23, antiderivative size = 453

$$
\begin{aligned}
&\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^6}\,dx \\
&= \frac{bc(24c^4d^2+19c^2de-31e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} \\
&\quad + \frac{bc(12c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25dx^4\sqrt{c^2x^2}} \\
&\quad - \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5} + \frac{2e(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{15d^2x^3} \\
&\quad - \frac{bc^2(24c^4d^2+19c^2de-31e^2)\,x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|\,-\frac{e}{c^2d}\right)}{225d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&\quad + \frac{b(c^2d+e)(24c^4d^2+7c^2de-30e^2)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{225d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}
$$

output

```
-1/5*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/d/x^5+2/15*e*(e*x^2+d)^(3/2)*(a+b*a
rcsec(c*x))/d^2/x^3-2/15*b*c*e^2*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^
2*x^2)^(1/2)+1/45*b*c*e*(2*c^2*d+e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/
(c^2*x^2)^(1/2)+1/75*b*c*(8*c^4*d^2+3*c^2*d*e-2*e^2)*(c^2*x^2-1)^(1/2)*(e*
x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)+1/25*b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2
)/x^4/(c^2*x^2)^(1/2)+1/45*b*c*e*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(
c^2*x^2)^(1/2)+1/75*b*c*(4*c^2*d+e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^
2/(c^2*x^2)^(1/2)+2/15*b*c^2*e^2*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x
^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2
/d)^(1/2)-1/45*b*c^2*e*(2*c^2*d+e)*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2
*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x
^2/d)^(1/2)-1/75*b*c^2*(8*c^4*d^2+3*c^2*d*e-2*e^2)*x*EllipticE(c*x,(-e/c^2
/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2
-1)^(1/2)/(1+e*x^2/d)^(1/2)+1/75*b*c^2*(8*c^2*d-e)*(c^2*d+e)*x*EllipticF(c
*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(c^2*x^2)^(1/2
)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)+2/45*b*c^2*e*(c^2*d+e)*x*EllipticF(c*x
,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(c^2*x^2)^(1/2)/
(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)-2/15*b*e^2*(c^2*d+e)*x*EllipticF(c*x,(-e
/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c
^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)
```

3.120.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.83 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^6}\,dx$$

$$= \frac{\sqrt{d+ex^2}\left(-15a(3d^2+dex^2-2e^2x^4)+bc\sqrt{1-\frac{1}{c^2x^2}}x(-31e^2x^4+dex^2(8+19c^2x^2)+3d^2(3+4c^2x^2+8\right.}{225d^2x^5}$$

$$- \frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(24c^4d^2+19c^2de-31e^2)\,E\left(i\operatorname{arcsinh}\left(\sqrt{-c^2}x\right)|-\frac{e}{c^2d}\right)+(-24c^6d^3-31c^4d^\right.}{225\sqrt{-c^2}d^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

input

```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^6,x]
```

output

```
(Sqrt[d + e*x^2]*(-15*a*(3*d^2 + d*e*x^2 - 2*e^2*x^4) + b*c*Sqrt[1 - 1/(c^
2*x^2)]*x*(-31*e^2*x^4 + d*e*x^2*(8 + 19*c^2*x^2) + 3*d^2*(3 + 4*c^2*x^2 +
 8*c^4*x^4)) - 15*b*(3*d^2 + d*e*x^2 - 2*e^2*x^4)*ArcSec[c*x]))/(225*d^2*x
^5) - ((I/225)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(24*
c^4*d^2 + 19*c^2*d*e - 31*e^2)*EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2
*d))] + (-24*c^6*d^3 - 31*c^4*d^2*e + 23*c^2*d*e^2 + 30*e^3)*EllipticF[I*A
rcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d^2*Sqrt[1 - c^2*x^2]*Sq
rt[d + e*x^2])
```

3.120.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 412, normalized size of antiderivative = 0.91, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {5761, 27, 442, 25, 442, 25, 445, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{x^6}\,dx$$

$$\downarrow 5761$$

$$-\frac{bcx\int -\frac{(3d-2ex^2)(ex^2+d)^{3/2}}{15d^2x^6\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\downarrow 27$$

$$\frac{bcx\int \frac{(3d-2ex^2)(ex^2+d)^{3/2}}{x^6\sqrt{c^2x^2-1}}dx}{15d^2\sqrt{c^2x^2}}+\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\downarrow 442$$

$$\frac{bcx\left(\frac{3d\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{5x^5}-\frac{1}{5}\int -\frac{\sqrt{ex^2+d}\left((3c^2d-10e)ex^2+d(12c^2d-e)\right)}{x^4\sqrt{c^2x^2-1}}dx\right)}{15d^2\sqrt{c^2x^2}}+$$
$$\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\downarrow 25$$

$$\frac{bcx\left(\frac{1}{5}\int\frac{\sqrt{ex^2+d}\left(\left(3c^2d-10e\right)ex^2+d\left(12c^2d-e\right)\right)}{x^4\sqrt{c^2x^2-1}}dx+\frac{3d\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{5x^5}\right)}{15d^2\sqrt{c^2x^2}}+$$
$$\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\downarrow 442$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{d\sqrt{c^2x^2-1}\left(12c^2d-e\right)\sqrt{d+ex^2}}{3x^3}-\frac{1}{3}\int-\frac{2e\left(6d^2c^4+4dec^2-15e^2\right)x^2+d\left(24d^2c^4+19dec^2-31e^2\right)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx\right)+\frac{3d\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{5x^5}\right)}{15d^2\sqrt{c^2x^2}}+$$
$$\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\downarrow 25$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\int\frac{2e\left(6d^2c^4+4dec^2-15e^2\right)x^2+d\left(24d^2c^4+19dec^2-31e^2\right)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{d\sqrt{c^2x^2-1}\left(12c^2d-e\right)\sqrt{d+ex^2}}{3x^3}\right)+\frac{3d\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{5x^5}\right)}{15d^2\sqrt{c^2x^2}}+$$
$$\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\downarrow 445$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{\int\frac{de\left(2\left(6d^2c^4+4dec^2-15e^2\right)-c^2\left(24d^2c^4+19dec^2-31e^2\right)x^2\right)}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{d}+\frac{\sqrt{c^2x^2-1}\left(24c^4d^2+19c^2de-31e^2\right)\sqrt{d+ex^2}}{x}\right)+\frac{d\sqrt{c^2x^2-1}\left(12c^2d\right.}{3x^3}\right.}{15d^2\sqrt{c^2x^2}}$$
$$\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\downarrow 27$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\int\frac{2\left(6d^2c^4+4dec^2-15e^2\right)-c^2\left(24d^2c^4+19dec^2-31e^2\right)x^2}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{\sqrt{c^2x^2-1}\left(24c^4d^2+19c^2de-31e^2\right)\sqrt{d+ex^2}}{x}\right)+\frac{d\sqrt{c^2x^2-1}\left(12c^2d\right.}{3x^3}\right.}{15d^2\sqrt{c^2x^2}}$$
$$\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\downarrow 399$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{(c^2d+e)(24c^4d^2+7c^2de-30e^2)\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{e}-\frac{c^2(24c^4d^2+19c^2de-31e^2)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-1}(24c^4d^2+19}{x}\right.\right.\right.}{15d^2\sqrt{c^2x^2}}$$

$$\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\downarrow 323$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{(c^2d+e)(24c^4d^2+7c^2de-30e^2)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}-\frac{c^2(24c^4d^2+19c^2de-31e^2)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-1}}{}\right.\right.\right.}{15d^2\sqrt{c^2x^2}}$$

$$\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\downarrow 323$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+e)(24c^4d^2+7c^2de-30e^2)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2(24c^4d^2+19c^2de-31e^2)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\right.\right.\right.}{15d^2\sqrt{c^2x^2}}$$

$$\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\downarrow 321$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+e)(24c^4d^2+7c^2de-30e^2)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2(24c^4d^2+19c^2de-31e^2)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)\right.\right.\right)}{15d^2\sqrt{c^2x^2}}$$

$$\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\downarrow 331$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+e)(24c^4d^2+7c^2de-30e^2)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\sqrt{1-c^2x^2}(24c^4d^2+19c^2de-31e^2)\int\frac{\sqrt{ex}}{\sqrt{1-}}\right.\right.\right.\right.}{e\sqrt{c^2x^2-1}}$$

$$15d^2\sqrt{c^2x^2}$$

$$\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\downarrow 330$$

$$bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}\,(c^2d+e)\,(24c^4d^2+7c^2de-30e^2)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\sqrt{1-c^2x^2}\,(24c^4d^2+19c^2de-31e^2)\sqrt{d+e}}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\right.\right.\right.$$

$$15d^2\sqrt{c^2}$$

$$\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\downarrow 327$$

$$\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}+$$

$$bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}\,(c^2d+e)\,(24c^4d^2+7c^2de-30e^2)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c\sqrt{1-c^2x^2}\,(24c^4d^2+19c^2de-31e^2)\sqrt{d+ex^2}}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\right.\right.\right.$$

$$15d^2\sqrt{c}$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^6,x]
```

output
```
-1/5*((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/(d*x^5) + (2*e*(d + e*x^2)^(3
/2)*(a + b*ArcSec[c*x]))/(15*d^2*x^3) + (b*c*x*((3*d*Sqrt[-1 + c^2*x^2]*(d
 + e*x^2)^(3/2))/(5*x^5) + ((d*(12*c^2*d - e)*Sqrt[-1 + c^2*x^2]*Sqrt[d +
e*x^2])/(3*x^3) + (((24*c^4*d^2 + 19*c^2*d*e - 31*e^2)*Sqrt[-1 + c^2*x^2]*
Sqrt[d + e*x^2])/x + e*(-((c*(24*c^4*d^2 + 19*c^2*d*e - 31*e^2)*Sqrt[1 - c
^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(e*Sqrt[-1 +
 c^2*x^2]*Sqrt[1 + (e*x^2)/d])) + ((c^2*d + e)*(24*c^4*d^2 + 7*c^2*d*e - 3
0*e^2)*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c
^2*d))])/(c*e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])))/3)/5))/(15*d^2*Sqrt[c^
2*x^2])
```

3.120.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])  u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])  Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.120.4 Maple [F]

$$\int \frac{(a+b\operatorname{arcsec}(cx))\sqrt{e\,x^2+d}}{x^6}dx$$

input
```
int((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^6,x)
```

output
```
int((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^6,x)
```

3.120.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^6}\,dx$$
$$= \frac{(30\,acde^2x^4 - 15\,acd^2ex^2 - 45\,acd^3 + 15\,(2\,bcde^2x^4 - bcd^2ex^2 - 3\,bcd^3)\operatorname{arcsec}(cx) + (9\,bcd^3 + (24\,bc^5d^3 -$$

input
```
integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
1/225*((30*a*c*d*e^2*x^4 - 15*a*c*d^2*e*x^2 - 45*a*c*d^3 + 15*(2*b*c*d*e^2
*x^4 - b*c*d^2*e*x^2 - 3*b*c*d^3)*arcsec(c*x) + (9*b*c*d^3 + (24*b*c^5*d^3
 + 19*b*c^3*d^2*e - 31*b*c*d*e^2)*x^4 + 4*(3*b*c^3*d^3 + 2*b*c*d^2*e)*x^2)
*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d) + ((24*b*c^8*d^3 + 19*b*c^6*d^2*e - 31
*b*c^4*d*e^2)*x^5*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (24*b*c^8*d^3 + (1
9*b*c^6 + 12*b*c^4)*d^2*e - (31*b*c^4 - 8*b*c^2)*d*e^2 - 30*b*e^3)*x^5*ell
iptic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(-d))/(c*d^3*x^5)
```

3.120.6 Sympy [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^6}\,dx = \int \frac{(a+b\,\text{asec}\,(cx))\sqrt{d+ex^2}}{x^6}\,dx$$

input
```
integrate((a+b*asec(c*x))*(e*x**2+d)**(1/2)/x**6,x)
```

output
```
Integral((a + b*asec(c*x))*sqrt(d + e*x**2)/x**6, x)
```

3.120.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.120.8 Giac [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^6}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\text{arcsec}\,(cx)+a)}{x^6}\,dx$$

input
```
integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/x^6, x)
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^6}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\text{acos}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

input
```
int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x^6,x)
```

output
```
int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x^6, x)
```

3.121 $\int x^3(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx$

3.121.1 Optimal result 904
3.121.2 Mathematica [C] (verified) 905
3.121.3 Rubi [A] (verified) 905
3.121.4 Maple [F] 910
3.121.5 Fricas [A] (verification not implemented) 911
3.121.6 Sympy [F(-1)] 911
3.121.7 Maxima [F(-2)] 912
3.121.8 Giac [F] 912
3.121.9 Mupad [F(-1)] 912

3.121.1 Optimal result

Integrand size = 23, antiderivative size = 374

$$\begin{aligned}
&\int x^3(d \\
&+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{b(3c^4d^2-38c^2de-25e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}} \\
&-\frac{b(13c^2d+25e)\,x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{840c^3e\sqrt{c^2x^2}} \\
&-\frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce\sqrt{c^2x^2}}-\frac{d(d+ex^2)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2} \\
&+\frac{(d+ex^2)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^2}-\frac{2bcd^{7/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{35e^2\sqrt{c^2x^2}} \\
&+\frac{b(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3)\,x\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{560c^6e^{3/2}\sqrt{c^2x^2}}
\end{aligned}$$

output

```
-1/5*d*(e*x^2+d)^(5/2)*(a+b*arcsec(c*x))/e^2+1/7*(e*x^2+d)^(7/2)*(a+b*arcs
ec(c*x))/e^2-2/35*b*c*d^(7/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)
^(1/2))/e^2/(c^2*x^2)^(1/2)+1/560*b*(35*c^6*d^3-35*c^4*d^2*e-63*c^2*d*e^2-
25*e^3)*x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^6/e^(3/2)
/(c^2*x^2)^(1/2)-1/840*b*(13*c^2*d+25*e)*x*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(1/
2)/c^3/e/(c^2*x^2)^(1/2)-1/42*b*x*(e*x^2+d)^(5/2)*(c^2*x^2-1)^(1/2)/c/e/(c
^2*x^2)^(1/2)+1/560*b*(3*c^4*d^2-38*c^2*d*e-25*e^2)*x*(c^2*x^2-1)^(1/2)*(e
*x^2+d)^(1/2)/c^5/e/(c^2*x^2)^(1/2)
```

3.121.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.58 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.82

$$\int x^3 \left(d+ex^2\right)^{3/2} (a$$

$$+b\sec^{-1}(cx))\ dx = \frac{96a(d+ex^2)^3(-2d+5ex^2) - \frac{2be\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)(75e^2+2c^2e(82d+25ex^2)+c^4(57d^2+106dex^2+40e^2x^4}{c^5}$$

input
```
Integrate[x^3*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]),x]
```

output
```
(96*a*(d + e*x^2)^3*(-2*d + 5*e*x^2) - (2*b*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d +
 e*x^2)*(75*e^2 + 2*c^2*e*(82*d + 25*e*x^2) + c^4*(57*d^2 + 106*d*e*x^2 +
40*e^2*x^4)))/c^5 + (3*b*(-32*c^4*d^4*Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1/2,
 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))] - (e*(35*c^6*d^3 - 35*c^4*d^2*e - 63*c
^2*d*e^2 - 25*e^3)*Sqrt[1 - 1/(c^2*x^2)]*x^4*Sqrt[1 + (e*x^2)/d]*AppellF1[
1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/Sqrt[1 - c^2*x^2]))/(c^5*x) + 96*b
*(d + e*x^2)^3*(-2*d + 5*e*x^2)*ArcSec[c*x])/(3360*e^2*Sqrt[d + e*x^2])
```

3.121.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.91, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {5761, 27, 435, 171, 27, 171, 27, 171, 27, 175, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)\ dx$$

$$\downarrow 5761$$

$$-\frac{bcx\int -\frac{(2d-5ex^2)(ex^2+d)^{5/2}}{35e^2x\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}} + \frac{(d+ex^2)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^2} - \frac{d(d+ex^2)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}$$

$$\downarrow 27$$

$$\frac{bcx\int\frac{(2d-5ex^2)(ex^2+d)^{5/2}}{x\sqrt{c^2x^2-1}}dx}{35e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{7/2}(a+b\sec^{-1}(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^2}$$

$$\downarrow 435$$

$$\frac{bcx\int\frac{(2d-5ex^2)(ex^2+d)^{5/2}}{x^2\sqrt{c^2x^2-1}}dx^2}{70e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{7/2}(a+b\sec^{-1}(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^2}$$

$$\downarrow 171$$

$$\frac{bcx\left(\frac{\int\frac{(ex^2+d)^{3/2}\left(12c^2d^2-e\left(13dc^2+25e\right)x^2\right)}{2x^2\sqrt{c^2x^2-1}}dx^2}{3c^2}-\frac{5e\sqrt{c^2x^2-1}(d+ex^2)^{5/2}}{3c^2}\right)}{70e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{7/2}(a+b\sec^{-1}(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^2}$$

$$\downarrow 27$$

$$\frac{bcx\left(\frac{\int\frac{(ex^2+d)^{3/2}\left(12c^2d^2-e\left(13dc^2+25e\right)x^2\right)}{x^2\sqrt{c^2x^2-1}}dx^2}{6c^2}-\frac{5e\sqrt{c^2x^2-1}(d+ex^2)^{5/2}}{3c^2}\right)}{70e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{7/2}(a+b\sec^{-1}(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^2}$$

$$\downarrow 171$$

$$\frac{bcx\left(\frac{\int\frac{3\sqrt{ex^2+d}\left(16d^3c^4+e\left(3d^2c^4-38dec^2-25e^2\right)x^2\right)}{2x^2\sqrt{c^2x^2-1}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(13c^2d+25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}}{6c^2}-\frac{5e\sqrt{c^2x^2-1}(d+ex^2)^{5/2}}{3c^2}\right)}{70e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{7/2}(a+b\sec^{-1}(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^2}$$

$$\downarrow 27$$

$$\frac{bcx\left(\frac{3\int\frac{\sqrt{ex^2+d}\left(16d^3c^4+e\left(3d^2c^4-38dec^2-25e^2\right)x^2\right)}{x^2\sqrt{c^2x^2-1}}dx^2}{4c^2}-\frac{e\sqrt{c^2x^2-1}\left(13c^2d+25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}}{6c^2}-\frac{5e\sqrt{c^2x^2-1}(d+ex^2)^{5/2}}{3c^2}\right)}{70e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{7/2}(a+b\sec^{-1}(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^2}$$

$$\downarrow 171$$

$$bcx\left(\frac{3\left(\frac{\int\frac{32d^4c^6+e\left(35d^3c^6-35d^2ec^4-63de^2c^2-25e^3\right)x^2}{2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{c^2}+\frac{e\sqrt{c^2x^2-1}\left(3c^4d^2-38c^2de-25e^2\right)\sqrt{d+ex^2}}{c^2}\right)}{4c^2}-\frac{e\sqrt{c^2x^2-1}\left(13c^2d+25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}-\frac{5e}{6c^2}\right.$$

$$\left.\frac{70e^2\sqrt{c^2x^2}}{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}\right)$$

$$\downarrow 27$$

$$bcx\left(\frac{3\left(\frac{\int\frac{32d^4c^6+e\left(35d^3c^6-35d^2ec^4-63de^2c^2-25e^3\right)x^2}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(3c^4d^2-38c^2de-25e^2\right)\sqrt{d+ex^2}}{c^2}\right)}{4c^2}-\frac{e\sqrt{c^2x^2-1}\left(13c^2d+25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}-\frac{5e}{6c^2}\right.$$

$$\left.\frac{70e^2\sqrt{c^2x^2}}{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}\right)$$

$$\downarrow 175$$

$$bcx\left(\frac{3\left(\frac{32c^6d^4\int\frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+e\left(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3\right)\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(3c^4d^2-38c^2de-25e^2\right)\sqrt{d+ex^2}}{c^2}\right)}{4c^2}-\frac{e}{6c^2}\right.$$

$$\left.\frac{70e^2\sqrt{c^2x^2}}{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}\right)$$

$$\downarrow 66$$

$$bcx\left(\frac{3\left(\frac{32c^6d^4\int\frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\left(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3\right)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(3c^4d^2-38c^2de-25e^2\right)\sqrt{d+ex^2}}{c^2}\right)}{4c^2}-\frac{e\sqrt{}}{6c^2}\right.$$

$$\left.\frac{70e^2\sqrt{c^2x^2}}{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}\right)$$

3.121. $\int x^3\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx$

$$\downarrow 104$$

$$bcx\left(\frac{3\left(\frac{64c^6d^4\int\frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+2e\left(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3\right)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(3c^4d^2-38c^2de-25e^2\right)\sqrt{d+ex^2}}{c^2}\right)}{4c^2}-\frac{e\sqrt{c^2x^2-1}}{6c^2}\right)$$

$$70e^2\sqrt{c^2x^2}$$

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}$$

$$\downarrow 217$$

$$bcx\left(\frac{3\left(\frac{2e\left(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3\right)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-64c^6d^{7/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(3c^4d^2-38c^2de-25e^2\right)\sqrt{d+ex^2}}{c^2}\right)}{4c^2}-\frac{e\sqrt{c^2}}{6c^2}\right)$$

$$70e^2\sqrt{c^2x^2}$$

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}$$

$$\downarrow 221$$

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}+$$

$$bcx\left(\frac{3\left(\frac{2\sqrt{e}\left(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3\right)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-64c^6d^{7/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(3c^4d^2-38c^2de-25e^2\right)\sqrt{d+ex^2}}{c^2}\right)}{4c^2}\right)$$

$$70e^2\sqrt{c^2x^2}$$

input
```
Int[x^3*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]),x]
```

output
```
-1/5*(d*(d + e*x^2)^(5/2)*(a + b*ArcSec[c*x]))/e^2 + ((d + e*x^2)^(7/2)*(a
 + b*ArcSec[c*x]))/(7*e^2) + (b*c*x*((-5*e*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^
(5/2))/(3*c^2) + (-1/2*(e*(13*c^2*d + 25*e)*Sqrt[-1 + c^2*x^2]*(d + e*x^2)
^(3/2))/c^2 + (3*((e*(3*c^4*d^2 - 38*c^2*d*e - 25*e^2)*Sqrt[-1 + c^2*x^2]*
Sqrt[d + e*x^2])/c^2 + (-64*c^6*d^(7/2)*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sq
rt[-1 + c^2*x^2])] + (2*Sqrt[e]*(35*c^6*d^3 - 35*c^4*d^2*e - 63*c^2*d*e^2
- 25*e^3)*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/c)/(2
*c^2)))/(4*c^2))/(6*c^2)))/(70*e^2*Sqrt[c^2*x^2])
```

3.121.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.121.4 Maple [F]

$$\int x^3 \left(e\,x^2+d\right)^{\frac{3}{2}} \left(a+b\,\operatorname{arcsec}\left(cx\right)\right) dx$$

input
```
int(x^3*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

output
```
int(x^3*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

3.121.5 Fricas [A] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 1701, normalized size of antiderivative = 4.55

$$\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)\, dx = \text{Too large to display}$$

input
```
integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
[1/6720*(96*b*c^7*sqrt(-d)*d^3*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c
^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2
 + d)*sqrt(-d) + 8*d^2)/x^4) - 3*(35*b*c^6*d^3 - 35*b*c^4*d^2*e - 63*b*c^2
*d*e^2 - 25*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^
4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqr
t(e*x^2 + d)*sqrt(e) + e^2) + 4*(240*a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 +
 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 + 48*(5*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x
^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*arcsec(c*x) - (40*b*c^5*e^3*x^4 + 57*b
*c^5*d^2*e + 164*b*c^3*d*e^2 + 75*b*c*e^3 + 2*(53*b*c^5*d*e^2 + 25*b*c^3*e
^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^7*e^2), -1/6720*(192*b*c^7
*d^(7/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2
+ d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + 3*(35*b*c^6*d^3
- 35*b*c^4*d^2*e - 63*b*c^2*d*e^2 - 25*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 +
c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d -
 c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 4*(240*a*c^7*e^3*
x^6 + 384*a*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 + 48*(5*b*c^
7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*arcsec(c*x)
 - (40*b*c^5*e^3*x^4 + 57*b*c^5*d^2*e + 164*b*c^3*d*e^2 + 75*b*c*e^3 + 2*(
53*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c
^7*e^2), 1/3360*(48*b*c^7*sqrt(-d)*d^3*log(((c^4*d^2 - 6*c^2*d*e + e^2)...
```

3.121.6 Sympy [F(-1)]

Timed out.

$$\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)\, dx = \text{Timed out}$$

input
```
integrate(x**3*(e*x**2+d)**(3/2)*(a+b*asec(c*x)),x)
```

output
```
Timed out
```

3.121.7 Maxima [F(-2)]

Exception generated.

$$\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.121.8 Giac [F]

$$\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)^{\frac{3}{2}} (b\,\text{arcsec}\,(cx)+a)x^3\,dx$$

input
```
integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)*x^3, x)
```

3.121.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)\,dx = \int x^3 \left(e\,x^2+d\right)^{3/2} \left(a+b\,\text{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^3*(d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))),x)
```

output
```
int(x^3*(d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))), x)
```

3.122 $\int x(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx$

3.122.1 Optimal result 913
3.122.2 Mathematica [C] (verified) 914
3.122.3 Rubi [A] (verified) 914
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3.122.5 Fricas [A] (verification not implemented) 918
3.122.6 Sympy [F] 919
3.122.7 Maxima [F] 920
3.122.8 Giac [F] 920
3.122.9 Mupad [F(-1)] 920

3.122.1 Optimal result

Integrand size = 21, antiderivative size = 262

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx = -\frac{b(7c^2d+3e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e} + \frac{bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{5e\sqrt{c^2x^2}} - \frac{b(15c^4d^2+10c^2de+3e^2)\,x\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{40c^4\sqrt{e}\sqrt{c^2x^2}}$$

output

```
1/5*(e*x^2+d)^(5/2)*(a+b*arcsec(c*x))/e+1/5*b*c*d^(5/2)*x*arctan((e*x^2+d)
^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e/(c^2*x^2)^(1/2)-1/40*b*(15*c^4*d^2+10*
c^2*d*e+3*e^2)*x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^4/
e^(1/2)/(c^2*x^2)^(1/2)-1/20*b*x*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(1/2)/c/(c^2*
x^2)^(1/2)-1/40*b*(7*c^2*d+3*e)*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c^3/(c
^2*x^2)^(1/2)
```

3.122.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.63 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.94

$$\int x\left(d+ex^2\right)^{3/2}(a+b\sec^{-1}(cx))\,dx = \frac{\frac{16a(d+ex^2)^3}{e} - \frac{2b\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)(3e+c^2(9d+2ex^2))}{c^3} + b\left(\frac{8c^2d^3\sqrt{1+\frac{d}{ex^2}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)}{e} + \frac{(15c^4}{}\right.}{80\sqrt{d+ex^2}}$$

input
```
Integrate[x*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]),x]
```

output
```
((16*a*(d + e*x^2)^3)/e - (2*b*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2)*(3*e +
c^2*(9*d + 2*e*x^2)))/c^3 + (b*((8*c^2*d^3*Sqrt[1 + d/(e*x^2)]*AppellF1[1,
 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))])/e + ((15*c^4*d^2 + 10*c^2*d*e +
3*e^2)*Sqrt[1 - 1/(c^2*x^2)]*x^4*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2,
 2, c^2*x^2, -((e*x^2)/d)])/Sqrt[1 - c^2*x^2]))/(c^3*x) + (16*b*(d + e*x^2
)^3*ArcSec[c*x])/e)/(80*Sqrt[d + e*x^2])
```

3.122.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.92, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {5759, 354, 113, 27, 171, 27, 175, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d+ex^2\right)^{3/2}(a+b\sec^{-1}(cx))\,dx$$

$$\downarrow 5759$$

$$\frac{\left(d+ex^2\right)^{5/2}(a+b\sec^{-1}(cx))}{5e} - \frac{bcx\int\frac{(ex^2+d)^{5/2}}{x\sqrt{c^2x^2-1}}dx}{5e\sqrt{c^2x^2}}$$

$$\downarrow 354$$

$$\frac{\left(d+ex^2\right)^{5/2}(a+b\sec^{-1}(cx))}{5e} - \frac{bcx\int\frac{(ex^2+d)^{5/2}}{x^2\sqrt{c^2x^2-1}}dx^2}{10e\sqrt{c^2x^2}}$$

$$\downarrow 113$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}-\frac{bcx\left(\frac{\int\frac{\sqrt{ex^2+d}\left(4c^2d^2+e\left(7dc^2+3e\right)x^2\right)}{2x^2\sqrt{c^2x^2-1}}dx^2}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{10e\sqrt{c^2x^2}}$$

$$\downarrow 27$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}-\frac{bcx\left(\frac{\int\frac{\sqrt{ex^2+d}\left(4c^2d^2+e\left(7dc^2+3e\right)x^2\right)}{x^2\sqrt{c^2x^2-1}}dx^2}{4c^2}+\frac{e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{10e\sqrt{c^2x^2}}$$

$$\downarrow 171$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}-\frac{bcx\left(\frac{\int\frac{8d^3c^4+e\left(15d^2c^4+10dec^2+3e^2\right)x^2}{2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{c^2}+\frac{e\sqrt{c^2x^2-1}\left(7c^2d+3e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{10e\sqrt{c^2x^2}}$$

$$\downarrow 27$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}-\frac{bcx\left(\frac{\int\frac{8d^3c^4+e\left(15d^2c^4+10dec^2+3e^2\right)x^2}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(7c^2d+3e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{10e\sqrt{c^2x^2}}$$

$$\downarrow 175$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}-\frac{bcx\left(\frac{8c^4d^3\int\frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+e\left(15c^4d^2+10c^2de+3e^2\right)\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(7c^2d+3e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{10e\sqrt{c^2x^2}}$$

$$\downarrow 66$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e} - bcx\left(\frac{\frac{8c^4d^3\int\frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\left(15c^4d^2+10c^2de+3e^2\right)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(7c^2d+3e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)$$

$$10e\sqrt{c^2x^2}$$

$$\downarrow 104$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e} - bcx\left(\frac{\frac{16c^4d^3\int\frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+2e\left(15c^4d^2+10c^2de+3e^2\right)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(7c^2d+3e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)$$

$$10e\sqrt{c^2x^2}$$

$$\downarrow 217$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e} - bcx\left(\frac{\frac{2e\left(15c^4d^2+10c^2de+3e^2\right)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-16c^4d^{5/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(7c^2d+3e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)$$

$$10e\sqrt{c^2x^2}$$

$$\downarrow 221$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e} - bcx\left(\frac{\frac{2\sqrt{e}\left(15c^4d^2+10c^2de+3e^2\right)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-16c^4d^{5/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(7c^2d+3e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)$$

$$10e\sqrt{c^2x^2}$$

input
```
Int[x*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]),x]
```

output
```
((d + e*x^2)^(5/2)*(a + b*ArcSec[c*x]))/(5*e) - (b*c*x*((e*Sqrt[-1 + c^2*x
^2]*(d + e*x^2)^(3/2))/(2*c^2) + ((e*(7*c^2*d + 3*e)*Sqrt[-1 + c^2*x^2]*Sq
rt[d + e*x^2])/c^2 + (-16*c^4*d^(5/2)*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt
[-1 + c^2*x^2])] + (2*Sqrt[e]*(15*c^4*d^2 + 10*c^2*d*e + 3*e^2)*ArcTanh[(S
qrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/c)/(2*c^2))/(4*c^2)))/(10
*e*Sqrt[c^2*x^2])
```

3.122. $\int x\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx$

3.122.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 113
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5759
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSec[c*x])/(2*e*(p + 1))), x
] - Simp[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2]))   Int[(d + e*x^2)^(p + 1)/(x*S
qrt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

3.122.4 Maple [F]

$$\int x\left(e\,x^2+d\right)^{\frac{3}{2}}\left(a+b\,\operatorname{arcsec}\left(cx\right)\right)dx$$

input
```
int(x*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

output
```
int(x*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

3.122.5 Fricas [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 1377, normalized size of antiderivative = 5.26

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
[1/160*(8*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2
*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 +
 d)*sqrt(-d) + 8*d^2)/x^4) + (15*b*c^4*d^2 + 10*b*c^2*d*e + 3*b*e^2)*sqrt(
e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4
*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e
^2) + 4*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 + 8*(b*c^5*e^2*x
^4 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)*arcsec(c*x) - (2*b*c^3*e^2*x^2 + 9*b*c^3
*d*e + 3*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e), 1/160*(16*b
*c^5*d^(5/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*
x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (15*b*c^4*d^
2 + 10*b*c^2*d*e + 3*b*e^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*
e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2
 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x
^2 + 8*a*c^5*d^2 + 8*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)*arcsec(
c*x) - (2*b*c^3*e^2*x^2 + 9*b*c^3*d*e + 3*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt
(e*x^2 + d))/(c^5*e), 1/80*(4*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e
 + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2
 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + (15*b*c^4*d^2 + 10*b*c^2*
d*e + 3*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2
- 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*...
```

3.122.6 Sympy [F]

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int x(a+b\,\text{asec}\,(cx))\left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(e*x**2+d)**(3/2)*(a+b*asec(c*x)),x)
```

output
```
Integral(x*(a + b*asec(c*x))*(d + e*x**2)**(3/2), x)
```

3.122.7 Maxima [F]

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int \left(ex^2+d\right)^{\frac{3}{2}}(b\operatorname{arcsec}(cx)+a)x\,dx$$

input
```
integrate(x*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
1/5*(e*x^2 + d)^(5/2)*a/e + 1/5*((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e*x^2 +
d)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 5*e*integrate((5*(c^2*e^2*x^5 + (
c^2*d*e - e^2)*x^3 - d*e*x + (c^2*e^2*x^5 + (c^2*d*e - e^2)*x^3 - d*e*x)*e
^(log(c*x + 1) + log(c*x - 1)))*sqrt(e*x^2 + d)*log(x) + (5*c^2*e^2*x^5*lo
g(c) + 5*(c^2*d*e*log(c) - e^2*log(c))*x^3 - 5*d*e*x*log(c) + ((5*c^2*log(
c) + c^2)*e^2*x^5 + ((5*c^2*log(c) + 2*c^2)*d*e - 5*e^2*log(c))*x^3 + (c^2
*d^2 - 5*d*e*log(c))*x)*e^(log(c*x + 1) + log(c*x - 1)))*sqrt(e*x^2 + d))/
(c^2*e*x^2 + (c^2*e*x^2 - e)*e^(log(c*x + 1) + log(c*x - 1)) - e), x))*b/e
```

3.122.8 Giac [F]

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int \left(ex^2+d\right)^{\frac{3}{2}}(b\operatorname{arcsec}(cx)+a)x\,dx$$

input
```
integrate(x*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)*x, x)
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int x\left(e\,x^2+d\right)^{3/2}\left(a+b\operatorname{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x*(d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))),x)
```

output
```
int(x*(d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))), x)
```

3.123 $\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x}\,dx$

3.123.1 Optimal result . . . 921
3.123.2 Mathematica [N/A] . . . 921
3.123.3 Rubi [N/A] . . . 922
3.123.4 Maple [N/A] (verified) . . . 922
3.123.5 Fricas [N/A] . . . 923
3.123.6 Sympy [N/A] . . . 923
3.123.7 Maxima [F(-2)] . . . 923
3.123.8 Giac [N/A] . . . 924
3.123.9 Mupad [N/A] . . . 924

3.123.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x}\,dx = \text{Int}\left(\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x}, x\right)$$

output
```
Unintegrable((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x,x)
```

3.123.2 Mathematica [N/A]

Not integrable

Time = 7.85 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x}\,dx$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x,x]
```

output
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x, x]
```

3.123.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x}\,dx$$
$$\downarrow 5771$$
$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x}dx$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x,x]
```

output
```
$Aborted
```

3.123.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrable[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.123.4 Maple [N/A] (verified)

Not integrable

Time = 2.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\left(e\,x^2+d\right)^{\frac{3}{2}}\left(a+b\,\operatorname{arcsec}\left(cx\right)\right)}{x}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x,x)
```

3.123.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}\left(b\operatorname{arcsec}(cx)+a\right)}{x}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsec(c*x))*sqrt(e*x^2 + d)/x,
x)
```

3.123.6 Sympy [N/A]

Not integrable

Time = 71.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x}\,dx = \int \frac{\left(a+b\operatorname{asec}(cx)\right)(d+ex^2)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*asec(c*x))/x,x)
```

output
```
Integral((a + b*asec(c*x))*(d + e*x**2)**(3/2)/x, x)
```

3.123.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.123.8 Giac [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\operatorname{arcsec}(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)/x, x)
```

3.123.9 Mupad [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x}\,dx = \int \frac{(e\,x^2+d)^{3/2}\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x,x)
```

output
```
int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x, x)
```

3.124 $\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^3}\,dx$

3.124.1 Optimal result . . . 925
3.124.2 Mathematica [N/A] . . . 925
3.124.3 Rubi [N/A] . . . 926
3.124.4 Maple [N/A] (verified) . . . 926
3.124.5 Fricas [N/A] . . . 927
3.124.6 Sympy [N/A] . . . 927
3.124.7 Maxima [F(-2)] . . . 927
3.124.8 Giac [N/A] . . . 928
3.124.9 Mupad [N/A] . . . 928

3.124.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^3}\,dx = \text{Int}\left(\frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^3}, x\right)$$

output
```
Unintegrable((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^3,x)
```

3.124.2 Mathematica [N/A]

Not integrable

Time = 6.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^3}\,dx$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^3,x]
```

output
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^3, x]
```

3.124.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^3}\,dx$$
$$\downarrow 5771$$
$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^3}dx$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^3,x]
```

output
```
$Aborted
```

3.124.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.124.4 Maple [N/A] (verified)

Not integrable

Time = 2.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\left(e\,x^2+d\right)^{\frac{3}{2}}\left(a+b\,\operatorname{arcsec}\left(cx\right)\right)}{x^3}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^3,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^3,x)
```

3.124.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^3}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}\left(b\operatorname{arcsec}(cx)+a\right)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsec(c*x))*sqrt(e*x^2 + d)/x^3
, x)
```

3.124.6 Sympy [N/A]

Not integrable

Time = 61.96 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^3}\,dx = \int \frac{\left(a+b\operatorname{asec}(cx)\right)(d+ex^2)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*asec(c*x))/x**3,x)
```

output
```
Integral((a + b*asec(c*x))*(d + e*x**2)**(3/2)/x**3, x)
```

3.124.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.124.8 Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^3}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\operatorname{arcsec}(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^3,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)/x^3, x)
```

3.124.9 Mupad [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^3}\,dx = \int \frac{(e\,x^2+d)^{3/2}\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^3,x)
```

output
```
int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^3, x)
```

3.125 $\int x^2(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx$

3.125.1 Optimal result 929
3.125.2 Mathematica [N/A] 929
3.125.3 Rubi [N/A] 930
3.125.4 Maple [N/A] (verified) 930
3.125.5 Fricas [N/A] 931
3.125.6 Sympy [F(-1)] 931
3.125.7 Maxima [F(-2)] 931
3.125.8 Giac [N/A] 932
3.125.9 Mupad [N/A] 932

3.125.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^2(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Int}\left(x^2(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right),x\right)$$

output
```
Unintegrable(x^2*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

3.125.2 Mathematica [N/A]

Not integrable

Time = 12.55 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^2(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int x^2(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx$$

input
```
Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]),x]
```

output
```
Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]), x]
```

3.125.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (d+ex^2)^{3/2} \left(a+b\sec^{-1}(cx)\right)\, dx$$

$$\downarrow 5771$$

$$\int x^2 (d+ex^2)^{3/2} \left(a+b\sec^{-1}(cx)\right) dx$$

input
```
Int[x^2*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]),x]
```

output
```
$Aborted
```

3.125.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrable[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.125.4 Maple [N/A] (verified)

Not integrable

Time = 1.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^2 \left(e\,x^2+d\right)^{\frac{3}{2}} \left(a+b\,\operatorname{arcsec}\left(cx\right)\right) dx$$

input
```
int(x^2*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

output
```
int(x^2*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

3.125.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

$$\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)\, dx = \int \left(ex^2+d\right)^{\frac{3}{2}} (b\operatorname{arcsec}(cx)+a)x^2\, dx$$

input
```
integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e*x^4 + a*d*x^2 + (b*e*x^4 + b*d*x^2)*arcsec(c*x))*sqrt(e*x^2
+ d), x)
```

3.125.6 Sympy [F(-1)]

Timed out.

$$\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)\, dx = \text{Timed out}$$

input
```
integrate(x**2*(e*x**2+d)**(3/2)*(a+b*asec(c*x)),x)
```

output
```
Timed out
```

3.125.7 Maxima [F(-2)]

Exception generated.

$$\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.125.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int \left(ex^2+d\right)^{\frac{3}{2}}(b\operatorname{arcsec}(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)*x^2, x)
```

3.125.9 Mupad [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int x^2\left(e\,x^2+d\right)^{3/2}\left(a+b\operatorname{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^2*(d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))),x)
```

output
```
int(x^2*(d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))), x)
```

3.126 $\int (d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx$

3.126.1 Optimal result 933
3.126.2 Mathematica [N/A] 933
3.126.3 Rubi [N/A] 934
3.126.4 Maple [N/A] (verified) 934
3.126.5 Fricas [N/A] 935
3.126.6 Sympy [F(-1)] 935
3.126.7 Maxima [F(-2)] 935
3.126.8 Giac [N/A] 936
3.126.9 Mupad [N/A] 936

3.126.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Int}\left((d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right),x\right)$$

output
```
Unintegrable((e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

3.126.2 Mathematica [N/A]

Not integrable

Time = 17.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int (d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx$$

input
```
Integrate[(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]),x]
```

output
```
Integrate[(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]), x]
```

3.126.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\downarrow 5771$$

$$\int \left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx$$

input
```
Int[(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]),x]
```

output
```
$Aborted
```

3.126.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrable[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.126.4 Maple [N/A] (verified)

Not integrable

Time = 1.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(e\,x^2+d\right)^{\frac{3}{2}}\left(a+b\,\text{arcsec}\left(cx\right)\right)dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

3.126.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)^{\frac{3}{2}}(b\operatorname{arcsec}(cx)+a)\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsec(c*x))*sqrt(e*x^2 + d), x)
```

3.126.6 Sympy [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*asec(c*x)),x)
```

output
```
Timed out
```

3.126.7 Maxima [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.126.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)^{\frac{3}{2}}(b\operatorname{arcsec}(cx)+a)\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a), x)
```

3.126.9 Mupad [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int \left(e\,x^2+d\right)^{3/2}\left(a+b\operatorname{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))),x)
```

output
```
int((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))), x)
```

3.127 $\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^2}\,dx$

3.127.1 Optimal result . . . 937
3.127.2 Mathematica [N/A] . . . 937
3.127.3 Rubi [N/A] . . . 938
3.127.4 Maple [N/A] (verified) . . . 938
3.127.5 Fricas [N/A] . . . 939
3.127.6 Sympy [N/A] . . . 939
3.127.7 Maxima [F(-2)] . . . 939
3.127.8 Giac [N/A] . . . 940
3.127.9 Mupad [N/A] . . . 940

3.127.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^2}\,dx = \text{Int}\left(\frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^2}, x\right)$$

output
```
Unintegrable((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^2,x)
```

3.127.2 Mathematica [N/A]

Not integrable

Time = 32.88 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^2}\,dx = \int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^2}\,dx$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^2,x]
```

output
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^2, x]
```

3.127.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^2}\,dx$$
$$\downarrow 5771$$
$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^2}dx$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^2,x]
```

output
```
$Aborted
```

3.127.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrable[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.127.4 Maple [N/A] (verified)

Not integrable

Time = 2.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\left(e\,x^2+d\right)^{\frac{3}{2}}\left(a+b\,\operatorname{arcsec}\left(cx\right)\right)}{x^2}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^2,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^2,x)
```

3.127.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^2}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}\left(b\operatorname{arcsec}(cx)+a\right)}{x^2}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^2,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsec(c*x))*sqrt(e*x^2 + d)/x^2
, x)
```

3.127.6 Sympy [N/A]

Not integrable

Time = 99.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^2}\,dx = \int \frac{\left(a+b\operatorname{asec}(cx)\right)(d+ex^2)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*asec(c*x))/x**2,x)
```

output
```
Integral((a + b*asec(c*x))*(d + e*x**2)**(3/2)/x**2, x)
```

3.127.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.127.8 Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^2}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}\left(b\operatorname{arcsec}(cx)+a\right)}{x^2}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^2,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)/x^2, x)
```

3.127.9 Mupad [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^2}\,dx = \int \frac{(e\,x^2+d)^{3/2}\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^2,x)
```

output
```
int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^2, x)
```

3.128 $\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^4}\,dx$

3.128.1 Optimal result . . . 941
3.128.2 Mathematica [N/A] . . . 941
3.128.3 Rubi [N/A] . . . 942
3.128.4 Maple [N/A] (verified) . . . 942
3.128.5 Fricas [N/A] . . . 943
3.128.6 Sympy [N/A] . . . 943
3.128.7 Maxima [F(-2)] . . . 943
3.128.8 Giac [N/A] . . . 944
3.128.9 Mupad [N/A] . . . 944

3.128.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^4}\,dx = \text{Int}\left(\frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^4}, x\right)$$

output
```
Unintegrable((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^4,x)
```

3.128.2 Mathematica [N/A]

Not integrable

Time = 8.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^4}\,dx = \int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^4}\,dx$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^4,x]
```

output
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^4, x]
```

3.128.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^4}\,dx$$
$$\downarrow 5771$$
$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^4}dx$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^4,x]
```

output
```
$Aborted
```

3.128.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.128.4 Maple [N/A] (verified)

Not integrable

Time = 0.38 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\left(e\,x^2+d\right)^{\frac{3}{2}}\left(a+b\,\operatorname{arcsec}\left(cx\right)\right)}{x^4}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^4,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^4,x)
```

3.128.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^4}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}\left(b\operatorname{arcsec}(cx)+a\right)}{x^4}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^4,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsec(c*x))*sqrt(e*x^2 + d)/x^4
, x)
```

3.128.6 Sympy [N/A]

Not integrable

Time = 67.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^4}\,dx = \int \frac{\left(a+b\operatorname{asec}(cx)\right)(d+ex^2)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*asec(c*x))/x**4,x)
```

output
```
Integral((a + b*asec(c*x))*(d + e*x**2)**(3/2)/x**4, x)
```

3.128.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.128.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^4}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}\left(b\operatorname{arcsec}(cx)+a\right)}{x^4}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^4,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)/x^4, x)
```

3.128.9 Mupad [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^4}\,dx = \int \frac{(e\,x^2+d)^{3/2}\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^4,x)
```

output
```
int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^4, x)
```

3.129 $\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^6}\,dx$

3.129.1 Optimal result . . . 945
3.129.2 Mathematica [C] (verified) . . . 946
3.129.3 Rubi [A] (verified) . . . 946
3.129.4 Maple [F] . . . 951
3.129.5 Fricas [A] (verification not implemented) . . . 952
3.129.6 Sympy [F(-1)] . . . 952
3.129.7 Maxima [F(-2)] . . . 952
3.129.8 Giac [F] . . . 953
3.129.9 Mupad [F(-1)] . . . 953

3.129.1 Optimal result

Integrand size = 23, antiderivative size = 416

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^6}\,dx = \frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}}$$
$$+\frac{4bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}}$$
$$+\frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25x^4\sqrt{c^2x^2}}-\frac{(d+ex^2)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$
$$-\frac{bc^2(8c^4d^2+23c^2de+23e^2)\,x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|\,-\frac{e}{c^2d}\right)}{75d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}$$
$$+\frac{b(c^2d+e)(8c^4d^2+19c^2de+15e^2)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{75d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

output

```
-1/5*(e*x^2+d)^(5/2)*(a+b*arcsec(c*x))/d/x^5+1/25*b*c*(e*x^2+d)^(3/2)*(c^2
*x^2-1)^(1/2)/x^4/(c^2*x^2)^(1/2)+1/75*b*c*(8*c^4*d^2+23*c^2*d*e+23*e^2)*(
c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/(c^2*x^2)^(1/2)+4/75*b*c*(c^2*d+2*e)*(c
^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/x^2/(c^2*x^2)^(1/2)-1/75*b*c^2*(8*c^4*d^2+
23*c^2*d*e+23*e^2)*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e
*x^2+d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)+1/75*b
*(c^2*d+e)*(8*c^4*d^2+19*c^2*d*e+15*e^2)*x*EllipticF(c*x,(-e/c^2/d)^(1/2))
*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/
(e*x^2+d)^(1/2)
```

3.129.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.89 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.73

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^6}\,dx = \frac{\sqrt{d+ex^2}\left(-15a(d+ex^2)^2+bc\sqrt{1-\frac{1}{c^2x^2}}x(23e^2x^4+dex^2(11+23c^2$$
$$-\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(8c^4d^2+23c^2de+23e^2)\,E\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)|-\frac{e}{c^2d}\right)-(8c^6d^3+27c^4d^2e+34\right.}{75\sqrt{-c^2}d\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^6,x]
```

output
```
(Sqrt[d + e*x^2]*(-15*a*(d + e*x^2)^2 + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(23*e^
2*x^4 + d*e*x^2*(11 + 23*c^2*x^2) + d^2*(3 + 4*c^2*x^2 + 8*c^4*x^4)) - 15*
b*(d + e*x^2)^2*ArcSec[c*x]))/(75*d*x^5) - ((I/75)*b*c*Sqrt[1 - 1/(c^2*x^2
)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*Elliptic
E[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] - (8*c^6*d^3 + 27*c^4*d^2*e + 34*
c^2*d*e^2 + 15*e^3)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sq
rt[-c^2]*d*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

3.129.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 381, normalized size of antiderivative = 0.92, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {5761, 27, 376, 25, 442, 25, 445, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^6}\,dx$$
$$\downarrow 5761$$
$$-\frac{bcx\int -\frac{(ex^2+d)^{5/2}}{5dx^6\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$
$$\downarrow 27$$

$$\frac{bcx\int\frac{(ex^2+d)^{5/2}}{x^6\sqrt{c^2x^2-1}}dx}{5d\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\downarrow 376$$

$$\frac{bcx\left(\frac{d\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{5x^5}-\frac{1}{5}\int-\frac{\sqrt{ex^2+d}\left(e\left(dc^2+5e\right)x^2+4d\left(dc^2+2e\right)\right)}{x^4\sqrt{c^2x^2-1}}dx\right)}{5d\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\downarrow 25$$

$$\frac{bcx\left(\frac{1}{5}\int\frac{\sqrt{ex^2+d}\left(e\left(dc^2+5e\right)x^2+4d\left(dc^2+2e\right)\right)}{x^4\sqrt{c^2x^2-1}}dx+\frac{d\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{5x^5}\right)}{5d\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\downarrow 442$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{4d\sqrt{c^2x^2-1}\left(c^2d+2e\right)\sqrt{d+ex^2}}{3x^3}-\frac{1}{3}\int-\frac{e\left(4d^2c^4+11dec^2+15e^2\right)x^2+d\left(8d^2c^4+23dec^2+23e^2\right)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx\right)+\frac{d\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{5x^5}\right)}{5d\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\downarrow 25$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\int\frac{e\left(4d^2c^4+11dec^2+15e^2\right)x^2+d\left(8d^2c^4+23dec^2+23e^2\right)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{4d\sqrt{c^2x^2-1}\left(c^2d+2e\right)\sqrt{d+ex^2}}{3x^3}\right)+\frac{d\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{5x^5}\right)}{5d\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\downarrow 445$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{\int\frac{de\left(4d^2c^4-\left(8d^2c^4+23dec^2+23e^2\right)x^2c^2+11dec^2+15e^2\right)}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{d}+\frac{\sqrt{c^2x^2-1}\left(8c^4d^2+23c^2de+23e^2\right)\sqrt{d+ex^2}}{x}\right)+\frac{4d\sqrt{c^2x^2-1}\left(c^2d+2e\right)}{3x^3}\right)}{5d\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\downarrow 27$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\int\frac{4d^2c^4-\left(8d^2c^4+23dec^2+23e^2\right)x^2c^2+11dec^2+15e^2}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{\sqrt{c^2x^2-1}\left(8c^4d^2+23c^2de+23e^2\right)\sqrt{d+ex^2}}{x}\right)+\frac{4d\sqrt{c^2x^2-1}\left(c^2d+2e\right)}{3x^3}\right.\right.}{5d\sqrt{c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\downarrow 399$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\left(c^2d+e\right)\left(8c^4d^2+19c^2de+15e^2\right)\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{e}-\frac{c^2\left(8c^4d^2+23c^2de+23e^2\right)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-1}\left(8c^4d^2+23c^2\right.\right.\right.}{5d\sqrt{c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\downarrow 323$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\left(c^2d+e\right)\left(8c^4d^2+19c^2de+15e^2\right)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}-\frac{c^2\left(8c^4d^2+23c^2de+23e^2\right)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-1}\left(\right.\right.\right.}{5d\sqrt{c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\downarrow 323$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}\left(c^2d+e\right)\left(8c^4d^2+19c^2de+15e^2\right)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\left(8c^4d^2+23c^2de+23e^2\right)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{}}{5d\sqrt{c^2x^2}}\right.\right.\right.}$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\downarrow 321$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}\left(c^2d+e\right)\left(8c^4d^2+19c^2de+15e^2\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\left(8c^4d^2+23c^2de+23e^2\right)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)\right.\right.\right.}{5d\sqrt{c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\downarrow 331$$

3.129. $\int\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^6}dx$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}\,(c^2d+e)\,(8c^4d^2+19c^2de+15e^2)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\sqrt{1-c^2x^2}\,(8c^4d^2+23c^2de+23e^2)\int\frac{\sqrt{ex^2}}{\sqrt{1-c^2}}}{e\sqrt{c^2x^2-1}}\right.\right.\right.}{5d\sqrt{c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\downarrow 330$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}\,(c^2d+e)\,(8c^4d^2+19c^2de+15e^2)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\sqrt{1-c^2x^2}\,(8c^4d^2+23c^2de+23e^2)\sqrt{d+ex}}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\right.\right.\right.}{5d\sqrt{c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\downarrow 327$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}\,(c^2d+e)\,(8c^4d^2+19c^2de+15e^2)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c\sqrt{1-c^2x^2}\,(8c^4d^2+23c^2de+23e^2)\sqrt{d+ex^2}E}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\right.\right.\right.}{5d\sqrt{c^2x}}$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^6,x]
```

output
```
-1/5*((d + e*x^2)^(5/2)*(a + b*ArcSec[c*x]))/(d*x^5) + (b*c*x*((d*Sqrt[-1
+ c^2*x^2]*(d + e*x^2)^(3/2))/(5*x^5) + ((4*d*(c^2*d + 2*e)*Sqrt[-1 + c^2*
x^2]*Sqrt[d + e*x^2])/(3*x^3) + (((8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*Sqrt[-
1 + c^2*x^2]*Sqrt[d + e*x^2])/x + e*(-((c*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2
)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/
(e*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d])) + ((c^2*d + e)*(8*c^4*d^2 + 19
*c^2*d*e + 15*e^2)*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[
c*x], -(e/(c^2*d))])/(c*e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2]))/3)/5))/(5*
d*Sqrt[c^2*x^2])
```

3.129. $\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^6}\,dx$

3.129.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.129.4 Maple [F]

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\operatorname{arcsec}(cx))}{x^6}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^6,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^6,x)
```

3.129.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\,\mathrm{sec}^{-1}(cx))}{x^6}\,dx =$$
$$-\frac{\left(15\,acde^2x^4+30\,acd^2ex^2+15\,acd^3+15\,(bcde^2x^4+2\,bcd^2ex^2+bcd^3)\,\mathrm{arcsec}\,(cx)-(3\,bcd^3+(8\,bc^5d^3+2\right.}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^6,x, algorithm="fricas")
```

output
```
-1/75*((15*a*c*d*e^2*x^4 + 30*a*c*d^2*e*x^2 + 15*a*c*d^3 + 15*(b*c*d*e^2*x
^4 + 2*b*c*d^2*e*x^2 + b*c*d^3)*arcsec(c*x) - (3*b*c*d^3 + (8*b*c^5*d^3 +
23*b*c^3*d^2*e + 23*b*c*d*e^2)*x^4 + (4*b*c^3*d^3 + 11*b*c*d^2*e)*x^2)*sqr
t(c^2*x^2 - 1))*sqrt(e*x^2 + d) - ((8*b*c^8*d^3 + 23*b*c^6*d^2*e + 23*b*c^
4*d*e^2)*x^5*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (8*b*c^8*d^3 + (23*b*c^
6 + 4*b*c^4)*d^2*e + (23*b*c^4 + 11*b*c^2)*d*e^2 + 15*b*e^3)*x^5*elliptic_
f(arcsin(c*x), -e/(c^2*d)))*sqrt(-d))/(c*d^2*x^5)
```

3.129.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\,\mathrm{sec}^{-1}(cx))}{x^6}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*asec(c*x))/x**6,x)
```

output
```
Timed out
```

3.129.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\,\mathrm{sec}^{-1}(cx))}{x^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^6,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.129.8 Giac [F]

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^6}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arcsec}\,(cx)+a)}{x^6}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^6,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)/x^6, x)
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^6}\,dx = \int \frac{(e\,x^2+d)^{3/2}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^6,x)
```

output
```
int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^6, x)
```

3.130 $\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^8}\,dx$

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3.130.2 Mathematica [C] (verified) . . . 955
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3.130.1 Optimal result

Integrand size = 23, antiderivative size = 554

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^8}\,dx = \frac{bc\left(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{c^2x^2}}$$
$$+\frac{bc\left(120c^4d^2+159c^2de-37e^2\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675dx^2\sqrt{c^2x^2}}$$
$$+\frac{bc\left(30c^2d+11e\right)\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{1225dx^4\sqrt{c^2x^2}}+\frac{bc\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{5/2}}{49dx^6\sqrt{c^2x^2}}$$
$$-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{7dx^7}+\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{35d^2x^5}$$
$$-\frac{bc^2\left(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3\right)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3675d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}$$
$$+\frac{2b\left(c^2d+e\right)\left(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3\right)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3675d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

output
```
-1/7*(e*x^2+d)^(5/2)*(a+b*arcsec(c*x))/d/x^7+2/35*e*(e*x^2+d)^(5/2)*(a+b*a
rcsec(c*x))/d^2/x^5+1/1225*b*c*(30*c^2*d+11*e)*(e*x^2+d)^(3/2)*(c^2*x^2-1)
^(1/2)/d/x^4/(c^2*x^2)^(1/2)+1/49*b*c*(e*x^2+d)^(5/2)*(c^2*x^2-1)^(1/2)/d/
x^6/(c^2*x^2)^(1/2)+1/3675*b*c*(240*c^6*d^3+528*c^4*d^2*e+193*c^2*d*e^2-24
7*e^3)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)+1/3675*b*c*(1
20*c^4*d^2+159*c^2*d*e-37*e^2)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(c^
2*x^2)^(1/2)-1/3675*b*c^2*(240*c^6*d^3+528*c^4*d^2*e+193*c^2*d*e^2-247*e^3
)*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2
/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)+2/3675*b*(c^2*d+e)*(1
20*c^6*d^3+204*c^4*d^2*e+17*c^2*d*e^2-105*e^3)*x*EllipticF(c*x,(-e/c^2/d)^
(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1
)^(1/2)/(e*x^2+d)^(1/2)
```

3.130.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.82 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^8}\,dx = \frac{\sqrt{d+ex^2}\Big(-105a(5d-2ex^2)\,(d+ex^2)^2+bc\sqrt{1-\frac{1}{c^2x^2}}x(-247e^3x^6+$$
$$-\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\,E\left(i\operatorname{arcsinh}\left(\sqrt{-c^2}x\right)|-\frac{e}{c^2d}\right)-2(12\right.}{3675\sqrt{-c^2}d^2\sqrt{1-c^2x^2}\sqrt{c}}$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^8,x]
```

output
```
(Sqrt[d + e*x^2]*(-105*a*(5*d - 2*e*x^2)*(d + e*x^2)^2 + b*c*Sqrt[1 - 1/(c
^2*x^2)]*x*(-247*e^3*x^6 + d*e^2*x^4*(71 + 193*c^2*x^2) + 3*d^2*e*x^2*(61
+ 83*c^2*x^2 + 176*c^4*x^4) + 15*d^3*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 16*c^6*x
^6)) - 105*b*(5*d - 2*e*x^2)*(d + e*x^2)^2*ArcSec[c*x]))/(3675*d^2*x^7) -
((I/3675)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(240*c^6*
d^3 + 528*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*EllipticE[I*ArcSinh[Sqrt[-c
^2]*x], -(e/(c^2*d))] - 2*(120*c^8*d^4 + 324*c^6*d^3*e + 221*c^4*d^2*e^2 -
 88*c^2*d*e^3 - 105*e^4)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))])
)/(Sqrt[-c^2]*d^2*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

3.130.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 504, normalized size of antiderivative = 0.91, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {5761, 27, 442, 25, 442, 25, 442, 25, 445, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^8}\,dx$$

$$\downarrow 5761$$

$$-\frac{bcx\int -\frac{(5d-2ex^2)(ex^2+d)^{5/2}}{35d^2x^8\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{7dx^7}$$

$$\downarrow 27$$

$$\frac{bcx\int \frac{(5d-2ex^2)(ex^2+d)^{5/2}}{x^8\sqrt{c^2x^2-1}}dx}{35d^2\sqrt{c^2x^2}}+\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{7dx^7}$$

$$\downarrow 442$$

$$\frac{bcx\left(\frac{5d\sqrt{c^2x^2-1}\left(d+ex^2\right)^{5/2}}{7x^7}-\frac{1}{7}\int -\frac{\left(ex^2+d\right)^{3/2}\left((5c^2d-14e)ex^2+d(30dc^2+11e)\right)}{x^6\sqrt{c^2x^2-1}}dx\right)}{35d^2\sqrt{c^2x^2}}+$$
$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{7dx^7}$$

$$\downarrow 25$$

$$\frac{bcx\left(\frac{1}{7}\int \frac{\left(ex^2+d\right)^{3/2}\left((5c^2d-14e)ex^2+d(30dc^2+11e)\right)}{x^6\sqrt{c^2x^2-1}}dx+\frac{5d\sqrt{c^2x^2-1}\left(d+ex^2\right)^{5/2}}{7x^7}\right)}{35d^2\sqrt{c^2x^2}}+$$
$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{7dx^7}$$

$$\downarrow 442$$

$$\frac{bcx\left(\frac{1}{7}\left(\frac{d\sqrt{c^2x^2-1}(30c^2d+11e)\left(d+ex^2\right)^{3/2}}{5x^5}-\frac{1}{5}\int -\frac{\sqrt{ex^2+d}\left(2e\left(15d^2c^4+18dec^2-35e^2\right)x^2+d\left(120d^2c^4+159dec^2-37e^2\right)\right)}{x^4\sqrt{c^2x^2-1}}dx\right)+\frac{5d\sqrt{c^2x^2}}{}\right.}{35d^2\sqrt{c^2x^2}}$$
$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{7dx^7}$$

$$\downarrow 25$$

$$\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\int\frac{\sqrt{ex^2+d}\left(2e\left(15d^2c^4+18dec^2-35e^2\right)x^2+d\left(120d^2c^4+159dec^2-37e^2\right)\right)}{x^4\sqrt{c^2x^2-1}}dx+\frac{d\sqrt{c^2x^2-1}\left(30c^2d+11e\right)\left(d+ex^2\right)^{3/2}}{5x^5}\right)+\frac{5d\sqrt{c^2x^2-}}{7}\right.}{35d^2\sqrt{c^2x^2}}$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{7dx^7}$$

$$\downarrow 442$$

$$\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{d\sqrt{c^2x^2-1}\left(120c^4d^2+159c^2de-37e^2\right)\sqrt{d+ex^2}}{3x^3}-\frac{1}{3}\int-\frac{e\left(120d^3c^6+249d^2ec^4+71de^2c^2-210e^3\right)x^2+d\left(240d^3c^6+528d^2ec^4+193de^2\right.}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}\right.\right.\right.}{35d^2\sqrt{c^2x^2}}$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{7dx^7}$$

$$\downarrow 25$$

$$\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int\frac{e\left(120d^3c^6+249d^2ec^4+71de^2c^2-210e^3\right)x^2+d\left(240d^3c^6+528d^2ec^4+193de^2c^2-247e^3\right)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{d\sqrt{c^2x^2-1}\left(120c^4d^2+159c^2de-37\right.}{3x^3}\right.\right.\right.}{35d^2\sqrt{c^2x^2}}$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{7dx^7}$$

$$\downarrow 445$$

$$\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\int\frac{de\left(120d^3c^6+249d^2ec^4+71de^2c^2-\left(240d^3c^6+528d^2ec^4+193de^2c^2-247e^3\right)x^2c^2-210e^3\right)}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx\right.\right.\right.\right.}{d}+\frac{\sqrt{c^2x^2-1}\left(240c^6d^3+528c^4d^2e+193c^\right.}{x}$$

$$35d$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{7dx^7}$$

$$\downarrow 27$$

$$\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\int\frac{120d^3c^6+249d^2ec^4+71de^2c^2-\left(240d^3c^6+528d^2ec^4+193de^2c^2-247e^3\right)x^2c^2-210e^3}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{\sqrt{c^2x^2-1}\left(240c^6d^3+528c^4d^2e+1\right.}{x}\right.\right.\right.\right.}{35d}$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{7dx^7}$$

$$\downarrow 399$$

$$bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{2(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{e}-\frac{c^2(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\int\frac{\sqrt{e}}{\sqrt{c}}}{e}\right.\right.\right.\right.\right.$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{7dx^7}$$

$$\downarrow 323$$

$$bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{2(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}-\frac{c^2(240c^6d^3+528c^4d^2e+193c^2de^2-2}{e}\right.\right.\right.\right.\right.$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{7dx^7}$$

$$\downarrow 323$$

$$bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{2\sqrt{1-c^2x^2}(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2(240c^6d^3+528c^4d^2e+193}{}\right.\right.\right.\right.\right.$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{7dx^7}$$

$$\downarrow 321$$

$$bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{2\sqrt{1-c^2x^2}(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2(240c^6d^3+528c^4d^2e}{}\right.\right.\right.\right.\right.$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{7dx^7}$$

$$\downarrow 331$$

$$bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{2\sqrt{1-c^2x^2}(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\sqrt{1-c^2x^2}(240c^6d^3+}{}\right.\right.\right.\right.\right.$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{7dx^7}$$

$$\downarrow 330$$

3.130. $\int\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{x^8}dx$

$$bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{2\sqrt{1-c^2x^2}\,(c^2d+e)\,(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\sqrt{1-c^2x^2}\,(240c^6d^3\right.\right.\right.\right.\right.$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{7dx^7}$$

$$\downarrow 327$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{7dx^7}+$$

$$bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{2\sqrt{1-c^2x^2}\,(c^2d+e)\,(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c\sqrt{1-c^2x^2}\,(240c^6d^3+5\right.\right.\right.\right.\right.$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^8,x]
```

output
```
-1/7*((d + e*x^2)^(5/2)*(a + b*ArcSec[c*x]))/(d*x^7) + (2*e*(d + e*x^2)^(5
/2)*(a + b*ArcSec[c*x]))/(35*d^2*x^5) + (b*c*x*((5*d*Sqrt[-1 + c^2*x^2]*(d
 + e*x^2)^(5/2))/(7*x^7) + ((d*(30*c^2*d + 11*e)*Sqrt[-1 + c^2*x^2]*(d + e
*x^2)^(3/2))/(5*x^5) + ((d*(120*c^4*d^2 + 159*c^2*d*e - 37*e^2)*Sqrt[-1 +
c^2*x^2]*Sqrt[d + e*x^2])/(3*x^3) + (((240*c^6*d^3 + 528*c^4*d^2*e + 193*c
^2*d*e^2 - 247*e^3)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/x + e*(-((c*(240*c
^6*d^3 + 528*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*Sqrt[1 - c^2*x^2]*Sqrt[d
 + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(e*Sqrt[-1 + c^2*x^2]*Sqrt
[1 + (e*x^2)/d])) + (2*(c^2*d + e)*(120*c^6*d^3 + 204*c^4*d^2*e + 17*c^2*d
*e^2 - 105*e^3)*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x
], -(e/(c^2*d))])/(c*e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2]))/3)/5)/7))/(35
*d^2*Sqrt[c^2*x^2])
```

3.130.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

3.130. $\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^8}\,dx$

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])  u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])  Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.130.4 Maple [F]

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\operatorname{arcsec}(cx))}{x^8}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^8,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^8,x)
```

3.130.5 Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 395, normalized size of antiderivative = 0.71

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^8}\,dx = \frac{(210\,acde^3x^6 - 105\,acd^2e^2x^4 - 840\,acd^3ex^2 - 525\,acd^4 + 105\,(2\,bcde^3$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^8,x, algorithm="fricas")
```

output
```
1/3675*((210*a*c*d*e^3*x^6 - 105*a*c*d^2*e^2*x^4 - 840*a*c*d^3*e*x^2 - 525
*a*c*d^4 + 105*(2*b*c*d*e^3*x^6 - b*c*d^2*e^2*x^4 - 8*b*c*d^3*e*x^2 - 5*b*
c*d^4)*arcsec(c*x) + ((240*b*c^7*d^4 + 528*b*c^5*d^3*e + 193*b*c^3*d^2*e^2
 - 247*b*c*d*e^3)*x^6 + 75*b*c*d^4 + (120*b*c^5*d^4 + 249*b*c^3*d^3*e + 71
*b*c*d^2*e^2)*x^4 + 3*(30*b*c^3*d^4 + 61*b*c*d^3*e)*x^2)*sqrt(c^2*x^2 - 1)
)*sqrt(e*x^2 + d) + ((240*b*c^10*d^4 + 528*b*c^8*d^3*e + 193*b*c^6*d^2*e^2
 - 247*b*c^4*d*e^3)*x^7*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (240*b*c^10*
d^4 + 24*(22*b*c^8 + 5*b*c^6)*d^3*e + (193*b*c^6 + 249*b*c^4)*d^2*e^2 - (2
47*b*c^4 - 71*b*c^2)*d*e^3 - 210*b*e^4)*x^7*elliptic_f(arcsin(c*x), -e/(c^
2*d)))*sqrt(-d))/(c*d^3*x^7)
```

3.130.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^8}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*asec(c*x))/x**8,x)
```

output
```
Timed out
```

3.130.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^8}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^8,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.130.8 Giac [F]

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^8}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\text{arcsec}\,(cx)+a)}{x^8}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^8,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)/x^8, x)
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^8}\,dx = \int \frac{(e\,x^2+d)^{3/2}\,(a+b\,\text{acos}\left(\frac{1}{c\,x}\right))}{x^8}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^8,x)
```

output
```
int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^8, x)
```

3.131 $\int \frac{x^5\left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

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3.131.1 Optimal result

Integrand size = 23, antiderivative size = 321

$$\begin{aligned}\int \frac{x^5(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx &= \frac{b(19c^2d-9e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{c^2x^2}} \\ &\quad - \frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{20ce^2\sqrt{c^2x^2}} + \frac{d^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3} \\ &\quad - \frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3} \\ &\quad + \frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3} + \frac{8bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{15e^3\sqrt{c^2x^2}} \\ &\quad - \frac{b(45c^4d^2-10c^2de+9e^2)\,x\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{120c^4e^{5/2}\sqrt{c^2x^2}}\end{aligned}$$

output

```
-2/3*d*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/e^3+1/5*(e*x^2+d)^(5/2)*(a+b*arcs
ec(c*x))/e^3+8/15*b*c*d^(5/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)
^(1/2))/e^3/(c^2*x^2)^(1/2)-1/120*b*(45*c^4*d^2-10*c^2*d*e+9*e^2)*x*arctan
h(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^4/e^(5/2)/(c^2*x^2)^(1/2)
-1/20*b*x*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(1/2)/c/e^2/(c^2*x^2)^(1/2)+d^2*(a+b
*arcsec(c*x))*(e*x^2+d)^(1/2)/e^3+1/120*b*(19*c^2*d-9*e)*x*(c^2*x^2-1)^(1/
2)*(e*x^2+d)^(1/2)/c^3/e^2/(c^2*x^2)^(1/2)
```

3.131.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 2.00 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.88

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$

$$= \frac{16a(d+ex^2)\left(8d^2-4dex^2+3e^2x^4\right) - \frac{2be\sqrt{1-\frac{1}{c^2x^2}}\,(d+ex^2)\left(9ex+c^2\left(-13dx+6ex^3\right)\right)}{c^3} + \frac{64bd^3\sqrt{1+\frac{d}{ex^2}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2}\right.}{cx}}{24}$$

input
```
Integrate[(x^5*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
(16*a*(d + e*x^2)*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4) - (2*b*e*Sqrt[1 - 1/(c^2
*x^2)]*(d + e*x^2)*(9*e*x + c^2*(-13*d*x + 6*e*x^3)))/c^3 + (64*b*d^3*Sqrt
[1 + d/(e*x^2)]*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))])/(c*x)
 + (b*e*(45*c^4*d^2 - 10*c^2*d*e + 9*e^2)*Sqrt[1 - 1/(c^2*x^2)]*x^3*Sqrt[1
 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/(c^3*Sqrt[1
 - c^2*x^2]) + 16*b*(d + e*x^2)*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4)*ArcSec[c*x
])/(240*e^3*Sqrt[d + e*x^2])
```

3.131.3 Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {5761, 27, 7282, 2118, 27, 171, 27, 175, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$

$$\downarrow 5761$$

$$-\frac{bcx\int \frac{\sqrt{ex^2+d}\left(3e^2x^4-4dex^2+8d^2\right)}{15e^3x\sqrt{c^2x^2-1}}\,dx}{\sqrt{c^2x^2}} + \frac{d^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3} + \frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3} - \frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$$\downarrow 27$$

$$-\frac{bcx\int\frac{\sqrt{ex^2+d}\left(3e^2x^4-4dex^2+8d^2\right)}{x\sqrt{c^2x^2-1}}dx}{15e^3\sqrt{c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$$\downarrow 7282$$

$$-\frac{bcx\int\frac{\sqrt{ex^2+d}\left(3e^2x^4-4dex^2+8d^2\right)}{x^2\sqrt{c^2x^2-1}}dx^2}{30e^3\sqrt{c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$$\downarrow 2118$$

$$-\frac{bcx\left(\frac{\int\frac{e\sqrt{ex^2+d}\left(32c^2d^2-\left(19c^2d-9e\right)ex^2\right)}{2x^2\sqrt{c^2x^2-1}}dx^2}{2c^2e}+\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3\sqrt{c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$$\downarrow 27$$

$$-\frac{bcx\left(\frac{\int\frac{\sqrt{ex^2+d}\left(32c^2d^2-\left(19c^2d-9e\right)ex^2\right)}{x^2\sqrt{c^2x^2-1}}dx^2}{4c^2}+\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3\sqrt{c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$$\downarrow 171$$

$$-\frac{bcx\left(\frac{\int\frac{64d^3c^4+e\left(45d^2c^4-10dec^2+9e^2\right)x^2}{2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{4c^2}-\frac{e\sqrt{c^2x^2-1}\left(19c^2d-9e\right)\sqrt{d+ex^2}}{c^2}+\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3\sqrt{c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$$\downarrow 27$$

$$-\frac{bcx\left(\frac{\frac{\int \frac{64d^3c^4+e\left(45d^2c^4-10dec^2+9e^2\right)x^2}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(19c^2d-9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3\sqrt{c^2x^2}}+$$
$$\frac{d^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$$\downarrow 175$$

$$-\frac{bcx\left(\frac{\frac{64c^4d^3\int\frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+e\left(45c^4d^2-10c^2de+9e^2\right)\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(19c^2d-9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/}}{2c^2}\right)}{30e^3\sqrt{c^2x^2}}$$
$$\frac{d^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$$\downarrow 66$$

$$-\frac{bcx\left(\frac{\frac{64c^4d^3\int\frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\left(45c^4d^2-10c^2de+9e^2\right)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(19c^2d-9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3\sqrt{c^2x^2}}$$
$$\frac{d^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$$\downarrow 104$$

$$-\frac{bcx\left(\frac{\frac{128c^4d^3\int\frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+2e\left(45c^4d^2-10c^2de+9e^2\right)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(19c^2d-9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3\sqrt{c^2x^2}}$$
$$\frac{d^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$$\downarrow 217$$

$$-\frac{bcx\left(\frac{\frac{2e\left(45c^4d^2-10c^2de+9e^2\right)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-128c^4d^{5/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(19c^2d-9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3\sqrt{c^2x^2}}$$
$$\frac{d^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

3.131. $\int \frac{x^5\left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

$$\downarrow 221$$

$$\frac{d^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}-$$

$$bcx\left(\frac{\frac{2\sqrt{e}\left(45c^4d^2-10c^2de+9e^2\right)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-128c^4d^{5/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(19c^2d-9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{c^2x^2-1}(d+ex^2}{2c^2}\right.$$

$$30e^3\sqrt{c^2x^2}$$

input
```
Int[(x^5*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
(d^2*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/e^3 - (2*d*(d + e*x^2)^(3/2)*(a
+ b*ArcSec[c*x]))/(3*e^3) + ((d + e*x^2)^(5/2)*(a + b*ArcSec[c*x]))/(5*e^3
) - (b*c*x*((3*e*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/(2*c^2) + (-(((19*c
^2*d - 9*e)*e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/c^2) + (-128*c^4*d^(5/2)
*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])] + (2*Sqrt[e]*(45*c^4
*d^2 - 10*c^2*d*e + 9*e^2)*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d
+ e*x^2])])/c)/(2*c^2))/(4*c^2)))/(30*e^3*Sqrt[c^2*x^2])
```

3.131.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])  u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])  Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]  Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

3.131.4 Maple [F]

$$\int \frac{x^5(a+b\,\text{arcsec}\,(cx))}{\sqrt{e\,x^2+d}}dx$$

input
```
int(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

3.131.5 Fricas [A] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 1385, normalized size of antiderivative = 4.31

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/480*(64*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^
2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2
+ d)*sqrt(-d) + 8*d^2)/x^4) + (45*b*c^4*d^2 - 10*b*c^2*d*e + 9*b*e^2)*sqrt
(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 -
4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) +
e^2) + 4*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 + 8*(3*b*c^5*
e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b*c^5*d^2)*arcsec(c*x) - (6*b*c^3*e^2*x^2 -
13*b*c^3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e^3), 1
/480*(128*b*c^5*d^(5/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2
*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (
45*b*c^4*d^2 - 10*b*c^2*d*e + 9*b*e^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2
 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*s
qrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(24*a*c^5*e^2*x^4 - 32
*a*c^5*d*e*x^2 + 64*a*c^5*d^2 + 8*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b
*c^5*d^2)*arcsec(c*x) - (6*b*c^3*e^2*x^2 - 13*b*c^3*d*e + 9*b*c*e^2)*sqrt(
c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e^3), 1/240*(32*b*c^5*sqrt(-d)*d^2*log
(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2
 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + (45
*b*c^4*d^2 - 10*b*c^2*d*e + 9*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^
2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*...
```

3.131.6 Sympy [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^5(a+b\,\text{asec}\,(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate(x**5*(a+b*asec(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**5*(a + b*asec(c*x))/sqrt(d + e*x**2), x)
```

3.131.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.131.8 Giac [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\,\text{arcsec}\,(cx)+a)x^5}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x^5/sqrt(e*x^2 + d), x)
```

3.131.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^5\left(a+b\,\text{acos}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

output
```
int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.132 $\int \frac{x^3\left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

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3.132.1 Optimal result

Integrand size = 23, antiderivative size = 225

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = -\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{c^2x^2}} - \frac{d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2} - \frac{2bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e^2\sqrt{c^2x^2}} + \frac{b(3c^2d-e)\,x\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2e^{3/2}\sqrt{c^2x^2}}$$

output

```
1/3*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/e^2-2/3*b*c*d^(3/2)*x*arctan((e*x^2+
d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e^2/(c^2*x^2)^(1/2)+1/6*b*(3*c^2*d-e)*
x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^2/e^(3/2)/(c^2*x^
2)^(1/2)-d*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/e^2-1/6*b*x*(c^2*x^2-1)^(1/2)
*(e*x^2+d)^(1/2)/c/e/(c^2*x^2)^(1/2)
```

3.132.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.36 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.08

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$
$$= \frac{-4bd^2\sqrt{1+\frac{d}{ex^2}}(-1+c^2x^2)\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)+b(3c^2d-e)\,e\sqrt{1-\frac{1}{c^2x^2}}x^4\sqrt{1-c^2x^2}\sqrt{1+}}{12ce^2}$$

input
```
Integrate[(x^3*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
(-4*b*d^2*Sqrt[1 + d/(e*x^2)]*(-1 + c^2*x^2)*AppellF1[1, 1/2, 1/2, 2, 1/(c
^2*x^2), -(d/(e*x^2))] + b*(3*c^2*d - e)*e*Sqrt[1 - 1/(c^2*x^2)]*x^4*Sqrt[
1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^
2)/d)] - 2*x*(-1 + c^2*x^2)*(d + e*x^2)*(4*a*c*d + b*e*Sqrt[1 - 1/(c^2*x^2
)]*x - 2*a*c*e*x^2 + 2*b*c*(2*d - e*x^2)*ArcSec[c*x]))/(12*c*e^2*x*(-1 + c
^2*x^2)*Sqrt[d + e*x^2])
```

3.132.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5761, 27, 435, 171, 27, 175, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$
$$\downarrow 5761$$
$$-\frac{bcx\int -\frac{(2d-ex^2)\sqrt{ex^2+d}}{3e^2x\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2}$$
$$\downarrow 27$$
$$\frac{bcx\int \frac{(2d-ex^2)\sqrt{ex^2+d}}{x\sqrt{c^2x^2-1}}dx}{3e^2\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2}$$

$$\downarrow 435$$

$$\frac{bcx\int\frac{(2d-ex^2)\sqrt{ex^2+d}}{x^2\sqrt{c^2x^2-1}}dx^2}{6e^2\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}-\frac{d\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^2}$$

$$\downarrow 171$$

$$\frac{bcx\left(\frac{\int\frac{4c^2d^2+\left(3c^2d-e\right)ex^2}{2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^2\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}-\frac{d\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^2}$$

$$\downarrow 27$$

$$\frac{bcx\left(\frac{\int\frac{4c^2d^2+\left(3c^2d-e\right)ex^2}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^2\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}-\frac{d\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^2}$$

$$\downarrow 175$$

$$\frac{bcx\left(\frac{4c^2d^2\int\frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+e\left(3c^2d-e\right)\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^2\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}-\frac{d\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^2}$$

$$\downarrow 66$$

$$\frac{bcx\left(\frac{4c^2d^2\int\frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\left(3c^2d-e\right)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^2\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}-\frac{d\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^2}$$

$$\downarrow 104$$

$$\frac{bcx\left(\frac{8c^2d^2\int\frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+2e\left(3c^2d-e\right)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^2\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}-\frac{d\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^2}$$

$$\downarrow 217$$

$$\frac{bcx\left(\frac{2e\left(3c^2d-e\right)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-8c^2d^{3/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^2\sqrt{c^2x^2}}+$$
$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}-\frac{d\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^2}$$

$$\downarrow 221$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^2}-\frac{d\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^2}+$$
$$\frac{bcx\left(\frac{\frac{2\sqrt{e}\left(3c^2d-e\right)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-8c^2d^{3/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^2\sqrt{c^2x^2}}$$

input
```
Int[(x^3*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
-((d*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/e^2) + ((d + e*x^2)^(3/2)*(a + b
*ArcSec[c*x]))/(3*e^2) + (b*c*x*(-((e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/
c^2) + (-8*c^2*d^(3/2)*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])
] + (2*(3*c^2*d - e)*Sqrt[e]*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[
d + e*x^2])])/c)/(2*c^2)))/(6*e^2*Sqrt[c^2*x^2])
```

3.132.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.132.4 Maple [F]

$$\int \frac{x^3(a+b\,\text{arcsec}\,(cx))}{\sqrt{e\,x^2+d}}dx$$

input
```
int(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

3.132.5 Fricas [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 1111, normalized size of antiderivative = 4.94

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/24*(4*b*c^3*sqrt(-d)*d*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^
2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)
*sqrt(-d) + 8*d^2)/x^4) - (3*b*c^2*d - b*e)*sqrt(e)*log(8*c^4*e^2*x^4 + c^
4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c
*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(2*a*c^3*e*x^2 -
4*a*c^3*d - sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 - 2*b*c^3*d)*arcsec(c
*x))*sqrt(e*x^2 + d))/(c^3*e^2), -1/24*(8*b*c^3*d^(3/2)*arctan(-1/2*sqrt(c
^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 +
 (c^2*d^2 - d*e)*x^2 - d^2)) + (3*b*c^2*d - b*e)*sqrt(e)*log(8*c^4*e^2*x^4
 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*
d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 4*(2*a*c^3*e*x
^2 - 4*a*c^3*d - sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 - 2*b*c^3*d)*arc
sec(c*x))*sqrt(e*x^2 + d))/(c^3*e^2), 1/12*(2*b*c^3*sqrt(-d)*d*log(((c^4*d
^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((
c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) - (3*b*c^2*d
- b*e)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqr
t(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(
2*a*c^3*e*x^2 - 4*a*c^3*d - sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 - 2*b
*c^3*d)*arcsec(c*x))*sqrt(e*x^2 + d))/(c^3*e^2), -1/12*(4*b*c^3*d^(3/2)*ar
ctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqr...
```

3.132.6 Sympy [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^3(a+b\operatorname{asec}(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate(x**3*(a+b*asec(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**3*(a + b*asec(c*x))/sqrt(d + e*x**2), x)
```

3.132.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.132.8 Giac [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x^3}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x^3/sqrt(e*x^2 + d), x)
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^3\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

output
```
int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.133 $\int \frac{x\left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

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3.133.1 Optimal result

Integrand size = 21, antiderivative size = 132

$$\int \frac{x(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e} + \frac{bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{e\sqrt{c^2x^2}} - \frac{bx\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{c^2x^2}}$$

output

```
b*c*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))*d^(1/2)/e/(c^2*x^2
)^(1/2)-b*x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/e^(1/2)/(
c^2*x^2)^(1/2)+(a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/e
```

3.133.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.36 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.82

$$\int \frac{x(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d+ex^2}\left(a - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}\,x\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},1,\frac{3}{2},\frac{e-c^2ex^2}{c^2d+e},1-c^2x^2\right)}{\sqrt{\frac{c^2\left(d+ex^2\right)}{c^2d+e}}} + b\sec^{-1}(cx)\right)}{e}$$

input
```
Integrate[(x*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
(Sqrt[d + e*x^2]*(a - (b*c*Sqrt[1 - 1/(c^2*x^2)]*x*AppellF1[1/2, -1/2, 1,
3/2, (e - c^2*e*x^2)/(c^2*d + e), 1 - c^2*x^2])/Sqrt[(c^2*(d + e*x^2))/(c^
2*d + e)] + b*ArcSec[c*x]))/e
```

3.133.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {5759, 354, 140, 27, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

$$\downarrow 5759$$

$$\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e} - \frac{bcx\int \frac{\sqrt{ex^2+d}}{x\sqrt{c^2x^2-1}}dx}{e\sqrt{c^2x^2}}$$

$$\downarrow 354$$

$$\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e} - \frac{bcx\int \frac{\sqrt{ex^2+d}}{x^2\sqrt{c^2x^2-1}}dx^2}{2e\sqrt{c^2x^2}}$$

$$\downarrow 140$$

$$\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e} - \frac{bcx\left(e\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2 + \int \frac{d}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2\right)}{2e\sqrt{c^2x^2}}$$

$$\downarrow 27$$

$$\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e} - \frac{bcx\left(e\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2 + d\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2\right)}{2e\sqrt{c^2x^2}}$$

$$\downarrow 66$$

$$\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e} - \frac{bcx\left(d\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2 + 2e\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}\right)}{2e\sqrt{c^2x^2}}$$

$$\downarrow 104$$

$$\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e}-\frac{bcx\left(2d\int\frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+2e\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}\right)}{2e\sqrt{c^2x^2}}$$

$$\downarrow 217$$

$$\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e}-\frac{bcx\left(2e\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-2\sqrt{d}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)\right)}{2e\sqrt{c^2x^2}}$$

$$\downarrow 221$$

$$\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e}-\frac{bcx\left(\frac{2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-2\sqrt{d}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)\right)}{2e\sqrt{c^2x^2}}$$

input
```
Int[(x*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/e - (b*c*x*(-2*Sqrt[d]*ArcTan[Sqrt[d
 + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])] + (2*Sqrt[e]*ArcTanh[(Sqrt[e]*Sqrt
[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/c))/(2*e*Sqrt[c^2*x^2])
```

3.133.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5759
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSec[c*x])/(2*e*(p + 1))), x
] - Simp[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2]))   Int[(d + e*x^2)^(p + 1)/(x*S
qrt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

3.133.4 Maple [F]

$$\int \frac{x(a+b\operatorname{arcsec}(cx))}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int(x*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int(x*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

3.133.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 869, normalized size of antiderivative = 6.58

$$\int \frac{x(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$
$$= \left[\frac{bc\sqrt{-d}\log\left(\frac{(c^4d^2-6c^2de+e^2)x^4-8(c^2d^2-de)x^2-4\sqrt{c^2x^2-1}((c^2d-e)x^2-2d)\sqrt{ex^2+d}\sqrt{-d}+8d^2}{x^4}\right)+b\sqrt{e}\log\left(8c^4e^2x^4+c^4\right.\right.}{}$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(b*c*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e
)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-
d) + 8*d^2)/x^4) + b*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(
c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*s
qrt(e*x^2 + d)*sqrt(e) + e^2) + 4*sqrt(e*x^2 + d)*(b*c*arcsec(c*x) + a*c))
/(c*e), 1/4*(2*b*c*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2
- 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2))
+ b*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2
)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sq
rt(e) + e^2) + 4*sqrt(e*x^2 + d)*(b*c*arcsec(c*x) + a*c))/(c*e), 1/4*(b*c*
sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*
sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2
)/x^4) + 2*b*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 -
1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2))
 + 4*sqrt(e*x^2 + d)*(b*c*arcsec(c*x) + a*c))/(c*e), 1/2*(b*c*sqrt(d)*arct
an(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/
(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + b*sqrt(-e)*arctan(1/2*(2*c^2*
e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4
 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*sqrt(e*x^2 + d)*(b*c*arcsec(c*x) +
a*c))/(c*e)]
```

3.133.6 Sympy [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x(a+b\,\text{asec}\,(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate(x*(a+b*asec(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x*(a + b*asec(c*x))/sqrt(d + e*x**2), x)
```

3.133.7 Maxima [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\,\text{arcsec}\,(cx)+a)x}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
-(e*integrate((c^2*e*x^3*log(c) - e*x*log(c) + ((c^2*log(c) + c^2)*e*x^3 +
 (c^2*d - e*log(c))*x)*e^(log(c*x + 1) + log(c*x - 1)) + (c^2*e*x^3 - e*x
+ (c^2*e*x^3 - e*x)*e^(log(c*x + 1) + log(c*x - 1)))*log(x))/((c^2*e*x^2 +
 (c^2*e*x^2 - e)*e^(log(c*x + 1) + log(c*x - 1)) - e)*sqrt(e*x^2 + d)), x)
 - sqrt(e*x^2 + d)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)))*b/e + sqrt(e*x^2 +
 d)*a/e
```

3.133.8 Giac [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\,\text{arcsec}\,(cx)+a)x}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x/sqrt(e*x^2 + d), x)
```

3.133.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

output
```
int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.134 $\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$

3.134.1 Optimal result 988
3.134.2 Mathematica [N/A] 988
3.134.3 Rubi [N/A] 989
3.134.4 Maple [N/A] (verified) 989
3.134.5 Fricas [N/A] 990
3.134.6 Sympy [N/A] 990
3.134.7 Maxima [F(-2)] 990
3.134.8 Giac [N/A] 991
3.134.9 Mupad [N/A] 991

3.134.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx = \text{Int}\left(\frac{a+b\sec^{-1}(cx)}{x\sqrt{d+ex^2}},x\right)$$

output
```
Unintegrable((a+b*arcsec(c*x))/x/(e*x^2+d)^(1/2),x)
```

3.134.2 Mathematica [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx = \int \frac{a+b\sec^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x*Sqrt[d + e*x^2]),x]
```

output
```
Integrate[(a + b*ArcSec[c*x])/(x*Sqrt[d + e*x^2]), x]
```

3.134.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \sec^{-1}(cx)}{x\sqrt{d + ex^2}}\, dx$$
$$\downarrow 5771$$
$$\int \frac{a + b \sec^{-1}(cx)}{x\sqrt{d + ex^2}} dx$$

input
```
Int[(a + b*ArcSec[c*x])/(x*Sqrt[d + e*x^2]),x]
```

output
```
$Aborted
```

3.134.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.134.4 Maple [N/A] (verified)

Not integrable

Time = 0.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a + b \operatorname{arcsec}(cx)}{x\sqrt{e\,x^2 + d}} dx$$

input
```
int((a+b*arcsec(c*x))/x/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arcsec(c*x))/x/(e*x^2+d)^(1/2),x)
```

3.134.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx = \int \frac{b\operatorname{arcsec}(cx)+a}{\sqrt{ex^2+d}x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/(e*x^3 + d*x), x)
```

3.134.6 Sympy [N/A]

Not integrable

Time = 7.82 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx = \int \frac{a+b\operatorname{asec}(cx)}{x\sqrt{d+ex^2}}\,dx$$

input
```
integrate((a+b*asec(c*x))/x/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asec(c*x))/(x*sqrt(d + e*x**2)), x)
```

3.134.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.134.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx = \int \frac{b\,\text{arcsec}\,(cx)+a}{\sqrt{ex^2+d}x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/(sqrt(e*x^2 + d)*x), x)
```

3.134.9 Mupad [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\text{acos}\left(\frac{1}{cx}\right)}{x\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^(1/2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^(1/2)), x)
```

3.135 $\int \frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$

3.135.1 Optimal result 992
3.135.2 Mathematica [N/A] 992
3.135.3 Rubi [N/A] 993
3.135.4 Maple [N/A] (verified) 993
3.135.5 Fricas [N/A] 994
3.135.6 Sympy [N/A] 994
3.135.7 Maxima [F(-2)] 994
3.135.8 Giac [N/A] 995
3.135.9 Mupad [N/A] 995

3.135.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \text{Int}\left(\frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}},x\right)$$

output
```
Unintegrable((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

3.135.2 Mathematica [N/A]

Not integrable

Time = 4.73 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x^3*Sqrt[d + e*x^2]),x]
```

output
```
Integrate[(a + b*ArcSec[c*x])/(x^3*Sqrt[d + e*x^2]), x]
```

3.135.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \sec^{-1}(cx)}{x^3 \sqrt{d + ex^2}}\, dx$$
$$\downarrow 5771$$
$$\int \frac{a + b \sec^{-1}(cx)}{x^3 \sqrt{d + ex^2}} dx$$

input
```
Int[(a + b*ArcSec[c*x])/(x^3*Sqrt[d + e*x^2]),x]
```

output
```
$Aborted
```

3.135.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.135.4 Maple [N/A] (verified)

Not integrable

Time = 0.82 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a + b \operatorname{arcsec}(cx)}{x^3 \sqrt{e\, x^2 + d}} dx$$

input
```
int((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

3.135.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{b\,\text{arcsec}\,(cx)+a}{\sqrt{ex^2+dx^3}}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/(e*x^5 + d*x^3), x)
```

3.135.6 Sympy [N/A]

Not integrable

Time = 27.73 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\text{asec}\,(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

input
```
integrate((a+b*asec(c*x))/x**3/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asec(c*x))/(x**3*sqrt(d + e*x**2)), x)
```

3.135.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.135.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{b\,\text{arcsec}\,(cx)+a}{\sqrt{ex^2+dx^3}}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/(sqrt(e*x^2 + d)*x^3), x)
```

3.135.9 Mupad [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\text{acos}\left(\frac{1}{cx}\right)}{x^3\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(x^3*(d + e*x^2)^(1/2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x^3*(d + e*x^2)^(1/2)), x)
```

3.136 $\int \frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$

3.136.1 Optimal result . . . 996
3.136.2 Mathematica [N/A] . . . 996
3.136.3 Rubi [N/A] . . . 997
3.136.4 Maple [N/A] (verified) . . . 997
3.136.5 Fricas [N/A] . . . 998
3.136.6 Sympy [N/A] . . . 998
3.136.7 Maxima [F(-2)] . . . 998
3.136.8 Giac [N/A] . . . 999
3.136.9 Mupad [N/A] . . . 999

3.136.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \text{Int}\left(\frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}},x\right)$$

output
```
Unintegrable(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

3.136.2 Mathematica [N/A]

Not integrable

Time = 32.38 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(x^2*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
Integrate[(x^2*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2], x]
```

3.136.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$
$$\downarrow 5771$$
$$\int \frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}dx$$

input
```
Int[(x^2*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

3.136.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.136.4 Maple [N/A] (verified)

Not integrable

Time = 0.45 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^2(a+b\ \text{arcsec}\,(cx))}{\sqrt{e\,x^2+d}}dx$$

input
```
int(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

3.136.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x^2}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x^2*arcsec(c*x) + a*x^2)/sqrt(e*x^2 + d), x)
```

3.136.6 Sympy [N/A]

Not integrable

Time = 54.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^2(a+b\operatorname{asec}(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate(x**2*(a+b*asec(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**2*(a + b*asec(c*x))/sqrt(d + e*x**2), x)
```

3.136.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.136.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x^2}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x^2/sqrt(e*x^2 + d), x)
```

3.136.9 Mupad [N/A]

Not integrable

Time = 1.47 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^2\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

output
```
int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.137 $\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$

3.137.1 Optimal result 1000
3.137.2 Mathematica [N/A] 1000
3.137.3 Rubi [N/A] 1001
3.137.4 Maple [N/A] (verified) 1001
3.137.5 Fricas [N/A] 1002
3.137.6 Sympy [N/A] 1002
3.137.7 Maxima [F(-2)] 1002
3.137.8 Giac [N/A] 1003
3.137.9 Mupad [N/A] 1003

3.137.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \text{Int}\left(\frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex^2}},x\right)$$

output
```
Unintegrable((a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

3.137.2 Mathematica [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(a + b*ArcSec[c*x])/Sqrt[d + e*x^2],x]
```

output
```
Integrate[(a + b*ArcSec[c*x])/Sqrt[d + e*x^2], x]
```

3.137.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \sec^{-1}(cx)}{\sqrt{d + ex^2}}\, dx$$
$$\downarrow 5771$$
$$\int \frac{a + b \sec^{-1}(cx)}{\sqrt{d + ex^2}} dx$$

input
```
Int[(a + b*ArcSec[c*x])/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

3.137.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.137.4 Maple [N/A] (verified)

Not integrable

Time = 0.47 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{a + b \operatorname{arcsec}(cx)}{\sqrt{e\, x^2 + d}} dx$$

input
```
int((a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

3.137.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a + b \sec^{-1}(cx)}{\sqrt{d + ex^2}}\, dx = \int \frac{b \operatorname{arcsec}(cx) + a}{\sqrt{ex^2 + d}}\, dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*arcsec(c*x) + a)/sqrt(e*x^2 + d), x)
```

3.137.6 Sympy [N/A]

Not integrable

Time = 15.39 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a + b \sec^{-1}(cx)}{\sqrt{d + ex^2}}\, dx = \int \frac{a + b \operatorname{asec}(cx)}{\sqrt{d + ex^2}}\, dx$$

input
```
integrate((a+b*asec(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asec(c*x))/sqrt(d + e*x**2), x)
```

3.137.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b \sec^{-1}(cx)}{\sqrt{d + ex^2}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.137.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{b\,\text{arcsec}\,(cx)+a}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/sqrt(e*x^2 + d), x)
```

3.137.9 Mupad [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\text{acos}\left(\frac{1}{c\,x}\right)}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(d + e*x^2)^(1/2),x)
```

output
```
int((a + b*acos(1/(c*x)))/(d + e*x^2)^(1/2), x)
```

3.138 $\int \frac{a+b\sec^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx$

3.138.1 Optimal result 1004
3.138.2 Mathematica [A] (verified) 1005
3.138.3 Rubi [A] (verified) 1005
3.138.4 Maple [F] 1009
3.138.5 Fricas [A] (verification not implemented) 1009
3.138.6 Sympy [F] 1010
3.138.7 Maxima [F(-2)] 1010
3.138.8 Giac [F] 1010
3.138.9 Mupad [F(-1)] 1011

3.138.1 Optimal result

Integrand size = 23, antiderivative size = 246

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{dx} - \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|\,-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} + \frac{b(c^2d+e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

output

```
-(a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/d/x+b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/
2)/d/(c^2*x^2)^(1/2)-b*c^2*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^
(1/2)*(e*x^2+d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2
)+b*(c^2*d+e)*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^
2/d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)
```

3.138.2 Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.58

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d+ex^2}\left(-a+bc\sqrt{1-\frac{1}{c^2x^2}}x-b\sec^{-1}(cx)\right)}{dx} - \frac{bce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}E\left(\arcsin\left(\sqrt{-\frac{e}{d}}x\right)|-\frac{c^2d}{e}\right)}{d\sqrt{-\frac{e}{d}}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

output
```
(Sqrt[d + e*x^2]*(-a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x - b*ArcSec[c*x]))/(d*x)
 - (b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*EllipticE[ArcSin[Sqr
t[-(e/d)]*x], -((c^2*d)/e)])/(d*Sqrt[-(e/d)]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*
x^2])
```

3.138.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {5761, 25, 27, 377, 27, 326, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx$$

$$\downarrow 5761$$

$$-\frac{bcx\int -\frac{\sqrt{ex^2+d}}{dx^2\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{dx}$$

$$\downarrow 25$$

$$\frac{bcx\int \frac{\sqrt{ex^2+d}}{dx^2\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{dx}$$

$$\downarrow 27$$

$$\frac{bcx\int\frac{\sqrt{ex^2+d}}{x^2\sqrt{c^2x^2-1}}dx}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{dx}$$

$$\downarrow 377$$

$$\frac{bcx\left(\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}-\int\frac{e\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}dx\right)}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{dx}$$

$$\downarrow 27$$

$$\frac{bcx\left(\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}-e\int\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}dx\right)}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{dx}$$

$$\downarrow 326$$

$$\frac{bcx\left(\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}-e\left(\frac{c^2\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}-\frac{\left(c^2d+e\right)\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{e}\right)\right)}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{dx}$$

$$\downarrow 323$$

$$\frac{bcx\left(\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}-e\left(\frac{c^2\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}-\frac{\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}\right)\right)}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{dx}$$

$$\downarrow 323$$

$$\frac{bcx\left(\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}-e\left(\frac{c^2\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}-\frac{\sqrt{1-c^2x^2}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}\right)\right)}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{dx}$$

$$\downarrow 321$$

$$\frac{bcx\left(\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}-e\left(\frac{c^2\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}-\frac{\sqrt{1-c^2x^2}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}\right)\right)}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{dx}$$

$$\downarrow 331$$

$$\frac{bcx\left(\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}-e\left(\frac{c^2\sqrt{1-c^2x^2}\int\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{c^2x^2-1}}-\frac{\sqrt{1-c^2x^2}(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}\right)\right)}{\frac{d\sqrt{c^2x^2}\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{dx}}-$$

$$\downarrow 330$$

$$\frac{bcx\left(\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}-e\left(\frac{c^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}-\frac{\sqrt{1-c^2x^2}(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}\right)\right)}{\frac{d\sqrt{c^2x^2}\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{dx}}-$$

$$\downarrow 327$$

$$\frac{bcx\left(\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}-e\left(\frac{c\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\middle|-\frac{e}{c^2d}\right)}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}-\frac{\sqrt{1-c^2x^2}(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}\right)\right)}{\frac{d\sqrt{c^2x^2}\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{dx}}-$$

input
```
Int[(a + b*ArcSec[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

output
```
-((Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/(d*x)) + (b*c*x*((Sqrt[-1 + c^2*x^
2]*Sqrt[d + e*x^2])/x - e*((c*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[
ArcSin[c*x], -(e/(c^2*d))])/(e*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) - (
(c^2*d + e)*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -
(e/(c^2*d))])/(c*e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])))/(d*Sqrt[c^2*x^2]
)
```

3.138.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.138.4 Maple [F]

$$\int \frac{a + b \operatorname{arcsec}(cx)}{x^2\sqrt{e\,x^2 + d}}dx$$

input
```
int((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

3.138.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.43

$$\int \frac{a + b\sec^{-1}(cx)}{x^2\sqrt{d + ex^2}}\,dx = -\frac{\left(bcd\operatorname{arcsec}(cx) - \sqrt{c^2x^2 - 1}bcd + acd\right)\sqrt{ex^2 + d} - \left(bc^4dxE(\arcsin(cx) \mid -\frac{e}{c^2d}\right) - (bc^4d + be)xF(\arcsi}{cd^2x}$$

input
```
integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
-((b*c*d*arcsec(c*x) - sqrt(c^2*x^2 - 1)*b*c*d + a*c*d)*sqrt(e*x^2 + d) -
(b*c^4*d*x*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (b*c^4*d + b*e)*x*ellipti
c_f(arcsin(c*x), -e/(c^2*d)))*sqrt(-d))/(c*d^2*x)
```

3.138.6 Sympy [F]

$$\int \frac{a + b \sec^{-1}(cx)}{x^2\sqrt{d + ex^2}}\,dx = \int \frac{a + b\,\text{asec}\,(cx)}{x^2\sqrt{d + ex^2}}\,dx$$

input
```
integrate((a+b*asec(c*x))/x**2/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asec(c*x))/(x**2*sqrt(d + e*x**2)), x)
```

3.138.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b \sec^{-1}(cx)}{x^2\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.138.8 Giac [F]

$$\int \frac{a + b \sec^{-1}(cx)}{x^2\sqrt{d + ex^2}}\,dx = \int \frac{b\,\text{arcsec}\,(cx) + a}{\sqrt{ex^2 + dx^2}}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/(sqrt(e*x^2 + d)*x^2), x)
```

3.138.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\text{acos}\left(\frac{1}{cx}\right)}{x^2\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)^(1/2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)^(1/2)), x)
```

3.139 $\int \frac{a+b\sec^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$

3.139.1 Optimal result 1012
3.139.2 Mathematica [C] (verified) 1013
3.139.3 Rubi [A] (verified) 1013
3.139.4 Maple [F] 1018
3.139.5 Fricas [A] (verification not implemented) 1018
3.139.6 Sympy [F] 1018
3.139.7 Maxima [F(-2)] 1019
3.139.8 Giac [F] 1019
3.139.9 Mupad [F(-1)] 1019

3.139.1 Optimal result

Integrand size = 23, antiderivative size = 362

$$\begin{aligned}
&\int \frac{a+b\sec^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx \\
&= \frac{bc(2c^2d-5e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}} \\
&\quad - \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{3dx^3} + \frac{2e\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{3d^2x} \\
&\quad - \frac{bc^2(2c^2d-5e)\,x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|\,-\frac{e}{c^2d}\right)}{9d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&\quad + \frac{2b(c^2d-3e)\,(c^2d+e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{9d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}$$

output

```
-1/3*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/d/x^3+2/3*e*(a+b*arcsec(c*x))*(e*x^
2+d)^(1/2)/d^2/x+1/9*b*c*(2*c^2*d-5*e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d
^2/(c^2*x^2)^(1/2)+1/9*b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(c^2*x^
2)^(1/2)-1/9*b*c^2*(2*c^2*d-5*e)*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x
^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2
/d)^(1/2)+2/9*b*(c^2*d-3*e)*(c^2*d+e)*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-
c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(
e*x^2+d)^(1/2)
```

3.139.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.52 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.69

$$\int \frac{a+b\sec^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$$
$$= \frac{\sqrt{d+ex^2}\left(bc\sqrt{1-\frac{1}{c^2x^2}}x(d+2c^2dx^2-5ex^2)-3a(d-2ex^2)-3b(d-2ex^2)\sec^{-1}(cx)\right)}{9d^2x^3}$$
$$-\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(2c^2d-5e)\,E\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)|-\frac{e}{c^2d}\right)+2(-c^4d^2+2c^2de+3e^2)\,\text{Elliptic}\right.}{9\sqrt{-c^2}d^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

output
```
(Sqrt[d + e*x^2]*(b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + 2*c^2*d*x^2 - 5*e*x^2)
 - 3*a*(d - 2*e*x^2) - 3*b*(d - 2*e*x^2)*ArcSec[c*x]))/(9*d^2*x^3) - ((I/9)
*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(2*c^2*d - 5*e)*El
lipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] + 2*(-(c^4*d^2) + 2*c^2*d*e
 + 3*e^2)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d
^2*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

3.139.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {5761, 27, 442, 25, 445, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$$
$$\downarrow 5761$$
$$-\frac{bcx\int -\frac{(d-2ex^2)\sqrt{ex^2+d}}{3d^2x^4\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{2e\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$
$$\downarrow 27$$

$$\frac{bcx\int\frac{(d-2ex^2)\sqrt{ex^2+d}}{x^4\sqrt{c^2x^2-1}}dx}{3d^2\sqrt{c^2x^2}}+\frac{2e\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

$$\downarrow 442$$

$$\frac{bcx\left(\frac{d\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{3x^3}-\frac{1}{3}\int-\frac{(c^2d-6e)ex^2+d(2c^2d-5e)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx\right)}{3d^2\sqrt{c^2x^2}}+\frac{2e\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

$$\downarrow 25$$

$$\frac{bcx\left(\frac{1}{3}\int\frac{(c^2d-6e)ex^2+d(2c^2d-5e)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{d\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{3x^3}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2e\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

$$\downarrow 445$$

$$\frac{bcx\left(\frac{1}{3}\left(\frac{\int\frac{de\left(-\left(\left(2c^2d-5e\right)x^2c^2\right)+dc^2-6e\right)}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{d}+\frac{\sqrt{c^2x^2-1}(2c^2d-5e)\sqrt{d+ex^2}}{x}\right)+\frac{d\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{3x^3}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2e\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

$$\downarrow 27$$

$$\frac{bcx\left(\frac{1}{3}\left(e\int\frac{-\left(\left(2c^2d-5e\right)x^2c^2\right)+dc^2-6e}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{\sqrt{c^2x^2-1}(2c^2d-5e)\sqrt{d+ex^2}}{x}\right)+\frac{d\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{3x^3}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2e\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

$$\downarrow 399$$

$$\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{2\left(c^2d-3e\right)\left(c^2d+e\right)\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{e}-\frac{c^2\left(2c^2d-5e\right)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-1}(2c^2d-5e)\sqrt{d+ex^2}}{x}\right)+\frac{d\sqrt{c^2x^2-1}\sqrt{d+}}{3x^3}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2e\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}$$

$$\downarrow 323$$

$$\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{2\left(c^2d-3e\right)\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}-\frac{c^2\left(2c^2d-5e\right)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-1}\left(2c^2d-5e\right)\sqrt{d+ex^2}}{x}\right)+\frac{d\sqrt{c}}{x}\right)}{\frac{2e\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3d^2x}-\frac{3d^2\sqrt{c^2x^2}\left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}}}$$

$$\downarrow 323$$

$$\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{2\sqrt{1-c^2x^2}\left(c^2d-3e\right)\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\left(2c^2d-5e\right)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-1}\left(2c^2d-5e\right)\sqrt{d+ex^2}}{x}\right)\right)}{\frac{2e\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3d^2x}-\frac{3d^2\sqrt{c^2x^2}\left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}}}$$

$$\downarrow 321$$

$$\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{2\sqrt{1-c^2x^2}\left(c^2d-3e\right)\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\left(2c^2d-5e\right)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-1}\left(2c^2d-5e\right)\sqrt{d+}}{x}\right)}{\frac{2e\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3d^2x}-\frac{3d^2\sqrt{c^2x^2}\left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}}}$$

$$\downarrow 331$$

$$\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{2\sqrt{1-c^2x^2}\left(c^2d-3e\right)\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\sqrt{1-c^2x^2}\left(2c^2d-5e\right)\int\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{c^2x^2-1}}\right)+\frac{\sqrt{c^2x^2-1}\left(2c^2d-\right.}{x}\right)}{\frac{2e\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3d^2x}-\frac{3d^2\sqrt{c^2x^2}\left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}}}$$

$$\downarrow 330$$

$$\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{2\sqrt{1-c^2x^2}\left(c^2d-3e\right)\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\sqrt{1-c^2x^2}\left(2c^2d-5e\right)\sqrt{d+ex^2}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\right)+\frac{\sqrt{c^2x^2}}{}\right.}{\frac{2e\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3d^2x}-\frac{3d^2\sqrt{c^2x^2}\left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}}}$$

$$\downarrow 327$$

$$\frac{2e\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{3dx^3}+$$
$$\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{2\sqrt{1-c^2x^2}\left(c^2d-3e\right)\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c\sqrt{1-c^2x^2}\left(2c^2d-5e\right)\sqrt{d+ex^2}E\left(\arcsin(cx)\middle|-\frac{e}{c^2d}\right)}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\right)+\frac{\sqrt{}}{}\right.\right.}{3d^2\sqrt{c^2x^2}}$$

input
```
Int[(a + b*ArcSec[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

output
```
-1/3*(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/(d*x^3) + (2*e*Sqrt[d + e*x^2]*
(a + b*ArcSec[c*x]))/(3*d^2*x) + (b*c*x*((d*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*
x^2])/(3*x^3) + (((2*c^2*d - 5*e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/x + 
e*(-((c*(2*c^2*d - 5*e)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin
[c*x], -(e/(c^2*d))])/(e*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d])) + (2*(c^
2*d - 3*e)*(c^2*d + e)*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[Arc
Sin[c*x], -(e/(c^2*d))])/(c*e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])))/3))/(3
*d^2*Sqrt[c^2*x^2])
```

3.139.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.139.4 Maple [F]

$$\int \frac{a + b \operatorname{arcsec}(cx)}{x^4 \sqrt{e\,x^2 + d}}\,dx$$

input
```
int((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

3.139.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.54

$$\int \frac{a + b \sec^{-1}(cx)}{x^4 \sqrt{d + ex^2}}\,dx$$
$$= \frac{\left(6\,acdex^2 - 3\,acd^2 + 3\left(2\,bcdex^2 - bcd^2\right)\operatorname{arcsec}(cx) + \left(bcd^2 + \left(2\,bc^3d^2 - 5\,bcde\right)x^2\right)\sqrt{c^2x^2 - 1}\right)\sqrt{ex^2 + d}}{9}$$

input
```
integrate((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
1/9*((6*a*c*d*e*x^2 - 3*a*c*d^2 + 3*(2*b*c*d*e*x^2 - b*c*d^2)*arcsec(c*x)
+ (b*c*d^2 + (2*b*c^3*d^2 - 5*b*c*d*e)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2
+ d) + ((2*b*c^6*d^2 - 5*b*c^4*d*e)*x^3*elliptic_e(arcsin(c*x), -e/(c^2*d)
) - (2*b*c^6*d^2 - (5*b*c^4 - b*c^2)*d*e - 6*b*e^2)*x^3*elliptic_f(arcsin(
c*x), -e/(c^2*d)))*sqrt(-d))/(c*d^3*x^3)
```

3.139.6 Sympy [F]

$$\int \frac{a + b \sec^{-1}(cx)}{x^4 \sqrt{d + ex^2}}\,dx = \int \frac{a + b \operatorname{asec}(cx)}{x^4 \sqrt{d + ex^2}}\,dx$$

input
```
integrate((a+b*asec(c*x))/x**4/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asec(c*x))/(x**4*sqrt(d + e*x**2)), x)
```

3.139.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.139.8 Giac [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx = \int \frac{b\,\text{arcsec}\,(cx)+a}{\sqrt{ex^2+dx^4}}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/(sqrt(e*x^2 + d)*x^4), x)
```

3.139.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\text{acos}\left(\frac{1}{c\,x}\right)}{x^4\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(x^4*(d + e*x^2)^(1/2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x^4*(d + e*x^2)^(1/2)), x)
```

3.140 $\int \frac{a+b\sec^{-1}(cx)}{x^6\sqrt{d+ex^2}}\,dx$

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3.140.1 Optimal result

Integrand size = 23, antiderivative size = 1006

$$
\begin{aligned}
&\int \frac{a+b\sec^{-1}(cx)}{x^6\sqrt{d+ex^2}}\,dx \\
&= \frac{8bce^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{15d^3\sqrt{c^2x^2}} - \frac{4bce(2c^2d+e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{45d^3\sqrt{c^2x^2}} \\
&\quad + \frac{bc(8c^4d^2+3c^2de-2e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d^3\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{25dx^4\sqrt{c^2x^2}} \\
&\quad - \frac{4bce\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{45d^2x^2\sqrt{c^2x^2}} + \frac{bc(4c^2d+e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d^2x^2\sqrt{c^2x^2}} \\
&\quad - \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{5dx^5} + \frac{4e\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{15d^2x^3} \\
&\quad - \frac{8e^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{15d^3x} - \frac{8bc^2e^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{15d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&\quad + \frac{4bc^2e(2c^2d+e)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{45d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&\quad - \frac{bc^2(8c^4d^2+3c^2de-2e^2)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{75d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&\quad + \frac{bc^2(8c^2d-e)(c^2d+e)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{75d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} \\
&\quad - \frac{8bc^2e(c^2d+e)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{45d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} \\
&\quad + \frac{8be^2(c^2d+e)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{15d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}
$$

output

```
-1/5*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/d/x^5+4/15*e*(a+b*arcsec(c*x))*(e*x
^2+d)^(1/2)/d^2/x^3-8/15*e^2*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/d^3/x+8/15*
b*c*e^2*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^3/(c^2*x^2)^(1/2)-4/45*b*c*e*(
2*c^2*d+e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^3/(c^2*x^2)^(1/2)+1/75*b*c*
(8*c^4*d^2+3*c^2*d*e-2*e^2)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^3/(c^2*x^2
)^(1/2)+1/25*b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^4/(c^2*x^2)^(1/2)-4
/45*b*c*e*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/x^2/(c^2*x^2)^(1/2)+1/75*b
*c*(4*c^2*d+e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/x^2/(c^2*x^2)^(1/2)-8
/15*b*c^2*e^2*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+
d)^(1/2)/d^3/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)+4/45*b*c^
2*e*(2*c^2*d+e)*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^
2+d)^(1/2)/d^3/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)-1/75*b*
c^2*(8*c^4*d^2+3*c^2*d*e-2*e^2)*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^
2+1)^(1/2)*(e*x^2+d)^(1/2)/d^3/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/
d)^(1/2)+1/75*b*c^2*(8*c^2*d-e)*(c^2*d+e)*x*EllipticF(c*x,(-e/c^2/d)^(1/2)
)*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/
2)/(e*x^2+d)^(1/2)-8/45*b*c^2*e*(c^2*d+e)*x*EllipticF(c*x,(-e/c^2/d)^(1/2)
)*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/
2)/(e*x^2+d)^(1/2)+8/15*b*e^2*(c^2*d+e)*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*
(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^3/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1...
```

3.140.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.64 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.33

$$\int \frac{a+b\sec^{-1}(cx)}{x^6\sqrt{d+ex^2}}\,dx$$

$$= \frac{\sqrt{d+ex^2}\left(-15a(3d^2-4dex^2+8e^2x^4)+bc\sqrt{1-\frac{1}{c^2x^2}}x(94e^2x^4-dex^2(17+31c^2x^2)+3d^2(3+4c^2x^2+8\right.}{225d^3x^5}$$

$$-\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(24c^4d^2-31c^2de+94e^2)\,E\left(i\operatorname{arcsinh}\left(\sqrt{-c^2}x\right)|-\frac{e}{c^2d}\right)-(24c^6d^3-19c^4d^2e\right.}{225\sqrt{-c^2}d^3\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

input

```
Integrate[(a + b*ArcSec[c*x])/(x^6*Sqrt[d + e*x^2]),x]
```

output

```
(Sqrt[d + e*x^2]*(-15*a*(3*d^2 - 4*d*e*x^2 + 8*e^2*x^4) + b*c*Sqrt[1 - 1/(
c^2*x^2)]*x*(94*e^2*x^4 - d*e*x^2*(17 + 31*c^2*x^2) + 3*d^2*(3 + 4*c^2*x^2
 + 8*c^4*x^4)) - 15*b*(3*d^2 - 4*d*e*x^2 + 8*e^2*x^4)*ArcSec[c*x]))/(225*d
^3*x^5) - ((I/225)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*
(24*c^4*d^2 - 31*c^2*d*e + 94*e^2)*EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/
(c^2*d))] - (24*c^6*d^3 - 19*c^4*d^2*e + 77*c^2*d*e^2 + 120*e^3)*EllipticF
[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d^3*Sqrt[1 - c^2*x^2
]*Sqrt[d + e*x^2])
```

3.140.3 Rubi [A] (verified)

Time = 1.99 (sec) , antiderivative size = 839, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5761, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x^6\sqrt{d+ex^2}}\,dx$$

$$\downarrow 5761$$

$$-\frac{bcx\int -\frac{\sqrt{ex^2+d}\left(8e^2x^4-4dex^2+3d^2\right)}{15d^3x^6\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}-\frac{8e^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{15d^3x}+\frac{4e\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{15d^2x^3}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\downarrow 27$$

$$\frac{bcx\int \frac{\sqrt{ex^2+d}\left(8e^2x^4-4dex^2+3d^2\right)}{x^6\sqrt{c^2x^2-1}}dx}{15d^3\sqrt{c^2x^2}}-\frac{8e^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{15d^3x}+\frac{4e\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{15d^2x^3}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\downarrow 7293$$

$$\frac{bcx\int \left(\frac{3\sqrt{ex^2+d}d^2}{x^6\sqrt{c^2x^2-1}}-\frac{4e\sqrt{ex^2+d}d}{x^4\sqrt{c^2x^2-1}}+\frac{8e^2\sqrt{ex^2+d}}{x^2\sqrt{c^2x^2-1}}\right)dx}{15d^3\sqrt{c^2x^2}}-\frac{8e^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{15d^3x}+\frac{4e\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{15d^2x^3}-\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}$$

$$\downarrow 2009$$

$$-\frac{8\sqrt{ex^2+d}\left(a+b\sec^{-1}(cx)\right)e^2}{15d^3x}+\frac{4\sqrt{ex^2+d}\left(a+b\sec^{-1}(cx)\right)e}{15d^2x^3}-\frac{\sqrt{ex^2+d}\left(a+b\sec^{-1}(cx)\right)}{5dx^5}+$$
$$bcx\left(\frac{3\sqrt{c^2x^2-1}\sqrt{ex^2+d}d^2}{5x^5}+\frac{c\left(8c^2d-e\right)\left(dc^2+e\right)\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)d}{5\sqrt{c^2x^2-1}\sqrt{ex^2+d}}-\frac{8ce\left(dc^2+e\right)\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}(\right.$$
$$\left.3\sqrt{c^2x^2-1}\sqrt{ex^2+d}\right.$$

input
```
Int[(a + b*ArcSec[c*x])/(x^6*Sqrt[d + e*x^2]),x]
```

output
```
-1/5*(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/(d*x^5) + (4*e*Sqrt[d + e*x^2]*
(a + b*ArcSec[c*x]))/(15*d^2*x^3) - (8*e^2*Sqrt[d + e*x^2]*(a + b*ArcSec[c
*x]))/(15*d^3*x) + (b*c*x*((3*d^2*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(5*x
^5) - (4*d*e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(3*x^3) + (d*(4*c^2*d + e
)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(5*x^3) + (8*e^2*Sqrt[-1 + c^2*x^2]*
Sqrt[d + e*x^2])/x - (4*e*(2*c^2*d + e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2]
)/(3*x) + ((8*c^4*d^2 + 3*c^2*d*e - 2*e^2)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x
^2])/(5*x) - (8*c*e^2*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c
*x], -(e/(c^2*d))])/(Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) + (4*c*e*(2*c
^2*d + e)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^
2*d))])/(3*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) - (c*(8*c^4*d^2 + 3*c^2
*d*e - 2*e^2)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e
/(c^2*d))])/(5*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) + (c*d*(8*c^2*d - e
)*(c^2*d + e)*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x],
 -(e/(c^2*d))])/(5*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2]) - (8*c*d*e*(c^2*d +
 e)*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*
d))])/(3*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2]) + (8*e^2*(c^2*d + e)*Sqrt[1 -
 c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(c*Sqr
t[-1 + c^2*x^2]*Sqrt[d + e*x^2]))/(15*d^3*Sqrt[c^2*x^2])
```

3.140.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])  u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])  Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.140.4 Maple [F]

$$\int \frac{a + b \operatorname{arcsec}(cx)}{x^6 \sqrt{e\,x^2 + d}}\,dx$$

input
```
int((a+b*arcsec(c*x))/x^6/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arcsec(c*x))/x^6/(e*x^2+d)^(1/2),x)
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.29

$$\int \frac{a + b \sec^{-1}(cx)}{x^6 \sqrt{d + ex^2}}\,dx =$$
$$-\frac{\left(120\,acde^2x^4 - 60\,acd^2ex^2 + 45\,acd^3 + 15\left(8\,bcde^2x^4 - 4\,bcd^2ex^2 + 3\,bcd^3\right)\operatorname{arcsec}(cx) - \left(9\,bcd^3 + (24\,b\right.}{}$$

input
```
integrate((a+b*arcsec(c*x))/x^6/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
-1/225*((120*a*c*d*e^2*x^4 - 60*a*c*d^2*e*x^2 + 45*a*c*d^3 + 15*(8*b*c*d*e
^2*x^4 - 4*b*c*d^2*e*x^2 + 3*b*c*d^3)*arcsec(c*x) - (9*b*c*d^3 + (24*b*c^5
*d^3 - 31*b*c^3*d^2*e + 94*b*c*d*e^2)*x^4 + (12*b*c^3*d^3 - 17*b*c*d^2*e)*
x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d) - ((24*b*c^8*d^3 - 31*b*c^6*d^2*e
+ 94*b*c^4*d*e^2)*x^5*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (24*b*c^8*d^3
- (31*b*c^6 - 12*b*c^4)*d^2*e + (94*b*c^4 - 17*b*c^2)*d*e^2 + 120*b*e^3)*x
^5*elliptic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(-d))/(c*d^4*x^5)
```

3.140.6 Sympy [F]

$$\int \frac{a + b \sec^{-1}(cx)}{x^6 \sqrt{d + ex^2}}\, dx = \int \frac{a + b \operatorname{asec}(cx)}{x^6 \sqrt{d + ex^2}}\, dx$$

input
```
integrate((a+b*asec(c*x))/x**6/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*asec(c*x))/(x**6*sqrt(d + e*x**2)), x)
```

3.140.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b \sec^{-1}(cx)}{x^6 \sqrt{d + ex^2}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/x^6/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.140.8 Giac [F]

$$\int \frac{a + b \sec^{-1}(cx)}{x^6 \sqrt{d + ex^2}}\, dx = \int \frac{b \operatorname{arcsec}(cx) + a}{\sqrt{ex^2 + d} x^6}\, dx$$

input
```
integrate((a+b*arcsec(c*x))/x^6/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/(sqrt(e*x^2 + d)*x^6), x)
```

3.140.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^6\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\text{acos}\left(\frac{1}{cx}\right)}{x^6\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(x^6*(d + e*x^2)^(1/2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x^6*(d + e*x^2)^(1/2)), x)
```

3.141 $\int \frac{x^5\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

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3.141.2 Mathematica [C] (verified) 1029
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3.141.8 Giac [F] 1035
3.141.9 Mupad [F(-1)] 1035

3.141.1 Optimal result

Integrand size = 23, antiderivative size = 252

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = -\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} + \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^3} - \frac{8bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e^3\sqrt{c^2x^2}} + \frac{b(9c^2d-e)\,x\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2e^{5/2}\sqrt{c^2x^2}}$$

output

```
1/3*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/e^3-8/3*b*c*d^(3/2)*x*arctan((e*x^2+
d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e^3/(c^2*x^2)^(1/2)+1/6*b*(9*c^2*d-e)*
x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^2/e^(5/2)/(c^2*x^
2)^(1/2)-d^2*(a+b*arcsec(c*x))/e^3/(e*x^2+d)^(1/2)-2*d*(a+b*arcsec(c*x))*(
e*x^2+d)^(1/2)/e^3-1/6*b*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c/e^2/(c^2*x^
2)^(1/2)
```

3.141.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.48 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.05

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{-16bd^2\sqrt{1+\frac{d}{ex^2}}(-1+c^2x^2)\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)+b(9c^2d-e)\,e\sqrt{}$$

input
```
Integrate[(x^5*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
(-16*b*d^2*Sqrt[1 + d/(e*x^2)]*(-1 + c^2*x^2)*AppellF1[1, 1/2, 1/2, 2, 1/(
c^2*x^2), -(d/(e*x^2))] + b*(9*c^2*d - e)*e*Sqrt[1 - 1/(c^2*x^2)]*x^4*Sqrt
[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x
^2)/d)] - 2*x*(-1 + c^2*x^2)*(b*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2) + 2*
a*c*(8*d^2 + 4*d*e*x^2 - e^2*x^4) + 2*b*c*(8*d^2 + 4*d*e*x^2 - e^2*x^4)*Ar
cSec[c*x]))/(12*c*e^3*x*(-1 + c^2*x^2)*Sqrt[d + e*x^2])
```

3.141.3 Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5761, 27, 7282, 2118, 27, 175, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

$$\downarrow 5761$$

$$-\frac{bcx\int -\frac{-e^2x^4+4dex^2+8d^2}{3e^3x\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{\sqrt{c^2x^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} + \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^3}$$

$$\downarrow 27$$

$$\frac{bcx\int\frac{-e^2x^4+4dex^2+8d^2}{x\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{3e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$$\downarrow 7282$$

$$\frac{bcx\int\frac{-e^2x^4+4dex^2+8d^2}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{6e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$$\downarrow 2118$$

$$\frac{bcx\left(\frac{\int\frac{e\left(16c^2d^2+\left(9c^2d-e\right)ex^2\right)}{2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{c^2e}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$$\downarrow 27$$

$$\frac{bcx\left(\frac{\int\frac{16c^2d^2+\left(9c^2d-e\right)ex^2}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$$\downarrow 175$$

$$\frac{bcx\left(\frac{16c^2d^2\int\frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+e\left(9c^2d-e\right)\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$$\downarrow 66$$

$$\frac{bcx\left(\frac{16c^2d^2\int\frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\left(9c^2d-e\right)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$$\downarrow 104$$

$$\frac{bcx\left(\frac{32c^2d^2\int\frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+2e\left(9c^2d-e\right)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^3\sqrt{c^2x^2}}-$$
$$\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$$\downarrow 217$$

$$\frac{bcx\left(\frac{2e\left(9c^2d-e\right)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-32c^2d^{3/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^3\sqrt{c^2x^2}}-$$
$$\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}$$

$$\downarrow 221$$

$$-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}+$$
$$\frac{bcx\left(\frac{\frac{2\sqrt{e}\left(9c^2d-e\right)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-32c^2d^{3/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^3\sqrt{c^2x^2}}$$

input
```
Int[(x^5*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
-((d^2*(a + b*ArcSec[c*x]))/(e^3*Sqrt[d + e*x^2])) - (2*d*Sqrt[d + e*x^2]*
(a + b*ArcSec[c*x]))/e^3 + ((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/(3*e^3)
 + (b*c*x*(-((e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/c^2) + (-32*c^2*d^(3/2
)*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])] + (2*(9*c^2*d - e)*
Sqrt[e]*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/c)/(2*c
^2)))/(6*e^3*Sqrt[c^2*x^2])
```

3.141.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])  u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])  Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]  Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

3.141.4 Maple [F]

$$\int \frac{x^5(a+b\operatorname{arcsec}(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

3.141.5 Fricas [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 1483, normalized size of antiderivative = 5.88

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[-1/24*((9*b*c^2*d^2 - b*d*e + (9*b*c^2*d*e - b*e^2)*x^2)*sqrt(e)*log(8*c^
4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x
^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 16*(b
*c^3*d*e*x^2 + b*c^3*d^2)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 -
8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e
*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) - 4*(2*a*c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 -
16*a*c^3*d^2 + 2*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*arcsec(c*
x) - (b*c*e^2*x^2 + b*c*d*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^3*e^4*
x^2 + c^3*d*e^3), -1/24*(32*(b*c^3*d*e*x^2 + b*c^3*d^2)*sqrt(d)*arctan(-1/
2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d
*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (9*b*c^2*d^2 - b*d*e + (9*b*c^2*d*e
 - b*e^2)*x^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*
e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*
x^2 + d)*sqrt(e) + e^2) - 4*(2*a*c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 - 16*a*c^3*
d^2 + 2*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*arcsec(c*x) - (b*c
*e^2*x^2 + b*c*d*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^3
*d*e^3), -1/12*((9*b*c^2*d^2 - b*d*e + (9*b*c^2*d*e - b*e^2)*x^2)*sqrt(-e)
*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sq
rt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) - 8*(b*c^3*d*e*x^2 +
 b*c^3*d^2)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 ...
```

3.141.6 Sympy [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^5(a+b\,\text{asec}\,(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5*(a+b*asec(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral(x**5*(a + b*asec(c*x))/(d + e*x**2)**(3/2), x)
```

3.141.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.141.8 Giac [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x^5}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x^5/(e*x^2 + d)^(3/2), x)
```

3.141.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^5\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

output
```
int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.142 $\int \frac{x^3\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

3.142.1 Optimal result 1036
3.142.2 Mathematica [C] (verified) 1036
3.142.3 Rubi [A] (verified) 1037
3.142.4 Maple [F] 1040
3.142.5 Fricas [A] (verification not implemented) 1040
3.142.6 Sympy [F] 1041
3.142.7 Maxima [F(-2)] 1041
3.142.8 Giac [F] 1041
3.142.9 Mupad [F(-1)] 1042

3.142.1 Optimal result

Integrand size = 23, antiderivative size = 157

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{d(a+b\sec^{-1}(cx))}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2} + \frac{2bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{e^2\sqrt{c^2x^2}} - \frac{bx\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{e^{3/2}\sqrt{c^2x^2}}$$

output
```
-b*x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/e^(3/2)/(c^2*x^2
)^(1/2)+2*b*c*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))*d^(1/2)/
e^2/(c^2*x^2)^(1/2)+d*(a+b*arcsec(c*x))/e^2/(e*x^2+d)^(1/2)+(a+b*arcsec(c*
x))*(e*x^2+d)^(1/2)/e^2
```

3.142.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.03 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.03

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{\frac{2bd\sqrt{1+\frac{d}{ex^2}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)}{cx} + \frac{bce\sqrt{1-\frac{1}{c^2x^2}}x^3\sqrt{1+\frac{ex^2}{d}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,c^2x^2,-\frac{ex^2}{d}\right)}{\sqrt{1-c^2x^2}}}{2e^2\sqrt{d+ex^2}}$$

input
```
Integrate[(x^3*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
((2*b*d*Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x
^2))])/(c*x) + (b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x^3*Sqrt[1 + (e*x^2)/d]*Appell
F1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/Sqrt[1 - c^2*x^2] + 2*(2*d + e*
x^2)*(a + b*ArcSec[c*x]))/(2*e^2*Sqrt[d + e*x^2])
```

3.142.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5761, 27, 435, 175, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

$$\downarrow 5761$$

$$-\frac{bcx\int \frac{ex^2+2d}{e^2x\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{\sqrt{c^2x^2}}+\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\sec^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}$$

$$\downarrow 27$$

$$-\frac{bcx\int \frac{ex^2+2d}{x\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{e^2\sqrt{c^2x^2}}+\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\sec^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}$$

$$\downarrow 435$$

$$-\frac{bcx\int \frac{ex^2+2d}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2e^2\sqrt{c^2x^2}}+\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\sec^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}$$

$$\downarrow 175$$

$$-\frac{bcx\left(e\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2d\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2\right)}{2e^2\sqrt{c^2x^2}}+\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\sec^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}$$

$$\downarrow 66$$

$$-\frac{bcx\left(2d\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}\right)}{2e^2\sqrt{c^2x^2}}+\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\sec^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}$$

$$\downarrow 104$$

$$-\frac{bcx\left(4d\int\frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+2e\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}\right)}{2e^2\sqrt{c^2x^2}}+\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\sec^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}$$

$$\downarrow 217$$

$$-\frac{bcx\left(2e\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-4\sqrt{d}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)\right)}{2e^2\sqrt{c^2x^2}}+\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\sec^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}$$

$$\downarrow 221$$

$$\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\sec^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}-\frac{bcx\left(\frac{2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-4\sqrt{d}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)\right)}{2e^2\sqrt{c^2x^2}}$$

input
```
Int[(x^3*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
(d*(a + b*ArcSec[c*x]))/(e^2*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*Ar
cSec[c*x]))/e^2 - (b*c*x*(-4*Sqrt[d]*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[
-1 + c^2*x^2])] + (2*Sqrt[e]*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[
d + e*x^2])])/c))/(2*e^2*Sqrt[c^2*x^2])
```

3.142.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.142.4 Maple [F]

$$\int \frac{x^3(a+b\,\text{arcsec}\,(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

3.142.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 1070, normalized size of antiderivative = 6.82

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((b*e*x^2 + b*d)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*
(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*
sqrt(e*x^2 + d)*sqrt(e) + e^2) + 2*(b*c*e*x^2 + b*c*d)*sqrt(-d)*log(((c^4*
d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*(
(c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + 4*(a*c*e*x
^2 + 2*a*c*d + (b*c*e*x^2 + 2*b*c*d)*arcsec(c*x))*sqrt(e*x^2 + d))/(c*e^3*
x^2 + c*d*e^2), 1/4*(4*(b*c*e*x^2 + b*c*d)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^
2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2
*d^2 - d*e)*x^2 - d^2)) + (b*e*x^2 + b*d)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*
d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e
)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(a*c*e*x^2 + 2*a*c*
d + (b*c*e*x^2 + 2*b*c*d)*arcsec(c*x))*sqrt(e*x^2 + d))/(c*e^3*x^2 + c*d*e
^2), 1/2*((b*e*x^2 + b*d)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sq
rt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e -
 c*e^2)*x^2)) + (b*c*e*x^2 + b*c*d)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e
^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2
*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + 2*(a*c*e*x^2 + 2*a*c*d + (b*c
*e*x^2 + 2*b*c*d)*arcsec(c*x))*sqrt(e*x^2 + d))/(c*e^3*x^2 + c*d*e^2), 1/2
*(2*(b*c*e*x^2 + b*c*d)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)
*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 ...
```

3.142.6 Sympy [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^3(a+b\operatorname{asec}(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*(a+b*asec(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral(x**3*(a + b*asec(c*x))/(d + e*x**2)**(3/2), x)
```

3.142.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.142.8 Giac [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x^3}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x^3/(e*x^2 + d)^(3/2), x)
```

3.142.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^3\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

output
```
int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.143 $\int \frac{x\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

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3.143.1 Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = -\frac{a+b\sec^{-1}(cx)}{e\sqrt{d+ex^2}} - \frac{bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{\sqrt{d}e\sqrt{c^2x^2}}$$

output
```
-b*c*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e/d^(1/2)/(c^2*x^
2)^(1/2)+(-a-b*arcsec(c*x))/e/(e*x^2+d)^(1/2)
```

3.143.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{-b\sqrt{1+\frac{d}{ex^2}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right) - 2cx(a+b\sec^{-1}(cx))}{2cex\sqrt{d+ex^2}}$$

input
```
Integrate[(x*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
(-(b*Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2)
)]) - 2*c*x*(a + b*ArcSec[c*x]))/(2*c*e*x*Sqrt[d + e*x^2])
```

3.143.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5759, 354, 104, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

$$\downarrow 5759$$

$$\frac{bcx\int \frac{1}{x\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{e\sqrt{c^2x^2}} - \frac{a+b\sec^{-1}(cx)}{e\sqrt{d+ex^2}}$$

$$\downarrow 354$$

$$\frac{bcx\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2e\sqrt{c^2x^2}} - \frac{a+b\sec^{-1}(cx)}{e\sqrt{d+ex^2}}$$

$$\downarrow 104$$

$$\frac{bcx\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}}{e\sqrt{c^2x^2}} - \frac{a+b\sec^{-1}(cx)}{e\sqrt{d+ex^2}}$$

$$\downarrow 217$$

$$-\frac{a+b\sec^{-1}(cx)}{e\sqrt{d+ex^2}} - \frac{bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{\sqrt{d}e\sqrt{c^2x^2}}$$

input
```
Int[(x*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
-((a + b*ArcSec[c*x])/(e*Sqrt[d + e*x^2])) - (b*c*x*ArcTan[Sqrt[d + e*x^2]
/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(Sqrt[d]*e*Sqrt[c^2*x^2])
```

3.143.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5759
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSec[c*x])/(2*e*(p + 1))), x
] - Simp[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2]))   Int[(d + e*x^2)^(p + 1)/(x*S
qrt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

3.143.4 Maple [F]

$$\int \frac{x(a+b\operatorname{arcsec}(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int(x*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

3.143.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.54

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \left[-\frac{(bex^2+bd)\sqrt{-d}\log\left(\frac{(c^4d^2-6c^2de+e^2)x^4-8(c^2d^2-de)x^2-4\sqrt{c^2x^2-1}((c^2d-e)x^2-2d)\sqrt{ex^2+}}{x^4}\right.}{4(de^2x^2+d^2e)}\right.$$
$$\left.-\frac{(bex^2+bd)\sqrt{d}\arctan\left(-\frac{\sqrt{c^2x^2-1}((c^2d-e)x^2-2d)\sqrt{ex^2+d}\sqrt{d}}{2(c^2dex^4+(c^2d^2-de)x^2-d^2)}\right)+2\sqrt{ex^2+d}(bd\,\mathrm{arcsec}(cx)+ad)}{2(de^2x^2+d^2e)}\right]$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((b*e*x^2 + b*d)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(
c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^
2 + d)*sqrt(-d) + 8*d^2)/x^4) + 4*sqrt(e*x^2 + d)*(b*d*arcsec(c*x) + a*d))
/(d*e^2*x^2 + d^2*e), -1/2*((b*e*x^2 + b*d)*sqrt(d)*arctan(-1/2*sqrt(c^2*x
^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^
2*d^2 - d*e)*x^2 - d^2)) + 2*sqrt(e*x^2 + d)*(b*d*arcsec(c*x) + a*d))/(d*e
^2*x^2 + d^2*e)]
```

3.143.6 Sympy [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x(a+b\,\mathrm{asec}(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*asec(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral(x*(a + b*asec(c*x))/(d + e*x**2)**(3/2), x)
```

3.143.7 Maxima [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
-(sqrt(e*x^2 + d)*e*integrate((c^2*e*x^3*log(c) - e*x*log(c) + ((c^2*log(c
) - c^2)*e*x^3 - (c^2*d + e*log(c))*x)*e^(log(c*x + 1) + log(c*x - 1)) + (
c^2*e*x^3 - e*x + (c^2*e*x^3 - e*x)*e^(log(c*x + 1) + log(c*x - 1)))*log(x
))/((c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e + (c^2*e^2*x^4 + (c^2*d*e - e
^2)*x^2 - d*e)*e^(log(c*x + 1) + log(c*x - 1)))*sqrt(e*x^2 + d)), x) + arc
tan(sqrt(c*x + 1)*sqrt(c*x - 1)))*b/(sqrt(e*x^2 + d)*e) - a/(sqrt(e*x^2 +
d)*e)
```

3.143.8 Giac [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x/(e*x^2 + d)^(3/2), x)
```

3.143.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

output
```
int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.144 $\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$

3.144.1 Optimal result 1048
3.144.2 Mathematica [N/A] 1048
3.144.3 Rubi [N/A] 1049
3.144.4 Maple [N/A] (verified) 1049
3.144.5 Fricas [N/A] 1050
3.144.6 Sympy [N/A] 1050
3.144.7 Maxima [F(-2)] 1050
3.144.8 Giac [N/A] 1051
3.144.9 Mupad [N/A] 1051

3.144.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx = \text{Int}\left(\frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}},x\right)$$

output
```
Unintegrable((a+b*arcsec(c*x))/x/(e*x^2+d)^(3/2),x)
```

3.144.2 Mathematica [N/A]

Not integrable

Time = 8.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x*(d + e*x^2)^(3/2)),x]
```

output
```
Integrate[(a + b*ArcSec[c*x])/(x*(d + e*x^2)^(3/2)), x]
```

3.144.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \sec^{-1}(cx)}{x \left(d + ex^2\right)^{3/2}} \, dx$$
$$\downarrow 5771$$
$$\int \frac{a + b \sec^{-1}(cx)}{x \left(d + ex^2\right)^{3/2}} dx$$

input
```
Int[(a + b*ArcSec[c*x])/(x*(d + e*x^2)^(3/2)),x]
```

output
```
$Aborted
```

3.144.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.144.4 Maple [N/A] (verified)

Not integrable

Time = 1.75 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a + b \operatorname{arcsec}(cx)}{x \left(e x^2 + d\right)^{\frac{3}{2}}} dx$$

input
```
int((a+b*arcsec(c*x))/x/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arcsec(c*x))/x/(e*x^2+d)^(3/2),x)
```

3.144.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{b\operatorname{arcsec}(cx)+a}{(ex^2+d)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x)
, x)
```

3.144.6 Sympy [N/A]

Not integrable

Time = 82.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{a+b\operatorname{asec}(cx)}{x\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asec(c*x))/x/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*asec(c*x))/(x*(d + e*x**2)**(3/2)), x)
```

3.144.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.144.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{b\operatorname{arcsec}(cx)+a}{(ex^2+d)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/((e*x^2 + d)^(3/2)*x), x)
```

3.144.9 Mupad [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{a+b\operatorname{acos}\left(\frac{1}{cx}\right)}{x\left(e\,x^2+d\right)^{3/2}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^(3/2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^(3/2)), x)
```

3.145 $\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$

3.145.1 Optimal result 1052
3.145.2 Mathematica [N/A] 1052
3.145.3 Rubi [N/A] 1053
3.145.4 Maple [N/A] (verified) 1053
3.145.5 Fricas [N/A] 1054
3.145.6 Sympy [F(-1)] 1054
3.145.7 Maxima [F(-2)] 1054
3.145.8 Giac [N/A] 1055
3.145.9 Mupad [N/A] 1055

3.145.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx = \text{Int}\left(\frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}},x\right)$$

output
```
Unintegrable((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

3.145.2 Mathematica [N/A]

Not integrable

Time = 10.62 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
```

output
```
Integrate[(a + b*ArcSec[c*x])/(x^3*(d + e*x^2)^(3/2)), x]
```

3.145.3 Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \sec^{-1}(cx)}{x^3 (d + ex^2)^{3/2}} \, dx$$
$$\downarrow 5771$$
$$\int \frac{a + b \sec^{-1}(cx)}{x^3 (d + ex^2)^{3/2}} dx$$

input
```
Int[(a + b*ArcSec[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
```

output
```
$Aborted
```

3.145.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.145.4 Maple [N/A] (verified)

Not integrable

Time = 2.42 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a + b \operatorname{arcsec}(cx)}{x^3 (e x^2 + d)^{\frac{3}{2}}} dx$$

input
```
int((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

3.145.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{b\operatorname{arcsec}(cx)+a}{(ex^2+d)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/(e^2*x^7 + 2*d*e*x^5 + d^2*x^
3), x)
```

3.145.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*asec(c*x))/x**3/(e*x**2+d)**(3/2),x)
```

output
```
Timed out
```

3.145.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.145.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{b\operatorname{arcsec}(cx)+a}{(ex^2+d)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/((e*x^2 + d)^(3/2)*x^3), x)
```

3.145.9 Mupad [N/A]

Not integrable

Time = 1.57 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{a+b\operatorname{acos}\left(\frac{1}{cx}\right)}{x^3\left(e\,x^2+d\right)^{3/2}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(x^3*(d + e*x^2)^(3/2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x^3*(d + e*x^2)^(3/2)), x)
```

3.146 $\int \frac{x^4\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

3.146.1 Optimal result 1056
3.146.2 Mathematica [N/A] 1056
3.146.3 Rubi [N/A] 1057
3.146.4 Maple [N/A] (verified) 1057
3.146.5 Fricas [N/A] 1058
3.146.6 Sympy [N/A] 1058
3.146.7 Maxima [F(-2)] 1058
3.146.8 Giac [N/A] 1059
3.146.9 Mupad [N/A] 1059

3.146.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Int}\left(\frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}},x\right)$$

output
```
Unintegrable(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

3.146.2 Mathematica [N/A]

Not integrable

Time = 14.41 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
Integrate[(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2), x]
```

3.146.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$
$$\downarrow 5771$$
$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}dx$$

input
```
Int[(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

3.146.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrable[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.146.4 Maple [N/A] (verified)

Not integrable

Time = 1.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^4(a+b\ \text{arcsec}\,(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

3.146.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x^4}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*x^4*arcsec(c*x) + a*x^4)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2
+ d^2), x)
```

3.146.6 Sympy [N/A]

Not integrable

Time = 144.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^4(a+b\operatorname{asec}(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4*(a+b*asec(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral(x**4*(a + b*asec(c*x))/(d + e*x**2)**(3/2), x)
```

3.146.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.146.8 Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x^4}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x^4/(e*x^2 + d)^(3/2), x)
```

3.146.9 Mupad [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^4\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((x^4*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

output
```
int((x^4*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.147 $\int \frac{x^2\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

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3.147.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Int}\left(\frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}},x\right)$$

output
```
Unintegrable(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

3.147.2 Mathematica [N/A]

Not integrable

Time = 5.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
Integrate[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2), x]
```

3.147.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$
$$\downarrow 5771$$
$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}dx$$

input
```
Int[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

3.147.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrable[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.147.4 Maple [N/A] (verified)

Not integrable

Time = 1.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^2(a+b\ \text{arcsec}\,(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

3.147.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x^2}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*x^2*arcsec(c*x) + a*x^2)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2
+ d^2), x)
```

3.147.6 Sympy [N/A]

Not integrable

Time = 35.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^2(a+b\operatorname{asec}(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(a+b*asec(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral(x**2*(a + b*asec(c*x))/(d + e*x**2)**(3/2), x)
```

3.147.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.147.8 Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x^2}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x^2/(e*x^2 + d)^(3/2), x)
```

3.147.9 Mupad [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^2\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

output
```
int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.148 $\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx$

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3.148.3 Rubi [A] (verified) 1065
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3.148.6 Sympy [F] 1067
3.148.7 Maxima [F(-2)] 1067
3.148.8 Giac [F] 1068
3.148.9 Mupad [F(-1)] 1068

3.148.1 Optimal result

Integrand size = 20, antiderivative size = 109

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \frac{x(a+b\sec^{-1}(cx))}{d\sqrt{d+ex^2}} - \frac{bx\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

output
```
x*(a+b*arcsec(c*x))/d/(e*x^2+d)^(1/2)-b*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*
(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(
e*x^2+d)^(1/2)
```

3.148.2 Mathematica [A] (verified)

Time = 1.71 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \frac{x(a+b\sec^{-1}(cx))}{d\sqrt{d+ex^2}} - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d\left(-c+c^3x^2\right)\sqrt{d+ex^2}}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(d + e*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcSec[c*x]))/(d*Sqrt[d + e*x^2]) - (b*c*Sqrt[1 - 1/(c^2*x^2)]*x
*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))
])/(d*(-c + c^3*x^2)*Sqrt[d + e*x^2])
```

3.148.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5751, 27, 323, 323, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx$$

$$\downarrow 5751$$

$$\frac{x\left(a+b\sec^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - \frac{bcx\int \frac{1}{d\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{\sqrt{c^2x^2}}$$

$$\downarrow 27$$

$$\frac{x\left(a+b\sec^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - \frac{bcx\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{d\sqrt{c^2x^2}}$$

$$\downarrow 323$$

$$\frac{x\left(a+b\sec^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - \frac{bcx\sqrt{\frac{ex^2}{d}+1}\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}dx}{d\sqrt{c^2x^2}\sqrt{d+ex^2}}$$

$$\downarrow 323$$

$$\frac{x\left(a+b\sec^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - \frac{bcx\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}$$

$$\downarrow 321$$

$$\frac{x\left(a+b\sec^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - \frac{bx\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}$$

input
```
Int[(a + b*ArcSec[c*x])/(d + e*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcSec[c*x]))/(d*Sqrt[d + e*x^2]) - (b*x*Sqrt[1 - c^2*x^2]*Sqrt[
1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(d*Sqrt[c^2*x^2]*Sqrt
[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

3.148.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 5751
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSec[c*x])   u,
x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 -
 1]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1
/2, 0])
```

3.148.4 Maple [F]

$$\int \frac{a + b \operatorname{arcsec}(cx)}{(e\,x^2 + d)^{\frac{3}{2}}} dx$$

input
```
int((a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

3.148.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.68

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \frac{(bex^2+bd)\sqrt{-d}F(\arcsin(cx)\mid -\frac{e}{c^2d}) + (bcdx\,\text{arcsec}(cx)+acdx)\sqrt{ex^2+d}}{cd^2ex^2+cd^3}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
((b*e*x^2 + b*d)*sqrt(-d)*elliptic_f(arcsin(c*x), -e/(c^2*d)) + (b*c*d*x*a
rcsec(c*x) + a*c*d*x)*sqrt(e*x^2 + d))/(c*d^2*e*x^2 + c*d^3)
```

3.148.6 Sympy [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\,\text{asec}(cx)}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*asec(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*asec(c*x))/(d + e*x**2)**(3/2), x)
```

3.148.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.148.8 Giac [F]

$$\int \frac{a + b \sec^{-1}(cx)}{(d + ex^2)^{3/2}}\, dx = \int \frac{b \operatorname{arcsec}(cx) + a}{(ex^2 + d)^{\frac{3}{2}}}\, dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/(e*x^2 + d)^(3/2), x)
```

3.148.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \sec^{-1}(cx)}{(d + ex^2)^{3/2}}\, dx = \int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{(e\,x^2 + d)^{3/2}}\, dx$$

input
```
int((a + b*acos(1/(c*x)))/(d + e*x^2)^(3/2),x)
```

output
```
int((a + b*acos(1/(c*x)))/(d + e*x^2)^(3/2), x)
```

3.149 $\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx$

3.149.1 Optimal result 1069
3.149.2 Mathematica [C] (verified) 1070
3.149.3 Rubi [A] (verified) 1070
3.149.4 Maple [F] 1074
3.149.5 Fricas [A] (verification not implemented) 1074
3.149.6 Sympy [F] 1075
3.149.7 Maxima [F(-2)] 1075
3.149.8 Giac [F] 1076
3.149.9 Mupad [F(-1)] 1076

3.149.1 Optimal result

Integrand size = 23, antiderivative size = 274

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx = \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}} - \frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex(a+b\sec^{-1}(cx))}{d^2\sqrt{d+ex^2}} - \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|\,-\frac{e}{c^2d}\right)}{d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} + \frac{b(c^2d+2e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

output

```
(-a-b*arcsec(c*x))/d/x/(e*x^2+d)^(1/2)-2*e*x*(a+b*arcsec(c*x))/d^2/(e*x^2+
d)^(1/2)+b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)-b*c^2*x
*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c
^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)+b*(c^2*d+2*e)*x*Elliptic
F(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)
^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)
```

3.149.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.82 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.77

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx = \frac{bc\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)-a(d+2ex^2)-b(d+2ex^2)\sec^{-1}(cx)}{d^2x\sqrt{d+ex^2}}$$
$$-\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2dE\left(i\operatorname{arcsinh}\left(\sqrt{-c^2}x\right)|-\frac{e}{c^2d}\right)-\left(c^2d+2e\right)\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{-c^2}x\right),-\frac{e}{c^2d}\right)\right.}{\sqrt{-c^2}d^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

output
```
(b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2) - a*(d + 2*e*x^2) - b*(d + 2*e*x^
2)*ArcSec[c*x])/(d^2*x*Sqrt[d + e*x^2]) - (I*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*S
qrt[1 + (e*x^2)/d]*(c^2*d*EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]
 - (c^2*d + 2*e)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[
-c^2]*d^2*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

3.149.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {5761, 25, 27, 445, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx$$
$$\downarrow 5761$$
$$-\frac{bcx\int -\frac{2ex^2+d}{d^2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{\sqrt{c^2x^2}} - \frac{2ex\left(a+b\sec^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} - \frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}}$$
$$\downarrow 25$$
$$\frac{bcx\int \frac{2ex^2+d}{d^2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{\sqrt{c^2x^2}} - \frac{2ex\left(a+b\sec^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} - \frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}}$$
$$\downarrow 27$$

$$\frac{bcx\int\frac{2ex^2+d}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{d^2\sqrt{c^2x^2}}-\frac{2ex\left(a+b\sec^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\downarrow 445$$

$$\frac{bcx\left(\frac{\int\frac{de\left(2-c^2x^2\right)}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{d}+\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{c^2x^2}}-\frac{2ex\left(a+b\sec^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\downarrow 27$$

$$\frac{bcx\left(e\int\frac{2-c^2x^2}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{c^2x^2}}-\frac{2ex\left(a+b\sec^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\downarrow 399$$

$$\frac{bcx\left(e\left(\frac{\left(c^2d+2e\right)\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{e}-\frac{c^2\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{c^2x^2}}-\frac{2ex\left(a+b\sec^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\downarrow 323$$

$$\frac{bcx\left(e\left(\frac{\left(c^2d+2e\right)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}-\frac{c^2\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{c^2x^2}}-\frac{2ex\left(a+b\sec^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\downarrow 323$$

$$\frac{bcx\left(e\left(\frac{\sqrt{1-c^2x^2}\left(c^2d+2e\right)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{c^2x^2}}-\frac{2ex\left(a+b\sec^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\downarrow 321$$

$$\frac{bcx\left(e\left(\frac{\sqrt{1-c^2x^2}\left(c^2d+2e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{c^2x^2}}-$$
$$\frac{2ex\left(a+b\sec^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\downarrow 331$$

$$\frac{bcx\left(e\left(\frac{\sqrt{1-c^2x^2}\left(c^2d+2e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\sqrt{1-c^2x^2}\int\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{c^2x^2-1}}\right)+\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{c^2x^2}}-$$
$$\frac{2ex\left(a+b\sec^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\downarrow 330$$

$$\frac{bcx\left(e\left(\frac{\sqrt{1-c^2x^2}\left(c^2d+2e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\right)+\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{c^2x^2}}-$$
$$\frac{2ex\left(a+b\sec^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\downarrow 327$$

$$-\frac{2ex\left(a+b\sec^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}}+$$
$$\frac{bcx\left(e\left(\frac{\sqrt{1-c^2x^2}\left(c^2d+2e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\middle|-\frac{e}{c^2d}\right)}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\right)+\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{c^2x^2}}$$

input
```
Int[(a + b*ArcSec[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

output
```
-((a + b*ArcSec[c*x])/(d*x*Sqrt[d + e*x^2])) - (2*e*x*(a + b*ArcSec[c*x]))
/(d^2*Sqrt[d + e*x^2]) + (b*c*x*((Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/x +
e*(-((c*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*
d))])/(e*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d])) + ((c^2*d + 2*e)*Sqrt[1
- c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(c*e*
Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])))))/(d^2*Sqrt[c^2*x^2])
```

3.149.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])  u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])  Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.149.4 Maple [F]

$$\int \frac{a + b \operatorname{arcsec}(cx)}{x^2 (e x^2 + d)^{\frac{3}{2}}} dx$$

input
```
int((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

3.149.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.69

$$\int \frac{a + b \sec^{-1}(cx)}{x^2 (d + ex^2)^{3/2}} dx =$$
$$-\frac{\left(2\,acdex^2 + acd^2 + \left(2\,bcdex^2 + bcd^2\right)\operatorname{arcsec}(cx) - \left(bcdex^2 + bcd^2\right)\sqrt{c^2x^2 - 1}\right)\sqrt{ex^2 + d} - \left(\left(bc^4dex^3 + bc\right.\right.}{cd^3ex^3 + cd^4}$$

input
```
integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
-((2*a*c*d*e*x^2 + a*c*d^2 + (2*b*c*d*e*x^2 + b*c*d^2)*arcsec(c*x) - (b*c*
d*e*x^2 + b*c*d^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d) - ((b*c^4*d*e*x^3 +
b*c^4*d^2*x)*elliptic_e(arcsin(c*x), -e/(c^2*d)) - ((b*c^4*d*e + 2*b*e^2)*
x^3 + (b*c^4*d^2 + 2*b*d*e)*x)*elliptic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(-
d))/(c*d^3*e*x^3 + c*d^4*x)
```

3.149.6 Sympy [F]

$$\int \frac{a + b \sec^{-1}(cx)}{x^2 \left(d + ex^2\right)^{3/2}} \, dx = \int \frac{a + b \operatorname{asec}(cx)}{x^2 \left(d + ex^2\right)^{\frac{3}{2}}} \, dx$$

input
```
integrate((a+b*asec(c*x))/x**2/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*asec(c*x))/(x**2*(d + e*x**2)**(3/2)), x)
```

3.149.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b \sec^{-1}(cx)}{x^2 \left(d + ex^2\right)^{3/2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.149.8 Giac [F]

$$\int \frac{a + b \sec^{-1}(cx)}{x^2 \left(d + ex^2\right)^{3/2}} \, dx = \int \frac{b \operatorname{arcsec}(cx) + a}{(ex^2 + d)^{\frac{3}{2}} x^2} \, dx$$

input
```
integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/((e*x^2 + d)^(3/2)*x^2), x)
```

3.149.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \sec^{-1}(cx)}{x^2 \left(d + ex^2\right)^{3/2}} \, dx = \int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{x^2 \left(e\,x^2 + d\right)^{3/2}} \, dx$$

input
```
int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)^(3/2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)^(3/2)), x)
```

3.150 $\int \frac{a+b\sec^{-1}(cx)}{x^4\left(d+ex^2\right)^{3/2}}\,dx$

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3.150.1 Optimal result

Integrand size = 23, antiderivative size = 701

$$\begin{aligned}
\int \frac{a+b\sec^{-1}(cx)}{x^4\left(d+ex^2\right)^{3/2}}\,dx &= \frac{2bc(c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^3\sqrt{c^2x^2}} \\
&- \frac{4bce\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3d^3\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^2x^2\sqrt{c^2x^2}} \\
&- \frac{a+b\sec^{-1}(cx)}{3dx^3\sqrt{d+ex^2}} + \frac{4e(a+b\sec^{-1}(cx))}{3d^2x\sqrt{d+ex^2}} + \frac{8e^2x(a+b\sec^{-1}(cx))}{3d^3\sqrt{d+ex^2}} \\
&- \frac{2bc^2(c^2d-e)\,x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|\,-\frac{e}{c^2d}\right)}{9d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&+ \frac{4bc^2ex\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|\,-\frac{e}{c^2d}\right)}{3d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&+ \frac{bc^2(2c^2d-e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{9d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} \\
&- \frac{4bc^2ex\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} \\
&- \frac{8be^2x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}$$

output
```
1/3*(-a-b*arcsec(c*x))/d/x^3/(e*x^2+d)^(1/2)+4/3*e*(a+b*arcsec(c*x))/d^2/x
/(e*x^2+d)^(1/2)+8/3*e^2*x*(a+b*arcsec(c*x))/d^3/(e*x^2+d)^(1/2)+2/9*b*c*(
c^2*d-e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^3/(c^2*x^2)^(1/2)-4/3*b*c*e*(
c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^3/(c^2*x^2)^(1/2)+1/9*b*c*(c^2*x^2-1)^(
1/2)*(e*x^2+d)^(1/2)/d^2/x^2/(c^2*x^2)^(1/2)-2/9*b*c^2*(c^2*d-e)*x*Ellipti
cE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^3/(c^2*x^2)^
(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)+4/3*b*c^2*e*x*EllipticE(c*x,(-e/
c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^3/(c^2*x^2)^(1/2)/(c^2*
x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)+1/9*b*c^2*(2*c^2*d-e)*x*EllipticF(c*x,(-e/c
^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2
*x^2-1)^(1/2)/(e*x^2+d)^(1/2)-4/3*b*c^2*e*x*EllipticF(c*x,(-e/c^2/d)^(1/2)
)*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/
2)/(e*x^2+d)^(1/2)-8/3*b*e^2*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1
)^(1/2)*(1+e*x^2/d)^(1/2)/d^3/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^
(1/2)
```

3.150.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.45 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.40

$$\int \frac{a+b\sec^{-1}(cx)}{x^4\left(d+ex^2\right)^{3/2}}\,dx = \frac{bc\sqrt{1-\frac{1}{c^2x^2}}x\left(d+2c^2dx^2-14ex^2\right)\left(d+ex^2\right)-3a(d^2-4dex^2-8e^2x^4)-3b(d^2-4}{9d^3x^3\sqrt{d+ex^2}}$$
$$-\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(2c^2d(c^2d-7e)\,E\left(i\operatorname{arcsinh}\left(\sqrt{-c^2}x\right)|-\frac{e}{c^2d}\right)+\left(-2c^4d^2+13c^2de+24e^2\right)\text{EllipticF}\right.}{9\sqrt{-c^2}d^3\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x^4*(d + e*x^2)^(3/2)),x]
```

output
```
(b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + 2*c^2*d*x^2 - 14*e*x^2)*(d + e*x^2) - 3*
a*(d^2 - 4*d*e*x^2 - 8*e^2*x^4) - 3*b*(d^2 - 4*d*e*x^2 - 8*e^2*x^4)*ArcSec
[c*x])/(9*d^3*x^3*Sqrt[d + e*x^2]) - ((I/9)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sq
rt[1 + (e*x^2)/d]*(2*c^2*d*(c^2*d - 7*e)*EllipticE[I*ArcSinh[Sqrt[-c^2]*x]
, -(e/(c^2*d))] + (-2*c^4*d^2 + 13*c^2*d*e + 24*e^2)*EllipticF[I*ArcSinh[S
qrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d^3*Sqrt[1 - c^2*x^2]*Sqrt[d + e
*x^2])
```

3.150.3 Rubi [A] (verified)

Time = 1.67 (sec) , antiderivative size = 591, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5761, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x^4\left(d+ex^2\right)^{3/2}}\,dx$$

$$\downarrow 5761$$

$$-\frac{bcx\int -\frac{-8e^2x^4-4dex^2+d^2}{3d^3x^4\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{\sqrt{c^2x^2}}+\frac{8e^2x\left(a+b\sec^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}}+\frac{4e\left(a+b\sec^{-1}(cx)\right)}{3d^2x\sqrt{d+ex^2}}-\frac{a+b\sec^{-1}(cx)}{3dx^3\sqrt{d+ex^2}}$$

$$\downarrow 27$$

$$\frac{bcx\int \frac{-8e^2x^4-4dex^2+d^2}{x^4\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{3d^3\sqrt{c^2x^2}}+\frac{8e^2x\left(a+b\sec^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}}+\frac{4e\left(a+b\sec^{-1}(cx)\right)}{3d^2x\sqrt{d+ex^2}}-\frac{a+b\sec^{-1}(cx)}{3dx^3\sqrt{d+ex^2}}$$

$$\downarrow 7293$$

$$\frac{bcx\int\left(\frac{d^2}{x^4\sqrt{c^2x^2-1}\sqrt{ex^2+d}}-\frac{4ed}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}-\frac{8e^2}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}\right)dx}{3d^3\sqrt{c^2x^2}}+\frac{8e^2x\left(a+b\sec^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}}+$$
$$\frac{4e\left(a+b\sec^{-1}(cx)\right)}{3d^2x\sqrt{d+ex^2}}-\frac{a+b\sec^{-1}(cx)}{3dx^3\sqrt{d+ex^2}}$$

$$\downarrow 2009$$

$$\frac{8e^2x\left(a+b\sec^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}}+\frac{4e\left(a+b\sec^{-1}(cx)\right)}{3d^2x\sqrt{d+ex^2}}-\frac{a+b\sec^{-1}(cx)}{3dx^3\sqrt{d+ex^2}}+$$
$$bcx\left(-\frac{8e^2\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{c\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{4cde\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}+\frac{cd\sqrt{1-c^2x^2}(2c^2d-e)\sqrt{}}{3\sqrt{c^}}\right.$$

input
```
Int[(a + b*ArcSec[c*x])/(x^4*(d + e*x^2)^(3/2)),x]
```

output
```
-1/3*(a + b*ArcSec[c*x])/(d*x^3*Sqrt[d + e*x^2]) + (4*e*(a + b*ArcSec[c*x]
))/(3*d^2*x*Sqrt[d + e*x^2]) + (8*e^2*x*(a + b*ArcSec[c*x]))/(3*d^3*Sqrt[d
 + e*x^2]) + (b*c*x*((d*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(3*x^3) + (2*(
c^2*d - e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(3*x) - (4*e*Sqrt[-1 + c^2*
x^2]*Sqrt[d + e*x^2])/x - (2*c*(c^2*d - e)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^
2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(3*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e
*x^2)/d]) + (4*c*e*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x]
, -(e/(c^2*d))])/(Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) + (c*d*(2*c^2*d
- e)*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2
*d))])/(3*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2]) - (4*c*d*e*Sqrt[1 - c^2*x^2]
*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(Sqrt[-1 + c^2*
x^2]*Sqrt[d + e*x^2]) - (8*e^2*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*Ellip
ticF[ArcSin[c*x], -(e/(c^2*d))])/(c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])))/
(3*d^3*Sqrt[c^2*x^2])
```

3.150.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.150.4 Maple [F]

$$\int \frac{a + b \operatorname{arcsec}(cx)}{x^4 \left(e x^2 + d\right)^{\frac{3}{2}}} dx$$

input
```
int((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(3/2),x)
```

3.150.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.47

$$\int \frac{a + b \sec^{-1}(cx)}{x^4 \left(d + ex^2\right)^{3/2}} dx = \frac{\left(24\, acde^2x^4 + 12\, acd^2ex^2 - 3\, acd^3 + 3\left(8\, bcde^2x^4 + 4\, bcd^2ex^2 - bcd^3\right) \operatorname{arcsec}(cx) + (\right.}{}$$

input
```
integrate((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
1/9*((24*a*c*d*e^2*x^4 + 12*a*c*d^2*e*x^2 - 3*a*c*d^3 + 3*(8*b*c*d*e^2*x^4
 + 4*b*c*d^2*e*x^2 - b*c*d^3)*arcsec(c*x) + (b*c*d^3 + 2*(b*c^3*d^2*e - 7*
b*c*d*e^2)*x^4 + (2*b*c^3*d^3 - 13*b*c*d^2*e)*x^2)*sqrt(c^2*x^2 - 1))*sqrt
(e*x^2 + d) + (2*((b*c^6*d^2*e - 7*b*c^4*d*e^2)*x^5 + (b*c^6*d^3 - 7*b*c^4
*d^2*e)*x^3)*elliptic_e(arcsin(c*x), -e/(c^2*d)) - ((2*b*c^6*d^2*e - (14*b
*c^4 - b*c^2)*d*e^2 - 24*b*e^3)*x^5 + (2*b*c^6*d^3 - (14*b*c^4 - b*c^2)*d^
2*e - 24*b*d*e^2)*x^3)*elliptic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(-d))/(c*d
^4*e*x^5 + c*d^5*x^3)
```

3.150.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b \sec^{-1}(cx)}{x^4 \left(d + ex^2\right)^{3/2}} dx = \text{Timed out}$$

input
```
integrate((a+b*asec(c*x))/x**4/(e*x**2+d)**(3/2),x)
```

output
```
Timed out
```

3.150.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x^4\left(d+ex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.150.8 Giac [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^4\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{b\operatorname{arcsec}(cx)+a}{(ex^2+d)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/((e*x^2 + d)^(3/2)*x^4), x)
```

3.150.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^4\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{a+b\operatorname{acos}\left(\frac{1}{cx}\right)}{x^4\left(e\,x^2+d\right)^{3/2}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(x^4*(d + e*x^2)^(3/2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x^4*(d + e*x^2)^(3/2)), x)
```

3.151 $\int \frac{x^5\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

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3.151.1 Optimal result

Integrand size = 23, antiderivative size = 244

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = -\frac{bcdx\sqrt{-1+c^2x^2}}{3e^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d(a+b\sec^{-1}(cx))}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} + \frac{8bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e^3\sqrt{c^2x^2}} - \frac{bx\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{e^{5/2}\sqrt{c^2x^2}}$$

output

```
-1/3*d^2*(a+b*arcsec(c*x))/e^3/(e*x^2+d)^(3/2)-b*x*arctanh(e^(1/2)*(c^2*x^
2-1)^(1/2)/c/(e*x^2+d)^(1/2))/e^(5/2)/(c^2*x^2)^(1/2)+8/3*b*c*x*arctan((e*
x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))*d^(1/2)/e^3/(c^2*x^2)^(1/2)+2*d*(a
+b*arcsec(c*x))/e^3/(e*x^2+d)^(1/2)-1/3*b*c*d*x*(c^2*x^2-1)^(1/2)/e^2/(c^2
*d+e)/(c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)+(a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/e^
3
```

3.151.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.30 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.99

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{-\frac{2bcde\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)}{c^2d+e}+2a(8d^2+12dex^2+3e^2x^4)+\frac{b(d+ex^2)\left(8d\sqrt{1+\frac{d}{ex^2}}\,\text{AppellF1}\left(1\right.\right.}{}$$

input
```
Integrate[(x^5*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
((-2*b*c*d*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2))/(c^2*d + e) + 2*a*(8*d^2
 + 12*d*e*x^2 + 3*e^2*x^4) + (b*(d + e*x^2)*(8*d*Sqrt[1 + d/(e*x^2)]*Appel
lF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))] + (3*c^2*e*Sqrt[1 - 1/(c^2*
x^2)]*x^4*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/
d)])/Sqrt[1 - c^2*x^2]))/(c*x) + 2*b*(8*d^2 + 12*d*e*x^2 + 3*e^2*x^4)*ArcS
ec[c*x])/(6*e^3*(d + e*x^2)^(3/2))
```

3.151.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.90, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5761, 27, 7282, 2117, 27, 175, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$$

$$\downarrow 5761$$

$$-\frac{bcx\int \frac{3e^2x^4+12dex^2+8d^2}{3e^3x\sqrt{c^2x^2-1}(ex^2+d)^{3/2}}dx}{\sqrt{c^2x^2}}-\frac{d^2(a+b\sec^{-1}(cx))}{3e^3(d+ex^2)^{3/2}}+\frac{2d(a+b\sec^{-1}(cx))}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3}$$

$$\downarrow 27$$

$$-\frac{bcx\int\frac{3e^2x^4+12dex^2+8d^2}{x\sqrt{c^2x^2-1}(ex^2+d)^{3/2}}dx}{3e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}$$

$$\downarrow 7282$$

$$-\frac{bcx\int\frac{3e^2x^4+12dex^2+8d^2}{x^2\sqrt{c^2x^2-1}(ex^2+d)^{3/2}}dx^2}{6e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}$$

$$\downarrow 2117$$

$$-\frac{bcx\left(\frac{2de\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{2\int-\frac{d\left(dc^2+e\right)\left(3ex^2+8d\right)}{2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{d(c^2d+e)}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}$$

$$\downarrow 27$$

$$-\frac{bcx\left(\int\frac{3ex^2+8d}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+\frac{2de\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}$$

$$\downarrow 175$$

$$-\frac{bcx\left(3e\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+8d\int\frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+\frac{2de\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}$$

$$\downarrow 66$$

$$-\frac{bcx\left(8d\int\frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+6e\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}+\frac{2de\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}$$

$$\downarrow 104$$

$$-\frac{bcx\left(6e\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}+16d\int\frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+\frac{2de\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}$$

$$\downarrow 217$$

$$-\frac{bcx\left(6e\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-16\sqrt{d}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)+\frac{2de\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}$$

$$\downarrow 221$$

$$-\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\sec^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)}{e^3}-\frac{bcx\left(-16\sqrt{d}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)+\frac{6\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}+\frac{2de\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^3\sqrt{c^2x^2}}$$

input
```
Int[(x^5*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
-1/3*(d^2*(a + b*ArcSec[c*x]))/(e^3*(d + e*x^2)^(3/2)) + (2*d*(a + b*ArcSe
c[c*x]))/(e^3*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/e^3
 - (b*c*x*((2*d*e*Sqrt[-1 + c^2*x^2])/((c^2*d + e)*Sqrt[d + e*x^2]) - 16*S
qrt[d]*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])] + (6*Sqrt[e]*A
rcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/c))/(6*e^3*Sqrt[
c^2*x^2])
```

3.151.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2117
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

3.151.4 Maple [F]

$$\int \frac{x^5(a+b\operatorname{arcsec}(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

3.151.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 515 vs. $2(206) = 412$.

Time $= 0.49$ (sec) , antiderivative size $= 2123$, normalized size of antiderivative $= 8.70$

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e
 + b*d*e^2)*x^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*
d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(
e*x^2 + d)*sqrt(e) + e^2) + 8*(b*c^3*d^3 + b*c*d^2*e + (b*c^3*d*e^2 + b*c*
e^3)*x^4 + 2*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(-d)*log(((c^4*d^2 - 6*c^2
*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)
*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + 4*(8*a*c^3*d^3 + 8*a*
c*d^2*e + 3*(a*c^3*d*e^2 + a*c*e^3)*x^4 + 12*(a*c^3*d^2*e + a*c*d*e^2)*x^2
 + (8*b*c^3*d^3 + 8*b*c*d^2*e + 3*(b*c^3*d*e^2 + b*c*e^3)*x^4 + 12*(b*c^3*
d^2*e + b*c*d*e^2)*x^2)*arcsec(c*x) - (b*c*d*e^2*x^2 + b*c*d^2*e)*sqrt(c^2
*x^2 - 1))*sqrt(e*x^2 + d))/(c^3*d^3*e^3 + c*d^2*e^4 + (c^3*d*e^5 + c*e^6)
*x^4 + 2*(c^3*d^2*e^4 + c*d*e^5)*x^2), 1/12*(16*(b*c^3*d^3 + b*c*d^2*e + (
b*c^3*d*e^2 + b*c*e^3)*x^4 + 2*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(d)*arct
an(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/
(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + 3*(b*c^2*d^3 + (b*c^2*d*e^2 +
 b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(e)*log(8*c^4*e
^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2
+ c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(8*a*c
^3*d^3 + 8*a*c*d^2*e + 3*(a*c^3*d*e^2 + a*c*e^3)*x^4 + 12*(a*c^3*d^2*e + a
*c*d*e^2)*x^2 + (8*b*c^3*d^3 + 8*b*c*d^2*e + 3*(b*c^3*d*e^2 + b*c*e^3)*...
```

3.151.6 Sympy [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^5(a+b\,\text{asec}\,(cx))}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**5*(a+b*asec(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Integral(x**5*(a + b*asec(c*x))/(d + e*x**2)**(5/2), x)
```

3.151.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.151.8 Giac [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\,\text{arcsec}\,(cx)+a)x^5}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x^5/(e*x^2 + d)^(5/2), x)
```

3.151.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^5\left(a+b\,\text{acos}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.152 $\int \frac{x^3\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

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3.152.1 Optimal result

Integrand size = 23, antiderivative size = 163

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{bcx\sqrt{-1+c^2x^2}}{3e\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{d(a+b\sec^{-1}(cx))}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\sec^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{2bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3\sqrt{d}e^2\sqrt{c^2x^2}}$$

output

```
1/3*d*(a+b*arcsec(c*x))/e^2/(e*x^2+d)^(3/2)-2/3*b*c*x*arctan((e*x^2+d)^(1/
2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e^2/d^(1/2)/(c^2*x^2)^(1/2)+(-a-b*arcsec(c*x
))/e^2/(e*x^2+d)^(1/2)+1/3*b*c*x*(c^2*x^2-1)^(1/2)/e/(c^2*d+e)/(c^2*x^2)^(
1/2)/(e*x^2+d)^(1/2)
```

3.152.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.46 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.84

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = -\frac{-\frac{bce\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)}{c^2d+e} + a(2d+3ex^2) + \frac{b\sqrt{1+\frac{d}{ex^2}}(d+ex^2)\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)}{cx} + b(2d+3ex^2)\sec^{-1}(cx)}{3e^2\left(d+ex^2\right)^{3/2}}$$

input
```
Integrate[(x^3*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
-1/3*(-((b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2))/(c^2*d + e)) + a*(2*d
+ 3*e*x^2) + (b*Sqrt[1 + d/(e*x^2)]*(d + e*x^2)*AppellF1[1, 1/2, 1/2, 2, 1
/(c^2*x^2), -(d/(e*x^2))])/(c*x) + b*(2*d + 3*e*x^2)*ArcSec[c*x])/(e^2*(d
+ e*x^2)^(3/2))
```

3.152.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5761, 27, 435, 169, 25, 27, 104, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

$$\downarrow 5761$$

$$-\frac{bcx\int -\frac{3ex^2+2d}{3e^2x\sqrt{c^2x^2-1}\left(ex^2+d\right)^{3/2}}dx}{\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\sec^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}$$

$$\downarrow 27$$

$$\frac{bcx\int \frac{3ex^2+2d}{x\sqrt{c^2x^2-1}\left(ex^2+d\right)^{3/2}}dx}{3e^2\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\sec^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}$$

$$\downarrow 435$$

$$\frac{bcx\int \frac{3ex^2+2d}{x^2\sqrt{c^2x^2-1}\left(ex^2+d\right)^{3/2}}dx^2}{6e^2\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\sec^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}$$

$$\downarrow 169$$

$$\frac{bcx\left(\frac{2e\sqrt{c^2x^2-1}}{\left(c^2d+e\right)\sqrt{d+ex^2}}-\frac{2\int -\frac{d\left(dc^2+e\right)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{d\left(c^2d+e\right)}\right)}{6e^2\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\sec^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}$$

$$\downarrow 25$$

$$\frac{bcx\left(\frac{2\int \frac{d\left(dc^2+e\right)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{d(c^2d+e)}+\frac{2e\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^2\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\sec^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}$$

$$\downarrow 27$$

$$\frac{bcx\left(2\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+\frac{2e\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^2\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\sec^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}$$

$$\downarrow 104$$

$$\frac{bcx\left(4\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+\frac{2e\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^2\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\sec^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}$$

$$\downarrow 217$$

$$-\frac{a+b\sec^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\sec^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}+\frac{bcx\left(\frac{2e\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{4\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{\sqrt{d}}\right)}{6e^2\sqrt{c^2x^2}}$$

input
```
Int[(x^3*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
(d*(a + b*ArcSec[c*x]))/(3*e^2*(d + e*x^2)^(3/2)) - (a + b*ArcSec[c*x])/(e
^2*Sqrt[d + e*x^2]) + (b*c*x*((2*e*Sqrt[-1 + c^2*x^2])/((c^2*d + e)*Sqrt[d
 + e*x^2]) - (4*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/Sqrt
[d]))/(6*e^2*Sqrt[c^2*x^2])
```

3.152.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.152.4 Maple [F]

$$\int \frac{x^3(a+b\,\text{arcsec}\,(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

3.152.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(137) = 274$.

Time $= 0.39$ (sec) , antiderivative size $= 664$, normalized size of antiderivative $= 4.07$

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \left[-\frac{(bc^2d^3+(bc^2de^2+be^3)x^4+bd^2e+2\,(bc^2d^2e+bde^2)x^2)\sqrt{-d}\log\left(\frac{(c^4d^2-6\,c^2de}{}\right.\right.$$

$$\left.\left.-\frac{(bc^2d^3+(bc^2de^2+be^3)x^4+bd^2e+2\,(bc^2d^2e+bde^2)x^2)\sqrt{d}\arctan\left(-\frac{\sqrt{c^2x^2-1}((c^2d-e)x^2-2\,d)\sqrt{ex^2+d}\sqrt{d}}{2\,(c^2dex^4+(c^2d^2-de)x^2-d^2)}\right)+(2}{3\,(c^2d^4e^2+d^3e^3-}\right]$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[-1/6*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e +
 b*d*e^2)*x^2)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2
- d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*s
qrt(-d) + 8*d^2)/x^4) + 2*(2*a*c^2*d^3 + 2*a*d^2*e + 3*(a*c^2*d^2*e + a*d*
e^2)*x^2 + (2*b*c^2*d^3 + 2*b*d^2*e + 3*(b*c^2*d^2*e + b*d*e^2)*x^2)*arcse
c(c*x) - (b*d*e^2*x^2 + b*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^2*
d^4*e^2 + d^3*e^3 + (c^2*d^2*e^4 + d*e^5)*x^4 + 2*(c^2*d^3*e^3 + d^2*e^4)*
x^2), -1/3*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^
2*e + b*d*e^2)*x^2)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2
 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2))
 + (2*a*c^2*d^3 + 2*a*d^2*e + 3*(a*c^2*d^2*e + a*d*e^2)*x^2 + (2*b*c^2*d^3
 + 2*b*d^2*e + 3*(b*c^2*d^2*e + b*d*e^2)*x^2)*arcsec(c*x) - (b*d*e^2*x^2 +
 b*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^2*d^4*e^2 + d^3*e^3 + (c^
2*d^2*e^4 + d*e^5)*x^4 + 2*(c^2*d^3*e^3 + d^2*e^4)*x^2)]
```

3.152.6 Sympy [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^3(a+b\operatorname{asec}(cx))}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3*(a+b*asec(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Integral(x**3*(a + b*asec(c*x))/(d + e*x**2)**(5/2), x)
```

3.152.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.152.8 Giac [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x^3}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x^3/(e*x^2 + d)^(5/2), x)
```

3.152.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^3\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.153 $\int \frac{x\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

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3.153.1 Optimal result

Integrand size = 21, antiderivative size = 138

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = -\frac{bcx\sqrt{-1+c^2x^2}}{3d\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{a+b\sec^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}} - \frac{bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3d^{3/2}e\sqrt{c^2x^2}}$$

output
```
1/3*(-a-b*arcsec(c*x))/e/(e*x^2+d)^(3/2)-1/3*b*c*x*arctan((e*x^2+d)^(1/2)/
d^(1/2)/(c^2*x^2-1)^(1/2))/d^(3/2)/e/(c^2*x^2)^(1/2)-1/3*b*c*x*(c^2*x^2-1)
^(1/2)/d/(c^2*d+e)/(c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)
```

3.153.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.45 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{-\frac{b\sqrt{1+\frac{d}{ex^2}}\left(d+ex^2\right)\operatorname{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)}{cdex} + \frac{2\left(-a-\frac{bce\sqrt{1-\frac{1}{c^2x^2}}x\left(d+ex^2\right)}{d\left(c^2d+e\right)}-b\sec^{-1}(cx)\right)}{e}}{6\left(d+ex^2\right)^{3/2}}$$

input
```
Integrate[(x*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
(-((b*Sqrt[1 + d/(e*x^2)]*(d + e*x^2)*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2)
, -(d/(e*x^2))])/(c*d*e*x)) + (2*(-a - (b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d +
 e*x^2))/(d*(c^2*d + e)) - b*ArcSec[c*x]))/e)/(6*(d + e*x^2)^(3/2))
```

3.153.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5759, 354, 107, 104, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

$$\downarrow 5759$$

$$\frac{bcx\int \frac{1}{x\sqrt{c^2x^2-1}\left(ex^2+d\right)^{3/2}}dx}{3e\sqrt{c^2x^2}} - \frac{a+b\sec^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}$$

$$\downarrow 354$$

$$\frac{bcx\int \frac{1}{x^2\sqrt{c^2x^2-1}\left(ex^2+d\right)^{3/2}}dx^2}{6e\sqrt{c^2x^2}} - \frac{a+b\sec^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}$$

$$\downarrow 107$$

$$\frac{bcx\left(\frac{\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{d} - \frac{2e\sqrt{c^2x^2-1}}{d\left(c^2d+e\right)\sqrt{d+ex^2}}\right)}{6e\sqrt{c^2x^2}} - \frac{a+b\sec^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}$$

$$\downarrow 104$$

$$\frac{bcx\left(\frac{2\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}}{d} - \frac{2e\sqrt{c^2x^2-1}}{d\left(c^2d+e\right)\sqrt{d+ex^2}}\right)}{6e\sqrt{c^2x^2}} - \frac{a+b\sec^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}$$

$$\downarrow 217$$

$$\frac{bcx\left(-\frac{2\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{d^{3/2}} - \frac{2e\sqrt{c^2x^2-1}}{d\left(c^2d+e\right)\sqrt{d+ex^2}}\right)}{6e\sqrt{c^2x^2}} - \frac{a+b\sec^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}$$

input
```
Int[(x*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
-1/3*(a + b*ArcSec[c*x])/(e*(d + e*x^2)^(3/2)) + (b*c*x*((-2*e*Sqrt[-1 + c
^2*x^2])/(d*(c^2*d + e)*Sqrt[d + e*x^2]) - (2*ArcTan[Sqrt[d + e*x^2]/(Sqrt
[d]*Sqrt[-1 + c^2*x^2])])/d^(3/2)))/(6*e*Sqrt[c^2*x^2])
```

3.153.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5759
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSec[c*x])/(2*e*(p + 1))), x
] - Simp[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2]))   Int[(d + e*x^2)^(p + 1)/(x*S
qrt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

3.153.4 Maple [F]

$$\int \frac{x(a+b\ \text{arcsec}\,(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int(x*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

3.153.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(114) = 228$.

Time $= 0.36$ (sec) , antiderivative size $= 571$, normalized size of antiderivative $= 4.14$

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \left[-\frac{(bc^2d^3+(bc^2de^2+be^3)x^4+bd^2e+2\,(bc^2d^2e+bde^2)x^2)\sqrt{-d}\log\left(\frac{(c^4d^2-6\,c^2de+}{}\right.\right.$$

$$\left.\left.-\frac{(bc^2d^3+(bc^2de^2+be^3)x^4+bd^2e+2\,(bc^2d^2e+bde^2)x^2)\sqrt{d}\arctan\left(-\frac{\sqrt{c^2x^2-1}((c^2d-e)x^2-2\,d)\sqrt{ex^2+d}\sqrt{d}}{2\,(c^2dex^4+(c^2d^2-de)x^2-d^2)}\right)+2\,(}{6\,(c^2d^5e+d^4e^2+(c^2d^3e^3+d^2e^4)x^4+2\,(c}\right]$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e
+ b*d*e^2)*x^2)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2
 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*
sqrt(-d) + 8*d^2)/x^4) + 4*(a*c^2*d^3 + a*d^2*e + (b*c^2*d^3 + b*d^2*e)*ar
csec(c*x) + (b*d*e^2*x^2 + b*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c
^2*d^5*e + d^4*e^2 + (c^2*d^3*e^3 + d^2*e^4)*x^4 + 2*(c^2*d^4*e^2 + d^3*e^
3)*x^2), -1/6*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2
*d^2*e + b*d*e^2)*x^2)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*
x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^
2)) + 2*(a*c^2*d^3 + a*d^2*e + (b*c^2*d^3 + b*d^2*e)*arcsec(c*x) + (b*d*e^
2*x^2 + b*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^2*d^5*e + d^4*e^2
+ (c^2*d^3*e^3 + d^2*e^4)*x^4 + 2*(c^2*d^4*e^2 + d^3*e^3)*x^2)]
```

3.153.6 Sympy [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x(a+b\,\mathrm{asec}\,(cx))}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*asec(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Integral(x*(a + b*asec(c*x))/(d + e*x**2)**(5/2), x)
```

3.153.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.153.8 Giac [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x/(e*x^2 + d)^(5/2), x)
```

3.153.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

output
```
int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.154 $\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$

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3.154.2 Mathematica [N/A] 1104
3.154.3 Rubi [N/A] 1105
3.154.4 Maple [N/A] (verified) 1105
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3.154.8 Giac [N/A] 1107
3.154.9 Mupad [N/A] 1107

3.154.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx = \text{Int}\left(\frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}},x\right)$$

output
```
Unintegrable((a+b*arcsec(c*x))/x/(e*x^2+d)^(5/2),x)
```

3.154.2 Mathematica [N/A]

Not integrable

Time = 14.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x*(d + e*x^2)^(5/2)),x]
```

output
```
Integrate[(a + b*ArcSec[c*x])/(x*(d + e*x^2)^(5/2)), x]
```

3.154.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \sec^{-1}(cx)}{x \left(d + ex^2\right)^{5/2}} \, dx$$
$$\downarrow 5771$$
$$\int \frac{a + b \sec^{-1}(cx)}{x \left(d + ex^2\right)^{5/2}} dx$$

input
```
Int[(a + b*ArcSec[c*x])/(x*(d + e*x^2)^(5/2)),x]
```

output
```
$Aborted
```

3.154.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrable[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.154.4 Maple [N/A] (verified)

Not integrable

Time = 1.82 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a + b \operatorname{arcsec}(cx)}{x \left(e x^2 + d\right)^{\frac{5}{2}}} dx$$

input
```
int((a+b*arcsec(c*x))/x/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arcsec(c*x))/x/(e*x^2+d)^(5/2),x)
```

3.154.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.30

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{b\operatorname{arcsec}(cx)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^
2*e*x^3 + d^3*x), x)
```

3.154.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*asec(c*x))/x/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.154.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.154.8 Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{b\operatorname{arcsec}(cx)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/((e*x^2 + d)^(5/2)*x), x)
```

3.154.9 Mupad [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{a+b\operatorname{acos}\left(\frac{1}{cx}\right)}{x\left(e\,x^2+d\right)^{5/2}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^(5/2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^(5/2)), x)
```

3.155 $\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$

3.155.1 Optimal result 1108
3.155.2 Mathematica [N/A] 1108
3.155.3 Rubi [N/A] 1109
3.155.4 Maple [N/A] (verified) 1109
3.155.5 Fricas [N/A] 1110
3.155.6 Sympy [F(-1)] 1110
3.155.7 Maxima [F(-2)] 1110
3.155.8 Giac [N/A] 1111
3.155.9 Mupad [N/A] 1111

3.155.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx = \text{Int}\left(\frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}},x\right)$$

output
```
Unintegrable((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

3.155.2 Mathematica [N/A]

Not integrable

Time = 15.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
```

output
```
Integrate[(a + b*ArcSec[c*x])/(x^3*(d + e*x^2)^(5/2)), x]
```

3.155.3 Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \sec^{-1}(cx)}{x^3 \left(d + ex^2\right)^{5/2}} \, dx$$
$$\downarrow 5771$$
$$\int \frac{a + b \sec^{-1}(cx)}{x^3 \left(d + ex^2\right)^{5/2}} dx$$

input
```
Int[(a + b*ArcSec[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
```

output
```
$Aborted
```

3.155.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.155.4 Maple [N/A] (verified)

Not integrable

Time = 2.33 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a + b \operatorname{arcsec}(cx)}{x^3 \left(e\, x^2 + d\right)^{\frac{5}{2}}} dx$$

input
```
int((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

3.155.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.39

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{b\operatorname{arcsec}(cx)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^
2*e*x^5 + d^3*x^3), x)
```

3.155.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*asec(c*x))/x**3/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.155.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.155.8 Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{b\,\text{arcsec}\,(cx)+a}{(ex^2+d)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/((e*x^2 + d)^(5/2)*x^3), x)
```

3.155.9 Mupad [N/A]

Not integrable

Time = 1.58 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{a+b\,\text{acos}\left(\frac{1}{c\,x}\right)}{x^3\left(e\,x^2+d\right)^{5/2}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(x^3*(d + e*x^2)^(5/2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x^3*(d + e*x^2)^(5/2)), x)
```

3.156 $\int \frac{x^6\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

3.156.1 Optimal result 1112
3.156.2 Mathematica [N/A] 1112
3.156.3 Rubi [N/A] 1113
3.156.4 Maple [N/A] (verified) 1113
3.156.5 Fricas [N/A] 1114
3.156.6 Sympy [F(-1)] 1114
3.156.7 Maxima [F(-2)] 1114
3.156.8 Giac [N/A] 1115
3.156.9 Mupad [N/A] 1115

3.156.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^6(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Int}\left(\frac{x^6(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}},x\right)$$

output
```
Unintegrable(x^6*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

3.156.2 Mathematica [N/A]

Not integrable

Time = 14.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^6(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^6(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$$

input
```
Integrate[(x^6*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
Integrate[(x^6*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2), x]
```

3.156.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$$
$$\downarrow 5771$$
$$\int \frac{x^6(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}dx$$

input
```
Int[(x^6*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
$Aborted
```

3.156.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrable[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.156.4 Maple [N/A] (verified)

Not integrable

Time = 1.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^6(a+b\ \text{arcsec}\,(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int(x^6*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^6*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

3.156.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.52

$$\int \frac{x^6(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x^6}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral((b*x^6*arcsec(c*x) + a*x^6)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^
4 + 3*d^2*e*x^2 + d^3), x)
```

3.156.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^6(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**6*(a+b*asec(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.156.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^6(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^6*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.156.8 Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^6(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x^6}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x^6/(e*x^2 + d)^(5/2), x)
```

3.156.9 Mupad [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^6(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^6\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((x^6*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^6*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.157 $\int \frac{x^4\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

3.157.1 Optimal result 1116
3.157.2 Mathematica [N/A] 1116
3.157.3 Rubi [N/A] 1117
3.157.4 Maple [N/A] (verified) 1117
3.157.5 Fricas [N/A] 1118
3.157.6 Sympy [F(-1)] 1118
3.157.7 Maxima [F(-2)] 1118
3.157.8 Giac [N/A] 1119
3.157.9 Mupad [N/A] 1119

3.157.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Int}\left(\frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}},x\right)$$

output
```
Unintegrable(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

3.157.2 Mathematica [N/A]

Not integrable

Time = 13.52 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$$

input
```
Integrate[(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
Integrate[(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2), x]
```

3.157.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$$
$$\downarrow 5771$$
$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}dx$$

input
```
Int[(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
$Aborted
```

3.157.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrable[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.157.4 Maple [N/A] (verified)

Not integrable

Time = 1.42 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^4(a+b\ \text{arcsec}\,(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

3.157.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.52

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x^4}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral((b*x^4*arcsec(c*x) + a*x^4)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^
4 + 3*d^2*e*x^2 + d^3), x)
```

3.157.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(a+b*asec(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.157.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.157.8 Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x^4}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x^4/(e*x^2 + d)^(5/2), x)
```

3.157.9 Mupad [N/A]

Not integrable

Time = 1.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^4\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((x^4*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^4*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.158 $\int \frac{x^2\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

3.158.1 Optimal result 1120
3.158.2 Mathematica [A] (verified) 1121
3.158.3 Rubi [A] (verified) 1121
3.158.4 Maple [F] 1125
3.158.5 Fricas [A] (verification not implemented) 1125
3.158.6 Sympy [F(-1)] 1126
3.158.7 Maxima [F] 1126
3.158.8 Giac [F] 1126
3.158.9 Mupad [F(-1)] 1127

3.158.1 Optimal result

Integrand size = 23, antiderivative size = 276

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = -\frac{bcx^2\sqrt{-1+c^2x^2}}{3d\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x^3(a+b\sec^{-1}(cx))}{3d\left(d+ex^2\right)^{3/2}} + \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3de\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} - \frac{bx\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3de\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

output

```
1/3*x^3*(a+b*arcsec(c*x))/d/(e*x^2+d)^(3/2)-1/3*b*c*x^2*(c^2*x^2-1)^(1/2)/
d/(c^2*d+e)/(c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)+1/3*b*c^2*x*EllipticE(c*x,(-e/
c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d/e/(c^2*d+e)/(c^2*x^2)^(
1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)-1/3*b*x*EllipticF(c*x,(-e/c^2/d)^
(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/e/(c^2*x^2)^(1/2)/(c^2*x^2-1
)^(1/2)/(e*x^2+d)^(1/2)
```

3.158.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.67

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{x^2\left(a(c^2d+e)\,x - bc\sqrt{1-\frac{1}{c^2x^2}}(d+ex^2) + b(c^2d+e)\,x\sec^{-1}(cx)\right)}{3d\,(c^2d+e)\,(d+ex^2)^{3/2}} + \frac{bc\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{1+\frac{ex^2}{d}}\,E\left(\arcsin\left(\sqrt{-\frac{e}{d}}x\right)|-\frac{c^2d}{e}\right)}{3d\sqrt{-\frac{e}{d}}\,(c^2d+e)\,\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
(x^2*(a*(c^2*d + e)*x - b*c*Sqrt[1 - 1/(c^2*x^2)]*(d + e*x^2) + b*(c^2*d +
 e)*x*ArcSec[c*x]))/(3*d*(c^2*d + e)*(d + e*x^2)^(3/2)) + (b*c*Sqrt[1 - 1/
(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*EllipticE[ArcSin[Sqrt[-(e/d)]*x], -((c^2*
d)/e)])/(3*d*Sqrt[-(e/d)]*(c^2*d + e)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

3.158.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.91, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5761, 27, 373, 326, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$$

$$\downarrow 5761$$

$$\frac{x^3(a+b\sec^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} - \frac{bcx\int \frac{x^2}{3d\sqrt{c^2x^2-1}(ex^2+d)^{3/2}}\,dx}{\sqrt{c^2x^2}}$$

$$\downarrow 27$$

$$\frac{x^3(a+b\sec^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} - \frac{bcx\int \frac{x^2}{\sqrt{c^2x^2-1}(ex^2+d)^{3/2}}\,dx}{3d\sqrt{c^2x^2}}$$

$$\downarrow 373$$

$$\frac{x^3\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bcx\left(\frac{x\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{\int\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}dx}{c^2d+e}\right)}{3d\sqrt{c^2x^2}}$$

$$\downarrow 326$$

$$\frac{x^3\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bcx\left(\frac{x\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{\frac{c^2\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}-\frac{\left(c^2d+e\right)\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{e}}{c^2d+e}\right)}{3d\sqrt{c^2x^2}}$$

$$\downarrow 323$$

$$\frac{x^3\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bcx\left(\frac{x\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{\frac{c^2\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}-\frac{\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}}{c^2d+e}\right)}{3d\sqrt{c^2x^2}}$$

$$\downarrow 323$$

$$\frac{x^3\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bcx\left(\frac{x\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{\frac{c^2\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}-\frac{\sqrt{1-c^2x^2}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}}{c^2d+e}\right)}{3d\sqrt{c^2x^2}}$$

$$\downarrow 321$$

$$\frac{x^3\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bcx\left(\frac{x\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{\frac{c^2\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}-\frac{\sqrt{1-c^2x^2}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}}{c^2d+e}\right)}{3d\sqrt{c^2x^2}}$$

$$\downarrow 331$$

$$\frac{x^3\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bcx\left(\frac{x\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{\frac{c^2\sqrt{1-c^2x^2}\int\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{c^2x^2-1}}-\frac{\sqrt{1-c^2x^2}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}}{c^2d+e}\right)}{3d\sqrt{c^2x^2}}$$

$$\downarrow 330$$

$$\frac{x^3\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bcx\left(\frac{x\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{\frac{c^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}-\frac{\sqrt{1-c^2x^2}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}}{c^2d+e}\right)}{3d\sqrt{c^2x^2}}$$

$$\downarrow 327$$

$$\frac{x^3\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bcx\left(\frac{x\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{\frac{c\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\middle|-\frac{e}{c^2d}\right)}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}-\frac{\sqrt{1-c^2x^2}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}}{c^2d+e}\right)}{3d\sqrt{c^2x^2}}$$

input
```
Int[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
(x^3*(a + b*ArcSec[c*x]))/(3*d*(d + e*x^2)^(3/2)) - (b*c*x*((x*Sqrt[-1 + c
^2*x^2])/((c^2*d + e)*Sqrt[d + e*x^2]) - ((c*Sqrt[1 - c^2*x^2]*Sqrt[d + e*
x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(e*Sqrt[-1 + c^2*x^2]*Sqrt[1 +
(e*x^2)/d]) - ((c^2*d + e)*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF
[ArcSin[c*x], -(e/(c^2*d))])/(c*e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2]))/(c^
2*d + e)))/(3*d*Sqrt[c^2*x^2])
```

3.158.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])  u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])  Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.158.4 Maple [F]

$$\int \frac{x^2(a+b\operatorname{arcsec}(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

3.158.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.04

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{((bc^3d^2e+bcde^2)x^3\operatorname{arcsec}(cx)+(ac^3d^2e+acde^2)x^3-(bcde^2x^3+bcd^2ex)\sqrt{c^2}}{}$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(((b*c^3*d^2*e + b*c*d*e^2)*x^3*arcsec(c*x) + (a*c^3*d^2*e + a*c*d*e^2
)*x^3 - (b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d) -
 ((b*c^4*d*e^2*x^4 + 2*b*c^4*d^2*e*x^2 + b*c^4*d^3)*elliptic_e(arcsin(c*x)
, -e/(c^2*d)) - (b*c^4*d^3 + (b*c^4*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^
4*d^2*e + b*d*e^2)*x^2)*elliptic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(-d))/(c^
3*d^5*e + c*d^4*e^2 + (c^3*d^3*e^3 + c*d^2*e^4)*x^4 + 2*(c^3*d^4*e^2 + c*d
^3*e^3)*x^2)
```

3.158.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*asec(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.158.7 Maxima [F]

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*a*(x/((e*x^2 + d)^(3/2)*e) - x/(sqrt(e*x^2 + d)*d*e)) + b*integrate(x
^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e
*x^2 + d)), x)
```

3.158.8 Giac [F]

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x^2/(e*x^2 + d)^(5/2), x)
```

3.158.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^2\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.159 $\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx$

3.159.1 Optimal result 1128
3.159.2 Mathematica [C] (verified) 1129
3.159.3 Rubi [A] (verified) 1129
3.159.4 Maple [F] 1133
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3.159.9 Mupad [F(-1)] 1135

3.159.1 Optimal result

Integrand size = 20, antiderivative size = 296

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{bcex^2\sqrt{-1+c^2x^2}}{3d^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x(a+b\sec^{-1}(cx))}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x(a+b\sec^{-1}(cx))}{3d^2\sqrt{d+ex^2}} - \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|\,-\frac{e}{c^2d}\right)}{3d^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} - \frac{2bx\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

output

```
1/3*x*(a+b*arcsec(c*x))/d/(e*x^2+d)^(3/2)+2/3*x*(a+b*arcsec(c*x))/d^2/(e*x
^2+d)^(1/2)+1/3*b*c*e*x^2*(c^2*x^2-1)^(1/2)/d^2/(c^2*d+e)/(c^2*x^2)^(1/2)/
(e*x^2+d)^(1/2)-1/3*b*c^2*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(
1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*d+e)/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*
x^2/d)^(1/2)-2/3*b*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1
+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)
```

3.159.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.13 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.84

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{x\left(bce\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)+a(c^2d+e)\left(3d+2ex^2\right)+b(c^2d+e)\left(3d+2ex^2\right)\sec^{-1}\right.}{3d^2\left(c^2d+e\right)\left(d+ex^2\right)^{3/2}}$$
$$-\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2dE\left(i\operatorname{arcsinh}\left(\sqrt{-c^2}x\right)|-\frac{e}{c^2d}\right)+2(c^2d+e)\,\text{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{-c^2}x\right),-\frac{e}{c^2d}\right)\right.}{3\sqrt{-c^2}d^2\left(c^2d+e\right)\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(d + e*x^2)^(5/2),x]
```

output
```
(x*(b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2) + a*(c^2*d + e)*(3*d + 2*e*x
^2) + b*(c^2*d + e)*(3*d + 2*e*x^2)*ArcSec[c*x]))/(3*d^2*(c^2*d + e)*(d +
e*x^2)^(3/2)) - ((I/3)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^
2*d*EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] + 2*(c^2*d + e)*Ellip
ticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d^2*(c^2*d + e)*
Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

3.159.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {5751, 27, 402, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx$$
$$\downarrow 5751$$
$$-\frac{bcx\int \frac{2ex^2+3d}{3d^2\sqrt{c^2x^2-1}(ex^2+d)^{3/2}}dx}{\sqrt{c^2x^2}}+\frac{2x\left(a+b\sec^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$
$$\downarrow 27$$
$$-\frac{bcx\int \frac{2ex^2+3d}{\sqrt{c^2x^2-1}(ex^2+d)^{3/2}}dx}{3d^2\sqrt{c^2x^2}}+\frac{2x\left(a+b\sec^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\downarrow 402$$

$$-\frac{bcx\left(\frac{\int \frac{d\left(ex^2c^2+3dc^2+2e\right)}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{d(c^2d+e)}-\frac{ex\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2x\left(a+b\sec^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\downarrow 27$$

$$-\frac{bcx\left(\frac{\int \frac{ex^2c^2+3dc^2+2e}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{c^2d+e}-\frac{ex\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2x\left(a+b\sec^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\downarrow 399$$

$$-\frac{bcx\left(\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx+2(c^2d+e)\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{c^2d+e}-\frac{ex\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2x\left(a+b\sec^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\downarrow 323$$

$$-\frac{bcx\left(\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx+\frac{2\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}dx}{\sqrt{d+ex^2}}}{c^2d+e}-\frac{ex\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2x\left(a+b\sec^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\downarrow 323$$

$$-\frac{bcx\left(\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx+\frac{2\sqrt{1-c^2x^2}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}}{c^2d+e}-\frac{ex\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2x\left(a+b\sec^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\downarrow 321$$

$$-\frac{bcx\left(\frac{c^2\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx+\frac{2\sqrt{1-c^2x^2}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{c\sqrt{c^2x^2-1}\sqrt{d+ex^2}}}{c^2d+e}-\frac{ex\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2\sqrt{c^2x^2}}+$$
$$\frac{2x\left(a+b\sec^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\downarrow 331$$

$$-\frac{bcx\left(\frac{\frac{c^2\sqrt{1-c^2x^2}\int\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{\sqrt{c^2x^2-1}}+\frac{2\sqrt{1-c^2x^2}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{c\sqrt{c^2x^2-1}\sqrt{d+ex^2}}}{c^2d+e}-\frac{ex\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2\sqrt{c^2x^2}}+$$
$$\frac{2x\left(a+b\sec^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\downarrow 330$$

$$-\frac{bcx\left(\frac{\frac{c^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-c^2x^2}}dx}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}+\frac{2\sqrt{1-c^2x^2}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{c\sqrt{c^2x^2-1}\sqrt{d+ex^2}}}{c^2d+e}-\frac{ex\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2\sqrt{c^2x^2}}+$$
$$\frac{2x\left(a+b\sec^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\downarrow 327$$

$$\frac{2x\left(a+b\sec^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sec^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-$$
$$\frac{bcx\left(\frac{2\sqrt{1-c^2x^2}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{c\sqrt{c^2x^2-1}\sqrt{d+ex^2}}+\frac{c\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\middle|-\frac{e}{c^2d}\right)}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}}{c^2d+e}-\frac{ex\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2\sqrt{c^2x^2}}$$

input
```
Int[(a + b*ArcSec[c*x])/(d + e*x^2)^(5/2),x]
```

output
```
(x*(a + b*ArcSec[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (2*x*(a + b*ArcSec[c*x])
)/(3*d^2*Sqrt[d + e*x^2]) - (b*c*x*(-((e*x*Sqrt[-1 + c^2*x^2])/((c^2*d + e
)*Sqrt[d + e*x^2])) + ((c*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcS
in[c*x], -(e/(c^2*d))])/(Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) + (2*(c^2
*d + e)*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(
c^2*d))])/(c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2]))/(c^2*d + e)))/(3*d^2*Sqr
t[c^2*x^2])
```

3.159.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 5751
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSec[c*x])   u,
x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 -
 1]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1
/2, 0])
```

3.159.4 Maple [F]

$$\int \frac{a + b \operatorname{arcsec}(cx)}{(e\,x^2 + d)^{\frac{5}{2}}} dx$$

input
```
int((a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

3.159.5 Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.18

$$\int \frac{a + b\sec^{-1}(cx)}{(d + ex^2)^{5/2}}\, dx = \frac{\left(2\left(ac^3d^2e + acde^2\right)x^3 + 3\left(ac^3d^3 + acd^2e\right)x + \left(2\left(bc^3d^2e + bcde^2\right)x^3 + 3\left(bc^3d^3 + bc\right.\right.\right.}{}$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
1/3*((2*(a*c^3*d^2*e + a*c*d*e^2)*x^3 + 3*(a*c^3*d^3 + a*c*d^2*e)*x + (2*(
b*c^3*d^2*e + b*c*d*e^2)*x^3 + 3*(b*c^3*d^3 + b*c*d^2*e)*x)*arcsec(c*x) +
(b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d) + ((b*c^4
*d*e^2*x^4 + 2*b*c^4*d^2*e*x^2 + b*c^4*d^3)*elliptic_e(arcsin(c*x), -e/(c^
2*d)) - (((b*c^4 - 3*b*c^2)*d*e^2 - 2*b*e^3)*x^4 + (b*c^4 - 3*b*c^2)*d^3 -
 2*b*d^2*e + 2*((b*c^4 - 3*b*c^2)*d^2*e - 2*b*d*e^2)*x^2)*elliptic_f(arcsi
n(c*x), -e/(c^2*d)))*sqrt(-d))/(c^3*d^6 + c*d^5*e + (c^3*d^4*e^2 + c*d^3*e
^3)*x^4 + 2*(c^3*d^5*e + c*d^4*e^2)*x^2)
```

3.159.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b \sec^{-1}(cx)}{(d + ex^2)^{5/2}}\, dx = \text{Timed out}$$

input
```
integrate((a+b*asec(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.159.7 Maxima [F]

$$\int \frac{a + b \sec^{-1}(cx)}{(d + ex^2)^{5/2}}\, dx = \int \frac{b \operatorname{arcsec}(cx) + a}{(ex^2 + d)^{\frac{5}{2}}}\, dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
1/3*a*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + b*integrate(
arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e*x^
2 + d)), x)
```

3.159.8 Giac [F]

$$\int \frac{a + b \sec^{-1}(cx)}{(d + ex^2)^{5/2}}\, dx = \int \frac{b \operatorname{arcsec}(cx) + a}{(ex^2 + d)^{\frac{5}{2}}}\, dx$$

input
```
integrate((a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/(e*x^2 + d)^(5/2), x)
```

3.159.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \sec^{-1}(cx)}{(d + ex^2)^{5/2}}\, dx = \int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{(e\,x^2 + d)^{5/2}}\, dx$$

input
```
int((a + b*acos(1/(c*x)))/(d + e*x^2)^(5/2),x)
```

output
```
int((a + b*acos(1/(c*x)))/(d + e*x^2)^(5/2), x)
```

3.160 $\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^{5/2}}\,dx$

3.160.1 Optimal result 1136
3.160.2 Mathematica [C] (verified) 1137
3.160.3 Rubi [A] (verified) 1138
3.160.4 Maple [F] 1140
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3.160.7 Maxima [F(-2)] 1141
3.160.8 Giac [F] 1141
3.160.9 Mupad [F(-1)] 1142

3.160.1 Optimal result

Integrand size = 23, antiderivative size = 631

$$\begin{aligned}
\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^{5/2}}\,dx &= -\frac{bce\sqrt{-1+c^2x^2}}{d^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} \\
&- \frac{4bce^2x^2\sqrt{-1+c^2x^2}}{3d^3\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^3\left(c^2d+e\right)\sqrt{c^2x^2}} \\
&- \frac{a+b\sec^{-1}(cx)}{dx\left(d+ex^2\right)^{3/2}} - \frac{4ex(a+b\sec^{-1}(cx))}{3d^2\left(d+ex^2\right)^{3/2}} - \frac{8ex(a+b\sec^{-1}(cx))}{3d^3\sqrt{d+ex^2}} \\
&+ \frac{4bc^2ex\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|\,-\frac{e}{c^2d}\right)}{3d^3\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&- \frac{bc^2(c^2d+2e)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|\,-\frac{e}{c^2d}\right)}{d^3\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&+ \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} \\
&+ \frac{8bex\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}$$

output
```
(-a-b*arcsec(c*x))/d/x/(e*x^2+d)^(3/2)-4/3*e*x*(a+b*arcsec(c*x))/d^2/(e*x^
2+d)^(3/2)-8/3*e*x*(a+b*arcsec(c*x))/d^3/(e*x^2+d)^(1/2)-b*c*e*(c^2*x^2-1)
^(1/2)/d^2/(c^2*d+e)/(c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)-4/3*b*c*e^2*x^2*(c^2*
x^2-1)^(1/2)/d^3/(c^2*d+e)/(c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)+b*c*(c^2*d+2*e)
*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^3/(c^2*d+e)/(c^2*x^2)^(1/2)+4/3*b*c^2
*e*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^
3/(c^2*d+e)/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)-b*c^2*(c^2
*d+2*e)*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/
2)/d^3/(c^2*d+e)/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)+b*c^2
*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^
2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)+8/3*b*e*x*EllipticF(c*
x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^3/(c^2*x^2)^(1/
2)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)
```

3.160.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.59 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.51

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^{5/2}}\,dx = \frac{-a(c^2d+e)\left(3d^2+12dex^2+8e^2x^4\right)+bc\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)\left(3c^2d(d+ex^2)+e(3\right.}{3d^3\left(c^2d+e\right)x\left(d+ex^2\right)^{3/2}}$$
$$-\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(3c^2d+2e)\,E\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)|-\frac{e}{c^2d}\right)-\left(3c^4d^2+11c^2de+8e^2\right)\text{EllipticF}\left(i\right.\right.}{3\sqrt{-c^2}d^3\left(c^2d+e\right)\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x^2*(d + e*x^2)^(5/2)),x]
```

output
```
(-(a*(c^2*d + e)*(3*d^2 + 12*d*e*x^2 + 8*e^2*x^4)) + b*c*Sqrt[1 - 1/(c^2*x
^2)]*x*(d + e*x^2)*(3*c^2*d*(d + e*x^2) + e*(3*d + 2*e*x^2)) - b*(c^2*d +
e)*(3*d^2 + 12*d*e*x^2 + 8*e^2*x^4)*ArcSec[c*x])/(3*d^3*(c^2*d + e)*x*(d +
 e*x^2)^(3/2)) - ((I/3)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c
^2*d*(3*c^2*d + 2*e)*EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] - (3
*c^4*d^2 + 11*c^2*d*e + 8*e^2)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2
*d))]))/(Sqrt[-c^2]*d^3*(c^2*d + e)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

3.160.3 Rubi [A] (verified)

Time = 1.67 (sec) , antiderivative size = 540, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5761, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^{5/2}}\,dx$$

$$\downarrow 5761$$

$$-\frac{bcx\int -\frac{8e^2x^4+12dex^2+3d^2}{3d^3x^2\sqrt{c^2x^2-1}\left(ex^2+d\right)^{3/2}}dx}{\sqrt{c^2x^2}}-\frac{8ex\left(a+b\sec^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}}-\frac{4ex\left(a+b\sec^{-1}(cx)\right)}{3d^2\left(d+ex^2\right)^{3/2}}-\frac{a+b\sec^{-1}(cx)}{dx\left(d+ex^2\right)^{3/2}}$$

$$\downarrow 27$$

$$\frac{bcx\int \frac{8e^2x^4+12dex^2+3d^2}{x^2\sqrt{c^2x^2-1}\left(ex^2+d\right)^{3/2}}dx}{3d^3\sqrt{c^2x^2}}-\frac{8ex\left(a+b\sec^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}}-\frac{4ex\left(a+b\sec^{-1}(cx)\right)}{3d^2\left(d+ex^2\right)^{3/2}}-\frac{a+b\sec^{-1}(cx)}{dx\left(d+ex^2\right)^{3/2}}$$

$$\downarrow 7293$$

$$\frac{bcx\int \left(\frac{3d^2}{x^2\sqrt{c^2x^2-1}\left(ex^2+d\right)^{3/2}}+\frac{12ed}{\sqrt{c^2x^2-1}\left(ex^2+d\right)^{3/2}}+\frac{8e^2x^2}{\sqrt{c^2x^2-1}\left(ex^2+d\right)^{3/2}}\right)dx}{3d^3\sqrt{c^2x^2}}-\frac{8ex\left(a+b\sec^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}}-\frac{4ex\left(a+b\sec^{-1}(cx)\right)}{3d^2\left(d+ex^2\right)^{3/2}}-\frac{a+b\sec^{-1}(cx)}{dx\left(d+ex^2\right)^{3/2}}$$

$$\downarrow 2009$$

$$-\frac{8ex\left(a+b\sec^{-1}(cx)\right)}{3d^3\sqrt{d+ex^2}}-\frac{4ex\left(a+b\sec^{-1}(cx)\right)}{3d^2\left(d+ex^2\right)^{3/2}}-\frac{a+b\sec^{-1}(cx)}{dx\left(d+ex^2\right)^{3/2}}+$$

$$\frac{bcx\left(\frac{8e\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{c\sqrt{c^2x^2-1}\sqrt{d+ex^2}}+\frac{3cd\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}+\frac{4ce\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsi\right.}{\sqrt{c^2x^2-1}(c^2d+e)\sqrt{}}\right.}{3d^3}$$

input
```
Int[(a + b*ArcSec[c*x])/(x^2*(d + e*x^2)^(5/2)),x]
```

output
```
-((a + b*ArcSec[c*x])/(d*x*(d + e*x^2)^(3/2))) - (4*e*x*(a + b*ArcSec[c*x]
))/(3*d^2*(d + e*x^2)^(3/2)) - (8*e*x*(a + b*ArcSec[c*x]))/(3*d^3*Sqrt[d +
 e*x^2]) + (b*c*x*((-3*d*e*Sqrt[-1 + c^2*x^2])/((c^2*d + e)*x*Sqrt[d + e*x
^2]) - (4*e^2*x*Sqrt[-1 + c^2*x^2])/((c^2*d + e)*Sqrt[d + e*x^2]) + (3*(c^
2*d + 2*e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/((c^2*d + e)*x) + (4*c*e*Sq
rt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/((c^
2*d + e)*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) - (3*c*(c^2*d + 2*e)*Sqrt
[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/((c^2*
d + e)*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) + (3*c*d*Sqrt[1 - c^2*x^2]*
Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(Sqrt[-1 + c^2*x
^2]*Sqrt[d + e*x^2]) + (8*e*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*Elliptic
F[ArcSin[c*x], -(e/(c^2*d))])/(c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])))/(3*
d^3*Sqrt[c^2*x^2])
```

3.160.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.160.4 Maple [F]

$$\int \frac{a + b \operatorname{arcsec}(cx)}{x^2 \left(e x^2 + d\right)^{\frac{5}{2}}} dx$$

input
```
int((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(5/2),x)
```

3.160.5 Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 536, normalized size of antiderivative = 0.85

$$\int \frac{a + b \sec^{-1}(cx)}{x^2 \left(d + e x^2\right)^{5/2}} dx =$$
$$-\frac{\left(3\,ac^3d^4 + 3\,acd^3e + 8\left(ac^3d^2e^2 + acde^3\right)x^4 + 12\left(ac^3d^3e + acd^2e^2\right)x^2 + \left(3\,bc^3d^4 + 3\,bcd^3e + 8\left(bc^3d^2e^2 + b\right.\right.\right.\right.}{}$$

input
```
integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*((3*a*c^3*d^4 + 3*a*c*d^3*e + 8*(a*c^3*d^2*e^2 + a*c*d*e^3)*x^4 + 12*
(a*c^3*d^3*e + a*c*d^2*e^2)*x^2 + (3*b*c^3*d^4 + 3*b*c*d^3*e + 8*(b*c^3*d^
2*e^2 + b*c*d*e^3)*x^4 + 12*(b*c^3*d^3*e + b*c*d^2*e^2)*x^2)*arcsec(c*x) -
 (3*b*c^3*d^4 + 3*b*c*d^3*e + (3*b*c^3*d^2*e^2 + 2*b*c*d*e^3)*x^4 + (6*b*c
^3*d^3*e + 5*b*c*d^2*e^2)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d) - (((3*b
*c^6*d^2*e^2 + 2*b*c^4*d*e^3)*x^5 + 2*(3*b*c^6*d^3*e + 2*b*c^4*d^2*e^2)*x^
3 + (3*b*c^6*d^4 + 2*b*c^4*d^3*e)*x)*elliptic_e(arcsin(c*x), -e/(c^2*d)) -
 ((3*b*c^6*d^2*e^2 + (2*b*c^4 + 9*b*c^2)*d*e^3 + 8*b*e^4)*x^5 + 2*(3*b*c^6
*d^3*e + (2*b*c^4 + 9*b*c^2)*d^2*e^2 + 8*b*d*e^3)*x^3 + (3*b*c^6*d^4 + (2*
b*c^4 + 9*b*c^2)*d^3*e + 8*b*d^2*e^2)*x)*elliptic_f(arcsin(c*x), -e/(c^2*d
)))*sqrt(-d))/((c^3*d^5*e^2 + c*d^4*e^3)*x^5 + 2*(c^3*d^6*e + c*d^5*e^2)*x
^3 + (c^3*d^7 + c*d^6*e)*x)
```

3.160.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b \sec^{-1}(cx)}{x^2 \left(d + ex^2\right)^{5/2}} \, dx = \text{Timed out}$$

input
```
integrate((a+b*asec(c*x))/x**2/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.160.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b \sec^{-1}(cx)}{x^2 \left(d + ex^2\right)^{5/2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.160.8 Giac [F]

$$\int \frac{a + b \sec^{-1}(cx)}{x^2 \left(d + ex^2\right)^{5/2}} \, dx = \int \frac{b \operatorname{arcsec}(cx) + a}{\left(ex^2 + d\right)^{\frac{5}{2}} x^2} \, dx$$

input
```
integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/((e*x^2 + d)^(5/2)*x^2), x)
```

3.160.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \sec^{-1}(cx)}{x^2 \left(d + ex^2\right)^{5/2}} \, dx = \int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{x^2 \left(e\,x^2 + d\right)^{5/2}} \, dx$$

input
```
int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)^(5/2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)^(5/2)), x)
```

3.161 $\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\sec^{-1}(cx)\right)\,dx$

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3.161.1 Optimal result

Integrand size = 23, antiderivative size = 589

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\sec^{-1}(cx)\right)\,dx =$$

$$-\frac{be\left(e^2(15+8m+m^2)^2+3c^2de(3+m)^2\left(42+13m+m^2\right)+3c^4d^2(840+638m+179m^2+22m^3+m^4\right.}{c^5f(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)\sqrt{c^2x^2}}$$

$$-\frac{be^2(e(5+m)^2+3c^2d(42+13m+m^2))\,x(fx)^{3+m}\sqrt{-1+c^2x^2}}{c^3f^3(4+m)(5+m)(6+m)(7+m)\sqrt{c^2x^2}}$$

$$-\frac{be^3x(fx)^{5+m}\sqrt{-1+c^2x^2}}{cf^5(6+m)(7+m)\sqrt{c^2x^2}}+\frac{d^3(fx)^{1+m}\left(a+b\sec^{-1}(cx)\right)}{f(1+m)}$$

$$+\frac{3d^2e(fx)^{3+m}\left(a+b\sec^{-1}(cx)\right)}{f^3(3+m)}$$

$$+\frac{3de^2(fx)^{5+m}\left(a+b\sec^{-1}(cx)\right)}{f^5(5+m)}+\frac{e^3(fx)^{7+m}\left(a+b\sec^{-1}(cx)\right)}{f^7(7+m)}$$

$$-\frac{b\left(\frac{c^6d^3(2+m)(4+m)(6+m)}{1+m}+\frac{e(1+m)\left(e^2\left(15+8m+m^2\right)^2+3c^2de(3+m)^2\left(42+13m+m^2\right)+3c^4d^2\left(840+638m+179m^2+22m^3+m^4\right)\right)}{(3+m)(5+m)(7+m)}\right)x}{c^5f(1+m)(2+m)(4+m)(6+m)\sqrt{c^2x^2}\sqrt{-1+}}$$

output
```
d^3*(f*x)^(1+m)*(a+b*arcsec(c*x))/f/(1+m)+3*d^2*e*(f*x)^(3+m)*(a+b*arcsec(
c*x))/f^3/(3+m)+3*d*e^2*(f*x)^(5+m)*(a+b*arcsec(c*x))/f^5/(5+m)+e^3*(f*x)^
(7+m)*(a+b*arcsec(c*x))/f^7/(7+m)-b*(c^6*d^3*(2+m)*(4+m)*(6+m)/(1+m)+e*(1+
m)*(e^2*(m^2+8*m+15)^2+3*c^2*d*e*(3+m)^2*(m^2+13*m+42)+3*c^4*d^2*(m^4+22*m
^3+179*m^2+638*m+840))/(m^3+15*m^2+71*m+105))*x*(f*x)^(1+m)*hypergeom([1/2
, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/c^5/f/(1+m)/(2+m)/(4+
m)/(6+m)/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)-b*e*(e^2*(m^2+8*m+15)^2+3*c^2*d
*e*(3+m)^2*(m^2+13*m+42)+3*c^4*d^2*(m^4+22*m^3+179*m^2+638*m+840))*x*(f*x)
^(1+m)*(c^2*x^2-1)^(1/2)/c^5/f/(6+m)/(m^2+6*m+8)/(m^3+15*m^2+71*m+105)/(c^
2*x^2)^(1/2)-b*e^2*(e*(5+m)^2+3*c^2*d*(m^2+13*m+42))*x*(f*x)^(3+m)*(c^2*x^
2-1)^(1/2)/c^3/f^3/(4+m)/(5+m)/(6+m)/(7+m)/(c^2*x^2)^(1/2)-b*e^3*x*(f*x)^(
5+m)*(c^2*x^2-1)^(1/2)/c/f^5/(6+m)/(7+m)/(c^2*x^2)^(1/2)
```

3.161.2 Mathematica [A] (verified)

Time = 3.01 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.68

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\sec^{-1}(cx)\right)\,dx$$

$$= x(fx)^m \left(\frac{ad^3}{1+m} + \frac{3ad^2ex^2}{3+m} + \frac{3ade^2x^4}{5+m} + \frac{ae^3x^6}{7+m} + \frac{bd^3\sec^{-1}(cx)}{1+m} + \frac{3bd^2ex^2\sec^{-1}(cx)}{3+m} + \frac{3bde^2x^4\sec^{-1}(cx)}{5+m} + \frac{be^3x^6\sec^{-1}(cx)}{7+m} + \frac{bcd^3\sqrt{1-\frac{1}{c^2x^2}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{(1+m)^2\sqrt{1-c^2x^2}} + \frac{3bcd^2e\sqrt{1-\frac{1}{c^2x^2}}x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2x^2\right)}{(3+m)^2\sqrt{1-c^2x^2}} + \frac{3bcde^2\sqrt{1-\frac{1}{c^2x^2}}x^5\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5+m}{2},\frac{7+m}{2},c^2x^2\right)}{(5+m)^2\sqrt{1-c^2x^2}} + \frac{bce^3\sqrt{1-\frac{1}{c^2x^2}}x^7\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7+m}{2},\frac{9+m}{2},c^2x^2\right)}{(7+m)^2\sqrt{1-c^2x^2}} \right)$$

input
```
Integrate[(f*x)^m*(d + e*x^2)^3*(a + b*ArcSec[c*x]),x]
```

output

```
x*(f*x)^m*((a*d^3)/(1 + m) + (3*a*d^2*e*x^2)/(3 + m) + (3*a*d*e^2*x^4)/(5
+ m) + (a*e^3*x^6)/(7 + m) + (b*d^3*ArcSec[c*x])/(1 + m) + (3*b*d^2*e*x^2*
ArcSec[c*x])/(3 + m) + (3*b*d*e^2*x^4*ArcSec[c*x])/(5 + m) + (b*e^3*x^6*Ar
cSec[c*x])/(7 + m) + (b*c*d^3*Sqrt[1 - 1/(c^2*x^2)]*x*Hypergeometric2F1[1/
2, (1 + m)/2, (3 + m)/2, c^2*x^2])/((1 + m)^2*Sqrt[1 - c^2*x^2]) + (3*b*c*
d^2*e*Sqrt[1 - 1/(c^2*x^2)]*x^3*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/
2, c^2*x^2])/((3 + m)^2*Sqrt[1 - c^2*x^2]) + (3*b*c*d*e^2*Sqrt[1 - 1/(c^2*
x^2)]*x^5*Hypergeometric2F1[1/2, (5 + m)/2, (7 + m)/2, c^2*x^2])/((5 + m)^
2*Sqrt[1 - c^2*x^2]) + (b*c*e^3*Sqrt[1 - 1/(c^2*x^2)]*x^7*Hypergeometric2F
1[1/2, (7 + m)/2, (9 + m)/2, c^2*x^2])/((7 + m)^2*Sqrt[1 - c^2*x^2]))
```

3.161.3 Rubi [A] (verified)

Time = 2.15 (sec) , antiderivative size = 542, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5761, 2340, 1590, 363, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^2)^3 (fx)^m \left(a+b\sec^{-1}(cx)\right)\, dx$$

$$\downarrow 5761$$

$$-\frac{bcx\int \frac{(fx)^m\left(\frac{e^3x^6}{m+7}+\frac{3de^2x^4}{m+5}+\frac{3d^2ex^2}{m+3}+\frac{d^3}{m+1}\right)}{\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{d^3(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\sec^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\sec^{-1}(cx)\right)}{f^7(m+7)}$$

$$\downarrow 2340$$

$$-\frac{bcx\left(\frac{\int \frac{(fx)^m\left(\frac{e^2\left(3d\left(m^2+13m+42\right)c^2+e(m+5)^2\right)x^4}{(m+5)(m+7)}+\frac{3c^2d^2e(m+6)x^2}{m+3}+\frac{c^2d^3(m+6)}{m+1}\right)}{\sqrt{c^2x^2-1}}dx}{c^2(m+6)}+\frac{e^3\sqrt{c^2x^2-1}(fx)^{m+5}}{c^2f^5(m+6)(m+7)}\right)}{\sqrt{c^2x^2}}+\frac{d^3(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\sec^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\sec^{-1}(cx)\right)}{f^7(m+7)}$$

$$\downarrow 1590$$

$$bcx\left(\frac{\int\frac{(fx)^m\left(\frac{d^3(m+4)(m+6)c^4}{m+1}+\frac{e\left(3d^2\left(m^4+22m^3+179m^2+638m+840\right)c^4+3de(m+3)^2\left(m^2+13m+42\right)c^2+e^2\left(m^2+8m+15\right)^2\right)x^2}{(m+3)(m+5)(m+7)}\right)}{\sqrt{c^2x^2-1}}dx}{c^2(m+4)}+\frac{e^2\sqrt{c^2x^2-1}(f\right.$$

$$\left.-\frac{}{c^2(m+6)}\right)$$

$$\frac{d^3(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\sec^{-1}(cx)\right)}{f^5(m+5)}+$$

$$\frac{\sqrt{c^2x^2}}{}$$

$$\frac{e^3(fx)^{m+7}\left(a+b\sec^{-1}(cx)\right)}{f^7(m+7)}$$

$$\downarrow 363$$

$$bcx\left(\frac{\left(\frac{c^4d^3(m+4)(m+6)}{m+1}+\frac{e(m+1)\left(3c^4d^2\left(m^4+22m^3+179m^2+638m+840\right)+3c^2de(m+3)^2\left(m^2+13m+42\right)+e^2\left(m^2+8m+15\right)^2\right)}{c^2(m+2)(m+3)(m+5)(m+7)}\right)\int\frac{(fx)^m}{\sqrt{c^2x^2-1}}dx+\frac{e\sqrt{c^2x^2}}{}\right.$$

$$\left.-\frac{}{c^2(m+4)}\right)$$

$$\frac{d^3(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\sec^{-1}(cx)\right)}{f^5(m+5)}+$$

$$\frac{e^3(fx)^{m+7}\left(a+b\sec^{-1}(cx)\right)}{f^7(m+7)}$$

$$\downarrow 279$$

$$bcx\left(\frac{\sqrt{1-c^2x^2}\left(\frac{c^4d^3(m+4)(m+6)}{m+1}+\frac{e(m+1)\left(3c^4d^2\left(m^4+22m^3+179m^2+638m+840\right)+3c^2de(m+3)^2\left(m^2+13m+42\right)+e^2\left(m^2+8m+15\right)^2\right)}{c^2(m+2)(m+3)(m+5)(m+7)}\right)}{\sqrt{c^2x^2-1}}\right.$$

$$\left.\int\frac{(fx)^m}{\sqrt{1-c^2x^2}}d\right.$$

$$\left.-\frac{}{c^2(m+4)}\right)$$

$$\frac{d^3(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\sec^{-1}(cx)\right)}{f^5(m+5)}+$$

$$\frac{e^3(fx)^{m+7}\left(a+b\sec^{-1}(cx)\right)}{f^7(m+7)}$$

$$\downarrow 278$$

3.161. $\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\sec^{-1}(cx)\right)\,dx$

$$\frac{d^3(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\sec^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\sec^{-1}(cx)\right)}{f^7(m+7)}-$$

$$bcx\left(\frac{e^3\sqrt{c^2x^2-1}(fx)^{m+5}}{c^2f^5(m+6)(m+7)}+\frac{\frac{e^2\sqrt{c^2x^2-1}(fx)^{m+3}\left(3c^2d\left(m^2+13m+42\right)+e(m+5)^2\right)}{c^2f^3(m+4)(m+5)(m+7)}+\frac{\frac{e\sqrt{c^2x^2-1}(fx)^{m+1}\left(3c^4d^2\left(m^4+22m^3+179m^2+638m+840\right)+3c^2\cdots\right.}{c^2f(m+2)(m+3)(m+5)(\cdots}}{\cdots}\right.$$

input
```
Int[(f*x)^m*(d + e*x^2)^3*(a + b*ArcSec[c*x]),x]
```

output
```
(d^3*(f*x)^(1 + m)*(a + b*ArcSec[c*x]))/(f*(1 + m)) + (3*d^2*e*(f*x)^(3 +
m)*(a + b*ArcSec[c*x]))/(f^3*(3 + m)) + (3*d*e^2*(f*x)^(5 + m)*(a + b*ArcS
ec[c*x]))/(f^5*(5 + m)) + (e^3*(f*x)^(7 + m)*(a + b*ArcSec[c*x]))/(f^7*(7
+ m)) - (b*c*x*((e^3*(f*x)^(5 + m)*Sqrt[-1 + c^2*x^2])/(c^2*f^5*(6 + m)*(7
 + m)) + ((e^2*(e*(5 + m)^2 + 3*c^2*d*(42 + 13*m + m^2))*(f*x)^(3 + m)*Sqr
t[-1 + c^2*x^2])/(c^2*f^3*(4 + m)*(5 + m)*(7 + m)) + ((e*(e^2*(15 + 8*m +
m^2)^2 + 3*c^2*d*e*(3 + m)^2*(42 + 13*m + m^2) + 3*c^4*d^2*(840 + 638*m +
179*m^2 + 22*m^3 + m^4))*(f*x)^(1 + m)*Sqrt[-1 + c^2*x^2])/(c^2*f*(2 + m)*
(3 + m)*(5 + m)*(7 + m)) + (((c^4*d^3*(4 + m)*(6 + m))/(1 + m) + (e*(1 + m
)*(e^2*(15 + 8*m + m^2)^2 + 3*c^2*d*e*(3 + m)^2*(42 + 13*m + m^2) + 3*c^4*
d^2*(840 + 638*m + 179*m^2 + 22*m^3 + m^4)))/(c^2*(2 + m)*(3 + m)*(5 + m)*
(7 + m)))*(f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2
, (3 + m)/2, c^2*x^2])/(f*(1 + m)*Sqrt[-1 + c^2*x^2]))/(c^2*(4 + m)))/(c^2
*(6 + m))))/Sqrt[c^2*x^2]
```

3.161. $\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\sec^{-1}(cx)\right) dx$

3.161.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.161.4 Maple [F]

$$\int (fx)^m \left(e\,x^2+d\right)^3 (a+b\,\operatorname{arcsec}(cx))\,dx$$

input
```
int((f*x)^m*(e*x^2+d)^3*(a+b*arcsec(c*x)),x)
```

output
```
int((f*x)^m*(e*x^2+d)^3*(a+b*arcsec(c*x)),x)
```

3.161.5 Fricas [F]

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\sec^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)^3 (b\,\operatorname{arcsec}(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^3*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arcsec(c*x))*(f*x)^m, x)
```

3.161.6 Sympy [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\sec^{-1}(cx)\right)\,dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(e*x**2+d)**3*(a+b*asec(c*x)),x)
```

output
```
Timed out
```

3.161.7 Maxima [F]

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\sec^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)^3 (b\operatorname{arcsec}(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^3*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
a*e^3*f^m*x^7*x^m/(m + 7) + 3*a*d*e^2*f^m*x^5*x^m/(m + 5) + 3*a*d^2*e*f^m*
x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d^3/(f*(m + 1)) + (((b*e^3*f^m*m^3 + 9*b
*e^3*f^m*m^2 + 23*b*e^3*f^m*m + 15*b*e^3*f^m)*x^7 + 3*(b*d*e^2*f^m*m^3 + 1
1*b*d*e^2*f^m*m^2 + 31*b*d*e^2*f^m*m + 21*b*d*e^2*f^m)*x^5 + 3*(b*d^2*e*f^
m*m^3 + 13*b*d^2*e*f^m*m^2 + 47*b*d^2*e*f^m*m + 35*b*d^2*e*f^m)*x^3 + (b*d
^3*f^m*m^3 + 15*b*d^3*f^m*m^2 + 71*b*d^3*f^m*m + 105*b*d^3*f^m)*x)*x^m*arc
tan(sqrt(c*x + 1)*sqrt(c*x - 1)) - (m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*i
ntegrate(-(b*d^3*f^m*m^3 + 15*b*d^3*f^m*m^2 + (b*e^3*f^m*m^3 + 9*b*e^3*f^m
*m^2 + 23*b*e^3*f^m*m + 15*b*e^3*f^m)*x^6 + 71*b*d^3*f^m*m + 105*b*d^3*f^m
 + 3*(b*d*e^2*f^m*m^3 + 11*b*d*e^2*f^m*m^2 + 31*b*d*e^2*f^m*m + 21*b*d*e^2
*f^m)*x^4 + 3*(b*d^2*e*f^m*m^3 + 13*b*d^2*e*f^m*m^2 + 47*b*d^2*e*f^m*m + 3
5*b*d^2*e*f^m)*x^2)*sqrt(c*x + 1)*sqrt(c*x - 1)*x^m/(m^4 + 16*m^3 - (c^2*m
^4 + 16*c^2*m^3 + 86*c^2*m^2 + 176*c^2*m + 105*c^2)*x^2 + 86*m^2 + 176*m +
 105), x))/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

3.161.8 Giac [F]

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\sec^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)^3 (b\operatorname{arcsec}(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^3*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^3*(b*arcsec(c*x) + a)*(f*x)^m, x)
```

3.161.9 Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\sec^{-1}(cx)\right)\,dx = \int (f\,x)^m \left(e\,x^2+d\right)^3 \left(a+b\,\text{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int((f*x)^m*(d + e*x^2)^3*(a + b*acos(1/(c*x))),x)
```

output
```
int((f*x)^m*(d + e*x^2)^3*(a + b*acos(1/(c*x))), x)
```

3.162 $\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx$

3.162.1 Optimal result 1152
3.162.2 Mathematica [A] (verified) 1153
3.162.3 Rubi [A] (verified) 1153
3.162.4 Maple [F] 1156
3.162.5 Fricas [F] 1157
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3.162.8 Giac [F] 1158
3.162.9 Mupad [F(-1)] 1158

3.162.1 Optimal result

Integrand size = 23, antiderivative size = 374

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx$$
$$= -\frac{be(e(3+m)^2+2c^2d(20+9m+m^2))\,x(fx)^{1+m}\sqrt{-1+c^2x^2}}{c^3f(2+m)(3+m)(4+m)(5+m)\sqrt{c^2x^2}}$$
$$-\frac{be^2x(fx)^{3+m}\sqrt{-1+c^2x^2}}{cf^3(4+m)(5+m)\sqrt{c^2x^2}}+\frac{d^2(fx)^{1+m}\left(a+b\sec^{-1}(cx)\right)}{f(1+m)}$$
$$+\frac{2de(fx)^{3+m}\left(a+b\sec^{-1}(cx)\right)}{f^3(3+m)}+\frac{e^2(fx)^{5+m}\left(a+b\sec^{-1}(cx)\right)}{f^5(5+m)}$$
$$-\frac{b(c^4d^2(2+m)(3+m)(4+m)(5+m)+e(1+m)^2\left(e(3+m)^2+2c^2d(20+9m+m^2)\right))\,x(fx)^{1+m}\sqrt{1-}}{c^3f(1+m)^2(2+m)(3+m)(4+m)(5+m)\sqrt{c^2x^2}\sqrt{-1+c}}$$

output

```
d^2*(f*x)^(1+m)*(a+b*arcsec(c*x))/f/(1+m)+2*d*e*(f*x)^(3+m)*(a+b*arcsec(c*
x))/f^3/(3+m)+e^2*(f*x)^(5+m)*(a+b*arcsec(c*x))/f^5/(5+m)-b*(c^4*d^2*(2+m)
*(3+m)*(4+m)*(5+m)+e*(1+m)^2*(e*(3+m)^2+2*c^2*d*(m^2+9*m+20)))*x*(f*x)^(1+
m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/c^3/
f/(1+m)^2/(2+m)/(3+m)/(4+m)/(5+m)/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)-b*e*(e
*(3+m)^2+2*c^2*d*(m^2+9*m+20))*x*(f*x)^(1+m)*(c^2*x^2-1)^(1/2)/c^3/f/(4+m)
/(5+m)/(m^2+5*m+6)/(c^2*x^2)^(1/2)-b*e^2*x*(f*x)^(3+m)*(c^2*x^2-1)^(1/2)/c
/f^3/(4+m)/(5+m)/(c^2*x^2)^(1/2)
```

3.162.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.78

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx$$
$$= x(fx)^m \left(\frac{ad^2}{1+m} + \frac{2adex^2}{3+m} + \frac{ae^2x^4}{5+m} + \frac{bd^2\sec^{-1}(cx)}{1+m} + \frac{2bdex^2\sec^{-1}(cx)}{3+m} \right.$$
$$+ \frac{be^2x^4\sec^{-1}(cx)}{5+m} + \frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{(1+m)^2\sqrt{1-c^2x^2}}$$
$$+ \frac{2bcde\sqrt{1-\frac{1}{c^2x^2}}x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2x^2\right)}{(3+m)^2\sqrt{1-c^2x^2}}$$
$$\left. + \frac{bce^2\sqrt{1-\frac{1}{c^2x^2}}x^5\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5+m}{2},\frac{7+m}{2},c^2x^2\right)}{(5+m)^2\sqrt{1-c^2x^2}} \right)$$

input
```
Integrate[(f*x)^m*(d + e*x^2)^2*(a + b*ArcSec[c*x]),x]
```

output
```
x*(f*x)^m*((a*d^2)/(1 + m) + (2*a*d*e*x^2)/(3 + m) + (a*e^2*x^4)/(5 + m) +
 (b*d^2*ArcSec[c*x])/(1 + m) + (2*b*d*e*x^2*ArcSec[c*x])/(3 + m) + (b*e^2*
x^4*ArcSec[c*x])/(5 + m) + (b*c*d^2*Sqrt[1 - 1/(c^2*x^2)]*x*Hypergeometric
2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/((1 + m)^2*Sqrt[1 - c^2*x^2]) + (
2*b*c*d*e*Sqrt[1 - 1/(c^2*x^2)]*x^3*Hypergeometric2F1[1/2, (3 + m)/2, (5 +
 m)/2, c^2*x^2])/((3 + m)^2*Sqrt[1 - c^2*x^2]) + (b*c*e^2*Sqrt[1 - 1/(c^2*
x^2)]*x^5*Hypergeometric2F1[1/2, (5 + m)/2, (7 + m)/2, c^2*x^2])/((5 + m)^
2*Sqrt[1 - c^2*x^2]))
```

3.162.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5761, 27, 1590, 363, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^2 (fx)^m \left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\downarrow 5761$$

$$-\frac{bcx\int\frac{(fx)^m\left(e^2(m+1)(m+3)x^4+2de(m+1)(m+5)x^2+d^2(m+3)(m+5)\right)}{(m^3+9m^2+23m+15)\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{d^2(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\sec^{-1}(cx)\right)}{f^5(m+5)}$$

$$\downarrow 27$$

$$-\frac{bcx\int\frac{(fx)^m\left(e^2(m+1)(m+3)x^4+2de(m+1)(m+5)x^2+d^2(m+3)(m+5)\right)}{\sqrt{c^2x^2-1}}dx}{(m^3+9m^2+23m+15)\sqrt{c^2x^2}}+\frac{d^2(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\sec^{-1}(cx)\right)}{f^5(m+5)}$$

$$\downarrow 1590$$

$$-\frac{bcx\left(\frac{\int\frac{(fx)^m\left(c^2(m+3)(m+4)(m+5)d^2+e(m+1)\left(2d\left(m^2+9m+20\right)c^2+e(m+3)^2\right)x^2\right)}{\sqrt{c^2x^2-1}}dx}{c^2(m+4)}+\frac{e^2(m+1)(m+3)\sqrt{c^2x^2-1}(fx)^{m+3}}{c^2f^3(m+4)}\right)}{(m^3+9m^2+23m+15)\sqrt{c^2x^2}}+\frac{d^2(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\sec^{-1}(cx)\right)}{f^5(m+5)}$$

$$\downarrow 363$$

$$-\frac{bcx\left(\frac{\left(c^4d^2(m+3)(m+4)(m+5)+\frac{e(m+1)^2\left(2c^2d\left(m^2+9m+20\right)+e(m+3)^2\right)}{m+2}\right)\int\frac{(fx)^m}{\sqrt{c^2x^2-1}}dx}{c^2}+\frac{e(m+1)\sqrt{c^2x^2-1}(fx)^{m+1}\left(2c^2d\left(m^2+9m+20\right)+e(m+3)^2\right)}{c^2f(m+2)}\right)}{c^2(m+4)}}{(m^3+9m^2+23m+15)\sqrt{c^2x^2}}+\frac{d^2(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\sec^{-1}(cx)\right)}{f^5(m+5)}$$

$$\downarrow 279$$

$$-\frac{bcx\left(\frac{\sqrt{1-c^2x^2}\left(c^4d^2(m+3)(m+4)(m+5)+\frac{e(m+1)^2\left(2c^2d\left(m^2+9m+20\right)+e(m+3)^2\right)}{m+2}\right)\int\frac{(fx)^m}{\sqrt{1-c^2x^2}}dx}{c^2\sqrt{c^2x^2-1}}+\frac{e(m+1)\sqrt{c^2x^2-1}(fx)^{m+1}\left(2c^2d\left(m^2+9m+20\right)+e\right.}{c^2f(m+2)}\right)}{c^2(m+4)}}{(m^3+9m^2+23m+15)\sqrt{c^2x^2}}+\frac{d^2(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\sec^{-1}(cx)\right)}{f^5(m+5)}$$

$$\downarrow 278$$

$$\frac{d^2(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\sec^{-1}(cx)\right)}{f^5(m+5)}-$$

$$bcx\left(\frac{e^2(m+1)(m+3)\sqrt{c^2x^2-1}(fx)^{m+3}}{c^2f^3(m+4)}+\frac{\frac{e(m+1)\sqrt{c^2x^2-1}(fx)^{m+1}\left(2c^2d\left(m^2+9m+20\right)+e(m+3)^2\right)}{c^2f(m+2)}+\frac{\sqrt{1-c^2x^2}(fx)^{m+1}\left(c^4d^2(m+3)(m+4)(m+5)+\right.}{c^2(m+4)}}{(m^3+9m^2+23m+15)\sqrt{c^2x^2}}\right.$$

input
```
Int[(f*x)^m*(d + e*x^2)^2*(a + b*ArcSec[c*x]),x]
```

output
```
(d^2*(f*x)^(1 + m)*(a + b*ArcSec[c*x]))/(f*(1 + m)) + (2*d*e*(f*x)^(3 + m)
*(a + b*ArcSec[c*x]))/(f^3*(3 + m)) + (e^2*(f*x)^(5 + m)*(a + b*ArcSec[c*x
]))/(f^5*(5 + m)) - (b*c*x*((e^2*(1 + m)*(3 + m)*(f*x)^(3 + m)*Sqrt[-1 + c
^2*x^2])/(c^2*f^3*(4 + m)) + ((e*(1 + m)*(e*(3 + m)^2 + 2*c^2*d*(20 + 9*m
+ m^2))*(f*x)^(1 + m)*Sqrt[-1 + c^2*x^2])/(c^2*f*(2 + m)) + ((c^4*d^2*(3 +
 m)*(4 + m)*(5 + m) + (e*(1 + m)^2*(e*(3 + m)^2 + 2*c^2*d*(20 + 9*m + m^2)
))/(2 + m))*(f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)
/2, (3 + m)/2, c^2*x^2])/(c^2*f*(1 + m)*Sqrt[-1 + c^2*x^2]))/(c^2*(4 + m))
))/((15 + 23*m + 9*m^2 + m^3)*Sqrt[c^2*x^2])
```

3.162.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))  Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))  Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])  u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])  Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.162.4 Maple [F]

$$\int (fx)^m \left(e\,x^2 + d\right)^2 (a + b\,\operatorname{arcsec}(cx))\,dx$$

input
```
int((f*x)^m*(e*x^2+d)^2*(a+b*arcsec(c*x)),x)
```

output
```
int((f*x)^m*(e*x^2+d)^2*(a+b*arcsec(c*x)),x)
```

3.162.5 Fricas [F]

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\, dx = \int \left(ex^2+d\right)^2 (b\operatorname{arcsec}(cx)+a)(fx)^m\, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arcsec(c*x))*(f*x)^m, x)
```

3.162.6 Sympy [F]

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\, dx = \int (fx)^m \left(a+b\operatorname{asec}(cx)\right) \left(d+ex^2\right)^2\, dx$$

input
```
integrate((f*x)**m*(e*x**2+d)**2*(a+b*asec(c*x)),x)
```

output
```
Integral((f*x)**m*(a + b*asec(c*x))*(d + e*x**2)**2, x)
```

3.162.7 Maxima [F]

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\, dx = \int \left(ex^2+d\right)^2 (b\operatorname{arcsec}(cx)+a)(fx)^m\, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
a*e^2*f^m*x^5*x^m/(m + 5) + 2*a*d*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*
d^2/(f*(m + 1)) + (((b*e^2*f^m*m^2 + 4*b*e^2*f^m*m + 3*b*e^2*f^m)*x^5 + 2*
(b*d*e*f^m*m^2 + 6*b*d*e*f^m*m + 5*b*d*e*f^m)*x^3 + (b*d^2*f^m*m^2 + 8*b*d
^2*f^m*m + 15*b*d^2*f^m)*x)*x^m*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - (m^3
 + 9*m^2 + 23*m + 15)*integrate(-(b*d^2*f^m*m^2 + 8*b*d^2*f^m*m + (b*e^2*f
^m*m^2 + 4*b*e^2*f^m*m + 3*b*e^2*f^m)*x^4 + 15*b*d^2*f^m + 2*(b*d*e*f^m*m^
2 + 6*b*d*e*f^m*m + 5*b*d*e*f^m)*x^2)*sqrt(c*x + 1)*sqrt(c*x - 1)*x^m/(m^3
 - (c^2*m^3 + 9*c^2*m^2 + 23*c^2*m + 15*c^2)*x^2 + 9*m^2 + 23*m + 15), x))
/(m^3 + 9*m^2 + 23*m + 15)
```

3.162.8 Giac [F]

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)^2 (b\,\text{arcsec}\,(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arcsec(c*x) + a)*(f*x)^m, x)
```

3.162.9 Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx = \int (f\,x)^m \left(e\,x^2+d\right)^2 \left(a+b\,\text{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int((f*x)^m*(d + e*x^2)^2*(a + b*acos(1/(c*x))),x)
```

output
```
int((f*x)^m*(d + e*x^2)^2*(a + b*acos(1/(c*x))), x)
```

3.163 $\int (fx)^m \left(d+ex^2\right) \left(a+b\sec^{-1}(cx)\right)\, dx$

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3.163.1 Optimal result

Integrand size = 21, antiderivative size = 178

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\sec^{-1}(cx)\right)\, dx$$
$$= -\frac{bex^{2+m}\sqrt{-1+c^2x^2}}{c\left(6+5m+m^2\right)\sqrt{c^2x^2}} + \frac{dx^{1+m}(a+b\sec^{-1}(cx))}{1+m} + \frac{ex^{3+m}(a+b\sec^{-1}(cx))}{3+m}$$
$$+\frac{b(e(1+m)^2+c^2d(2+m)(3+m))\,x^{2+m}\sqrt{-1+c^2x^2}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{3+m}{2},c^2x^2\right)}{c(1+m)^2(2+m)(3+m)\sqrt{c^2x^2}}$$

output

```
d*x^(1+m)*(a+b*arcsec(c*x))/(1+m)+e*x^(3+m)*(a+b*arcsec(c*x))/(3+m)-b*e*x^
(2+m)*(c^2*x^2-1)^(1/2)/c/(m^2+5*m+6)/(c^2*x^2)^(1/2)+b*(e*(1+m)^2+c^2*d*(
2+m)*(3+m))*x^(2+m)*hypergeom([1, 1+1/2*m],[3/2+1/2*m],c^2*x^2)*(c^2*x^2-1
)^(1/2)/c/(1+m)^2/(2+m)/(3+m)/(c^2*x^2)^(1/2)
```

3.163.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.95

$$\int (fx)^m \left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$= x(fx)^m\left(\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{(1+m)^2\sqrt{1-c^2x^2}} + \frac{\frac{(3+m)\left(d(3+m)+e(1+m)x^2\right)\left(a+b\sec^{-1}(cx)\right)}{1+m} + \frac{bce\sqrt{1-\frac{1}{c^2x^2}}x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2x^2\right)}{\sqrt{1-c^2x^2}}}{(3+m)^2}\right)$$

input
```
Integrate[(f*x)^m*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

output
```
x*(f*x)^m*((b*c*d*Sqrt[1 - 1/(c^2*x^2)]*x*Hypergeometric2F1[1/2, (1 + m)/2
, (3 + m)/2, c^2*x^2])/((1 + m)^2*Sqrt[1 - c^2*x^2]) + (((3 + m)*(d*(3 + m
) + e*(1 + m)*x^2)*(a + b*ArcSec[c*x]))/(1 + m) + (b*c*e*Sqrt[1 - 1/(c^2*x
^2)]*x^3*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, c^2*x^2])/Sqrt[1 - c
^2*x^2])/(3 + m)^2)
```

3.163.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5761, 27, 363, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)(fx)^m\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\downarrow 5761$$

$$-\frac{bcx\int \frac{(fx)^m\left(e(m+1)x^2+d(m+3)\right)}{(m^2+4m+3)\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}} + \frac{d(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)} + \frac{e(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}$$

$$\downarrow 27$$

$$-\frac{bcx\int\frac{(fx)^m(e(m+1)x^2+d(m+3))}{\sqrt{c^2x^2-1}}dx}{(m^2+4m+3)\sqrt{c^2x^2}}+\frac{d(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}$$

$$\downarrow 363$$

$$-\frac{bcx\left(\left(\frac{e(m+1)^2}{c^2(m+2)}+d(m+3)\right)\int\frac{(fx)^m}{\sqrt{c^2x^2-1}}dx+\frac{e(m+1)\sqrt{c^2x^2-1}(fx)^{m+1}}{c^2f(m+2)}\right)}{(m^2+4m+3)\sqrt{c^2x^2}}+\frac{d(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}$$

$$\downarrow 279$$

$$-\frac{bcx\left(\frac{\sqrt{1-c^2x^2}\left(\frac{e(m+1)^2}{c^2(m+2)}+d(m+3)\right)\int\frac{(fx)^m}{\sqrt{1-c^2x^2}}dx}{\sqrt{c^2x^2-1}}+\frac{e(m+1)\sqrt{c^2x^2-1}(fx)^{m+1}}{c^2f(m+2)}\right)}{(m^2+4m+3)\sqrt{c^2x^2}}+\frac{d(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}$$

$$\downarrow 278$$

$$\frac{d(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)}-\frac{bcx\left(\frac{\sqrt{1-c^2x^2}(fx)^{m+1}\left(\frac{e(m+1)^2}{c^2(m+2)}+d(m+3)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)}{f(m+1)\sqrt{c^2x^2-1}}+\frac{e(m+1)\sqrt{c^2x^2-1}(fx)^{m+1}}{c^2f(m+2)}\right)}{(m^2+4m+3)\sqrt{c^2x^2}}$$

input
```
Int[(f*x)^m*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

output
```
(d*(f*x)^(1 + m)*(a + b*ArcSec[c*x]))/(f*(1 + m)) + (e*(f*x)^(3 + m)*(a +
b*ArcSec[c*x]))/(f^3*(3 + m)) - (b*c*x*((e*(1 + m)*(f*x)^(1 + m)*Sqrt[-1 +
 c^2*x^2])/(c^2*f*(2 + m)) + (((e*(1 + m)^2)/(c^2*(2 + m)) + d*(3 + m))*(f
*x)^(1 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2,
 c^2*x^2])/(f*(1 + m)*Sqrt[-1 + c^2*x^2])))/((3 + 4*m + m^2)*Sqrt[c^2*x^2]
)
```

3.163.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.163.4 Maple [F]

$$\int (fx)^m \left(e\,x^2 + d\right) (a + b\,\operatorname{arcsec}(cx))\,dx$$

input
```
int((f*x)^m*(e*x^2+d)*(a+b*arcsec(c*x)),x)
```

output
```
int((f*x)^m*(e*x^2+d)*(a+b*arcsec(c*x)),x)
```

3.163.5 Fricas [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\sec^{-1}(cx)\right)\, dx = \int \left(ex^2+d\right)(b\operatorname{arcsec}(cx)+a)(fx)^m\, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsec(c*x))*(f*x)^m, x)
```

3.163.6 Sympy [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\sec^{-1}(cx)\right)\, dx = \int (fx)^m \left(a+b\operatorname{asec}(cx)\right) \left(d+ex^2\right)\, dx$$

input
```
integrate((f*x)**m*(e*x**2+d)*(a+b*asec(c*x)),x)
```

output
```
Integral((f*x)**m*(a + b*asec(c*x))*(d + e*x**2), x)
```

3.163.7 Maxima [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\sec^{-1}(cx)\right)\, dx = \int \left(ex^2+d\right)(b\operatorname{arcsec}(cx)+a)(fx)^m\, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
a*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d/(f*(m + 1)) + (((b*e*f^m*m + b
*e*f^m)*x^3 + (b*d*f^m*m + 3*b*d*f^m)*x)*x^m*arctan(sqrt(c*x + 1)*sqrt(c*x
 - 1)) - (m^2 + 4*m + 3)*integrate((b*d*f^m*m + 3*b*d*f^m + (b*e*f^m*m + b
*e*f^m)*x^2)*sqrt(c*x + 1)*sqrt(c*x - 1)*x^m/((c^2*m^2 + 4*c^2*m + 3*c^2)*
x^2 - m^2 - 4*m - 3), x))/(m^2 + 4*m + 3)
```

3.163.8 Giac [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\sec^{-1}(cx)\right)\, dx = \int \left(ex^2+d\right)(b\operatorname{arcsec}(cx)+a)(fx)^m\, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arcsec(c*x) + a)*(f*x)^m, x)
```

3.163.9 Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\sec^{-1}(cx)\right)\, dx = \int (f\,x)^m \left(e\,x^2+d\right) \left(a+b\operatorname{acos}\left(\frac{1}{c\,x}\right)\right)\, dx$$

input
```
int((f*x)^m*(d + e*x^2)*(a + b*acos(1/(c*x))),x)
```

output
```
int((f*x)^m*(d + e*x^2)*(a + b*acos(1/(c*x))), x)
```

3.164 $\int \frac{(fx)^m(a+b\sec^{-1}(cx))}{d+ex^2}\,dx$

3.164.1 Optimal result 1165
3.164.2 Mathematica [N/A] 1165
3.164.3 Rubi [N/A] 1166
3.164.4 Maple [N/A] (verified) 1166
3.164.5 Fricas [N/A] 1167
3.164.6 Sympy [N/A] 1167
3.164.7 Maxima [N/A] 1167
3.164.8 Giac [N/A] 1168
3.164.9 Mupad [N/A] 1168

3.164.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{d+ex^2}\,dx = \text{Int}\left(\frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{d+ex^2}, x\right)$$

output
```
Unintegrable((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d),x)
```

3.164.2 Mathematica [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{d+ex^2}\,dx = \int \frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{d+ex^2}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcSec[c*x]))/(d + e*x^2),x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcSec[c*x]))/(d + e*x^2), x]
```

3.164.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m \left(a + b \sec^{-1}(cx)\right)}{d + ex^2} \, dx$$
$$\downarrow 5771$$
$$\int \frac{(fx)^m \left(a + b \sec^{-1}(cx)\right)}{d + ex^2} dx$$

input
```
Int[((f*x)^m*(a + b*ArcSec[c*x]))/(d + e*x^2),x]
```

output
```
$Aborted
```

3.164.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.164.4 Maple [N/A] (verified)

Not integrable

Time = 2.82 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a + b \operatorname{arcsec}(cx)\right)}{e\,x^2 + d} dx$$

input
```
int((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d),x)
```

output
```
int((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d),x)
```

3.164.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m \left(a + b \sec^{-1}(cx)\right)}{d + ex^2} \, dx = \int \frac{(b \operatorname{arcsec}(cx) + a)(fx)^m}{ex^2 + d} \, dx$$

input
```
integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arcsec(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

3.164.6 Sympy [N/A]

Not integrable

Time = 39.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(fx)^m \left(a + b \sec^{-1}(cx)\right)}{d + ex^2} \, dx = \int \frac{(fx)^m \left(a + b \operatorname{asec}(cx)\right)}{d + ex^2} \, dx$$

input
```
integrate((f*x)**m*(a+b*asec(c*x))/(e*x**2+d),x)
```

output
```
Integral((f*x)**m*(a + b*asec(c*x))/(d + e*x**2), x)
```

3.164.7 Maxima [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m \left(a + b \sec^{-1}(cx)\right)}{d + ex^2} \, dx = \int \frac{(b \operatorname{arcsec}(cx) + a)(fx)^m}{ex^2 + d} \, dx$$

input
```
integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
integrate((b*arcsec(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

3.164.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m \left(a+b\sec^{-1}(cx)\right)}{d+ex^2}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)(fx)^m}{ex^2+d}\,dx$$

input
```
integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

3.164.9 Mupad [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{(fx)^m \left(a+b\sec^{-1}(cx)\right)}{d+ex^2}\,dx = \int \frac{(f\,x)^m \left(a+b\operatorname{acos}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2+d}\,dx$$

input
```
int(((f*x)^m*(a + b*acos(1/(c*x))))/(d + e*x^2),x)
```

output
```
int(((f*x)^m*(a + b*acos(1/(c*x))))/(d + e*x^2), x)
```

3.165 $\int \frac{(fx)^m(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx$

3.165.1 Optimal result 1169
3.165.2 Mathematica [N/A] 1169
3.165.3 Rubi [N/A] 1170
3.165.4 Maple [N/A] (verified) 1170
3.165.5 Fricas [N/A] 1171
3.165.6 Sympy [F(-1)] 1171
3.165.7 Maxima [N/A] 1171
3.165.8 Giac [N/A] 1172
3.165.9 Mupad [N/A] 1172

3.165.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \text{Int}\left(\frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^2}, x\right)$$

output
```
Unintegrable((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^2,x)
```

3.165.2 Mathematica [N/A]

Not integrable

Time = 4.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int \frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcSec[c*x]))/(d + e*x^2)^2, x]
```

3.165.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m \left(a + b \sec^{-1}(cx)\right)}{\left(d + ex^2\right)^2} dx$$
$$\downarrow 5771$$
$$\int \frac{(fx)^m \left(a + b \sec^{-1}(cx)\right)}{\left(d + ex^2\right)^2} dx$$

input
```
Int[((f*x)^m*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

output
```
$Aborted
```

3.165.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrable[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.165.4 Maple [N/A] (verified)

Not integrable

Time = 2.53 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a + b \operatorname{arcsec}(cx)\right)}{\left(e x^2 + d\right)^2} dx$$

input
```
int((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^2,x)
```

output
```
int((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^2,x)
```

3.165.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(fx)^m \left(a + b \sec^{-1}(cx)\right)}{\left(d + ex^2\right)^2} \, dx = \int \frac{(b \operatorname{arcsec}(cx) + a)(fx)^m}{\left(ex^2 + d\right)^2} \, dx$$

input
```
integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arcsec(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.165.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m \left(a + b \sec^{-1}(cx)\right)}{\left(d + ex^2\right)^2} \, dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(a+b*asec(c*x))/(e*x**2+d)**2,x)
```

output
```
Timed out
```

3.165.7 Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m \left(a + b \sec^{-1}(cx)\right)}{\left(d + ex^2\right)^2} \, dx = \int \frac{(b \operatorname{arcsec}(cx) + a)(fx)^m}{\left(ex^2 + d\right)^2} \, dx$$

input
```
integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
integrate((b*arcsec(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

3.165.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m \left(a + b \sec^{-1}(cx)\right)}{\left(d + ex^2\right)^2} \, dx = \int \frac{(b \operatorname{arcsec}(cx) + a)(fx)^m}{\left(ex^2 + d\right)^2} \, dx$$

input
```
integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

3.165.9 Mupad [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{(fx)^m \left(a + b \sec^{-1}(cx)\right)}{\left(d + ex^2\right)^2} \, dx = \int \frac{(f\,x)^m \left(a + b \operatorname{acos}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2 + d\right)^2} \, dx$$

input
```
int(((f*x)^m*(a + b*acos(1/(c*x))))/(d + e*x^2)^2,x)
```

output
```
int(((f*x)^m*(a + b*acos(1/(c*x))))/(d + e*x^2)^2, x)
```

3.166 $\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)\, dx$

3.166.1 Optimal result 1173
3.166.2 Mathematica [N/A] 1173
3.166.3 Rubi [N/A] 1174
3.166.4 Maple [N/A] (verified) 1174
3.166.5 Fricas [N/A] 1175
3.166.6 Sympy [F(-1)] 1175
3.166.7 Maxima [N/A] 1175
3.166.8 Giac [N/A] 1176
3.166.9 Mupad [N/A] 1176

3.166.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)\, dx = \text{Int}\left((fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right), x\right)$$

output
```
Unintegrable((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

3.166.2 Mathematica [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)\, dx = \int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)\, dx$$

input
```
Integrate[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]),x]
```

output
```
Integrate[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]), x]
```

3.166.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^{3/2} (fx)^m \left(a+b\sec^{-1}(cx)\right)\, dx$$

$$\downarrow 5771$$

$$\int \left(d+ex^2\right)^{3/2} (fx)^m \left(a+b\sec^{-1}(cx)\right) dx$$

input
```
Int[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]),x]
```

output
```
$Aborted
```

3.166.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrable[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.166.4 Maple [N/A] (verified)

Not integrable

Time = 2.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (fx)^m \left(e\,x^2+d\right)^{\frac{3}{2}} \left(a+b\,\operatorname{arcsec}\left(cx\right)\right) dx$$

input
```
int((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

output
```
int((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

3.166.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.68

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)\, dx = \int \left(ex^2+d\right)^{\frac{3}{2}} (b\operatorname{arcsec}(cx)+a)(fx)^m\, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsec(c*x))*sqrt(e*x^2 + d)*(f*
x)^m, x)
```

3.166.6 Sympy [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)\, dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(e*x**2+d)**(3/2)*(a+b*asec(c*x)),x)
```

output
```
Timed out
```

3.166.7 Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)\, dx = \int \left(ex^2+d\right)^{\frac{3}{2}} (b\operatorname{arcsec}(cx)+a)(fx)^m\, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)*(f*x)^m, x)
```

3.166.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)^{\frac{3}{2}} (b\operatorname{arcsec}(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)*(f*x)^m, x)
```

3.166.9 Mupad [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)\,dx = \int (f\,x)^m \left(e\,x^2+d\right)^{3/2} \left(a+b\operatorname{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int((f*x)^m*(d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))),x)
```

output
```
int((f*x)^m*(d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))), x)
```

3.167 $\int (fx)^m \sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx$

3.167.1 Optimal result 1177
3.167.2 Mathematica [N/A] 1177
3.167.3 Rubi [N/A] 1178
3.167.4 Maple [N/A] (verified) 1178
3.167.5 Fricas [N/A] 1179
3.167.6 Sympy [N/A] 1179
3.167.7 Maxima [N/A] 1179
3.167.8 Giac [N/A] 1180
3.167.9 Mupad [N/A] 1180

3.167.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (fx)^m \sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx = \text{Int}\left((fx)^m \sqrt{d+ex^2}(a+b\sec^{-1}(cx)),x\right)$$

output
```
Unintegrable((f*x)^m*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x)
```

3.167.2 Mathematica [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (fx)^m \sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx = \int (fx)^m \sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx$$

input
```
Integrate[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
```

output
```
Integrate[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]), x]
```

3.167.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex^2}(fx)^m\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$\downarrow 5771$$

$$\int \sqrt{d+ex^2}(fx)^m\left(a+b\sec^{-1}(cx)\right)dx$$

input
```
Int[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
```

output
```
$Aborted
```

3.167.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrable[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.167.4 Maple [N/A] (verified)

Not integrable

Time = 0.90 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (fx)^m\sqrt{e\,x^2+d}\,(a+b\,\text{arcsec}\,(cx))\,dx$$

input
```
int((f*x)^m*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x)
```

output
```
int((f*x)^m*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x)
```

3.167.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m \sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\operatorname{arcsec}(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)*(f*x)^m, x)
```

3.167.6 Sympy [N/A]

Not integrable

Time = 52.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int (fx)^m \sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int (fx)^m\left(a+b\operatorname{asec}(cx)\right)\sqrt{d+ex^2}\,dx$$

input
```
integrate((f*x)**m*(e*x**2+d)**(1/2)*(a+b*asec(c*x)),x)
```

output
```
Integral((f*x)**m*(a + b*asec(c*x))*sqrt(d + e*x**2), x)
```

3.167.7 Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m \sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\operatorname{arcsec}(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)*(f*x)^m, x)
```

3.167.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m \sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\operatorname{arcsec}(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)*(f*x)^m, x)
```

3.167.9 Mupad [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int (fx)^m \sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int (f\,x)^m \sqrt{e\,x^2+d}\left(a+b\operatorname{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int((f*x)^m*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))),x)
```

output
```
int((f*x)^m*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))), x)
```

3.168 $\int \frac{(fx)^m(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$

3.168.1 Optimal result 1181
3.168.2 Mathematica [N/A] 1181
3.168.3 Rubi [N/A] 1182
3.168.4 Maple [N/A] (verified) 1182
3.168.5 Fricas [N/A] 1183
3.168.6 Sympy [N/A] 1183
3.168.7 Maxima [N/A] 1183
3.168.8 Giac [N/A] 1184
3.168.9 Mupad [N/A] 1184

3.168.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \text{Int}\left(\frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}},x\right)$$

output
```
Unintegrable((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

3.168.2 Mathematica [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \int \frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2], x]
```

3.168.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m \left(a + b \sec^{-1}(cx)\right)}{\sqrt{d + ex^2}}\, dx$$
$$\downarrow 5771$$
$$\int \frac{(fx)^m \left(a + b \sec^{-1}(cx)\right)}{\sqrt{d + ex^2}}\, dx$$

input
```
Int[((f*x)^m*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

3.168.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.168.4 Maple [N/A] (verified)

Not integrable

Time = 1.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(fx)^m \left(a + b \operatorname{arcsec}(cx)\right)}{\sqrt{e\,x^2 + d}}\, dx$$

input
```
int((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

3.168.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a + b \sec^{-1}(cx)\right)}{\sqrt{d + ex^2}}\, dx = \int \frac{(b \operatorname{arcsec}(cx) + a)(fx)^m}{\sqrt{ex^2 + d}}\, dx$$

input
```
integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*arcsec(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

3.168.6 Sympy [N/A]

Not integrable

Time = 22.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(fx)^m \left(a + b \sec^{-1}(cx)\right)}{\sqrt{d + ex^2}}\, dx = \int \frac{(fx)^m \left(a + b \operatorname{asec}(cx)\right)}{\sqrt{d + ex^2}}\, dx$$

input
```
integrate((f*x)**m*(a+b*asec(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral((f*x)**m*(a + b*asec(c*x))/sqrt(d + e*x**2), x)
```

3.168.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a + b \sec^{-1}(cx)\right)}{\sqrt{d + ex^2}}\, dx = \int \frac{(b \operatorname{arcsec}(cx) + a)(fx)^m}{\sqrt{ex^2 + d}}\, dx$$

input
```
integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsec(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

3.168.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a + b \sec^{-1}(cx)\right)}{\sqrt{d + ex^2}}\, dx = \int \frac{(b \operatorname{arcsec}(cx) + a)(fx)^m}{\sqrt{ex^2 + d}}\, dx$$

input
```
integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

3.168.9 Mupad [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{(fx)^m \left(a + b \sec^{-1}(cx)\right)}{\sqrt{d + ex^2}}\, dx = \int \frac{(f\,x)^m \left(a + b \operatorname{acos}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2 + d}}\, dx$$

input
```
int(((f*x)^m*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

output
```
int(((f*x)^m*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.169 $\int \frac{(fx)^m(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$

3.169.1 Optimal result 1185
3.169.2 Mathematica [N/A] 1185
3.169.3 Rubi [N/A] 1186
3.169.4 Maple [N/A] (verified) 1186
3.169.5 Fricas [N/A] 1187
3.169.6 Sympy [N/A] 1187
3.169.7 Maxima [N/A] 1187
3.169.8 Giac [N/A] 1188
3.169.9 Mupad [N/A] 1188

3.169.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \text{Int}\left(\frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}},x\right)$$

output
```
Unintegrable((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

3.169.2 Mathematica [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2), x]
```

3.169.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m \left(a + b \sec^{-1}(cx)\right)}{\left(d + ex^2\right)^{3/2}} dx$$
$$\downarrow 5771$$
$$\int \frac{(fx)^m \left(a + b \sec^{-1}(cx)\right)}{\left(d + ex^2\right)^{3/2}} dx$$

input
```
Int[((f*x)^m*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

3.169.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrable[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.169.4 Maple [N/A] (verified)

Not integrable

Time = 2.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(fx)^m \left(a + b \operatorname{arcsec}(cx)\right)}{\left(e x^2 + d\right)^{\frac{3}{2}}} dx$$

input
```
int((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

3.169.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.80

$$\int \frac{(fx)^m \left(a + b \sec^{-1}(cx)\right)}{\left(d + ex^2\right)^{3/2}} \, dx = \int \frac{(b \operatorname{arcsec}(cx) + a)(fx)^m}{(ex^2 + d)^{\frac{3}{2}}} \, dx$$

input
```
integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2
+ d^2), x)
```

3.169.6 Sympy [N/A]

Not integrable

Time = 140.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(fx)^m \left(a + b \sec^{-1}(cx)\right)}{\left(d + ex^2\right)^{3/2}} \, dx = \int \frac{(fx)^m \left(a + b \operatorname{asec}(cx)\right)}{(d + ex^2)^{\frac{3}{2}}} \, dx$$

input
```
integrate((f*x)**m*(a+b*asec(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral((f*x)**m*(a + b*asec(c*x))/(d + e*x**2)**(3/2), x)
```

3.169.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a + b \sec^{-1}(cx)\right)}{\left(d + ex^2\right)^{3/2}} \, dx = \int \frac{(b \operatorname{arcsec}(cx) + a)(fx)^m}{(ex^2 + d)^{\frac{3}{2}}} \, dx$$

input
```
integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arcsec(c*x) + a)*(f*x)^m/(e*x^2 + d)^(3/2), x)
```

3.169.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a + b \sec^{-1}(cx)\right)}{\left(d + ex^2\right)^{3/2}} \, dx = \int \frac{(b \operatorname{arcsec}(cx) + a)(fx)^m}{(ex^2 + d)^{\frac{3}{2}}} \, dx$$

input
```
integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*(f*x)^m/(e*x^2 + d)^(3/2), x)
```

3.169.9 Mupad [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{(fx)^m \left(a + b \sec^{-1}(cx)\right)}{\left(d + ex^2\right)^{3/2}} \, dx = \int \frac{(f\,x)^m \left(a + b \operatorname{acos}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2 + d\right)^{3/2}} \, dx$$

input
```
int(((f*x)^m*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

output
```
int(((f*x)^m*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.170 $\int \frac{x^{11}(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$

3.170.1 Optimal result . . . 1189
3.170.2 Mathematica [A] (verified) . . . 1190
3.170.3 Rubi [A] (verified) . . . 1190
3.170.4 Maple [F] . . . 1193
3.170.5 Fricas [A] (verification not implemented) . . . 1193
3.170.6 Sympy [F(-1)] . . . 1194
3.170.7 Maxima [F] . . . 1194
3.170.8 Giac [F(-2)] . . . 1194
3.170.9 Mupad [F(-1)] . . . 1195

3.170.1 Optimal result

Integrand size = 26, antiderivative size = 401

$$\begin{aligned}\int \frac{x^{11}(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx &= \frac{4b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{15c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{7b\sqrt{1-c^2x^2}(1+c^2x^2)^{3/2}}{90c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} \\ &+ \frac{13b\sqrt{1-c^2x^2}(1+c^2x^2)^{5/2}}{150c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{3b\sqrt{1-c^2x^2}(1+c^2x^2)^{7/2}}{70c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} \\ &+ \frac{b\sqrt{1-c^2x^2}(1+c^2x^2)^{9/2}}{90c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^{12}} \\ &+ \frac{(1-c^4x^4)^{3/2}(a+b\sec^{-1}(cx))}{3c^{12}} \\ &- \frac{(1-c^4x^4)^{5/2}(a+b\sec^{-1}(cx))}{10c^{12}} \\ &- \frac{4b\sqrt{1-c^2x^2}\operatorname{arctanh}\left(\sqrt{1+c^2x^2}\right)}{15c^{13}\sqrt{1-\frac{1}{c^2x^2}}x}\end{aligned}$$

output
```
1/3*(-c^4*x^4+1)^(3/2)*(a+b*arcsec(c*x))/c^12-1/10*(-c^4*x^4+1)^(5/2)*(a+b
*arcsec(c*x))/c^12-7/90*b*(c^2*x^2+1)^(3/2)*(-c^2*x^2+1)^(1/2)/c^13/x/(1-1
/c^2/x^2)^(1/2)+13/150*b*(c^2*x^2+1)^(5/2)*(-c^2*x^2+1)^(1/2)/c^13/x/(1-1/
c^2/x^2)^(1/2)-3/70*b*(c^2*x^2+1)^(7/2)*(-c^2*x^2+1)^(1/2)/c^13/x/(1-1/c^2
/x^2)^(1/2)+1/90*b*(c^2*x^2+1)^(9/2)*(-c^2*x^2+1)^(1/2)/c^13/x/(1-1/c^2/x^
2)^(1/2)-4/15*b*arctanh((c^2*x^2+1)^(1/2))*(-c^2*x^2+1)^(1/2)/c^13/x/(1-1/
c^2/x^2)^(1/2)+4/15*b*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/c^13/x/(1-1/c^2
/x^2)^(1/2)-1/2*(a+b*arcsec(c*x))*(-c^4*x^4+1)^(1/2)/c^12
```

3.170.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.48

$$\int \frac{x^{11}(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$$
$$= \frac{-105a\sqrt{1-c^4x^4}(8+4c^4x^4+3c^8x^8)+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1-c^4x^4}(768+36c^2x^2+78c^4x^4+5c^6x^6+35c^8x^8)}{-1+c^2x^2}-105b\sqrt{1-c^4x^4}(}{3150c^{12}}$$

input
```
Integrate[(x^11*(a + b*ArcSec[c*x]))/Sqrt[1 - c^4*x^4],x]
```

output
```
(-105*a*Sqrt[1 - c^4*x^4]*(8 + 4*c^4*x^4 + 3*c^8*x^8) + (b*c*Sqrt[1 - 1/(c
^2*x^2)]*x*Sqrt[1 - c^4*x^4]*(768 + 36*c^2*x^2 + 78*c^4*x^4 + 5*c^6*x^6 +
35*c^8*x^8))/(-1 + c^2*x^2) - 105*b*Sqrt[1 - c^4*x^4]*(8 + 4*c^4*x^4 + 3*c
^8*x^8)*ArcSec[c*x] + 840*b*ArcTan[(c*Sqrt[1 - 1/(c^2*x^2)]*x)/Sqrt[1 - c^
4*x^4]])/(3150*c^12)
```

3.170.3 Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.57, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {5769, 27, 7272, 1388, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$$

$$\downarrow 5769$$

$$-\frac{b\int -\frac{\sqrt{1-c^4x^4}\left(3c^8x^8+4c^4x^4+8\right)}{30c^{12}\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{c}-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^{12}}$$

$$\downarrow 27$$

$$\frac{b\int \frac{\sqrt{1-c^4x^4}\left(3c^8x^8+4c^4x^4+8\right)}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{30c^{13}}-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^{12}}$$

$$\downarrow 7272$$

$$\frac{b\sqrt{1-c^2x^2}\int \frac{\sqrt{1-c^4x^4}\left(3c^8x^8+4c^4x^4+8\right)}{x\sqrt{1-c^2x^2}}dx}{30c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^{12}}$$

$$\downarrow 1388$$

$$\frac{b\sqrt{1-c^2x^2}\int \frac{\sqrt{c^2x^2+1}\left(3c^8x^8+4c^4x^4+8\right)}{x}dx}{30c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^{12}}$$

$$\downarrow 2331$$

$$\frac{b\sqrt{1-c^2x^2}\int \frac{\sqrt{c^2x^2+1}\left(3c^8x^8+4c^4x^4+8\right)}{x^2}dx^2}{60c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^{12}}$$

$$\downarrow 2123$$

$$\frac{b\sqrt{1-c^2x^2}\int \left(3c^2\left(c^2x^2+1\right)^{7/2}-9c^2\left(c^2x^2+1\right)^{5/2}+13c^2\left(c^2x^2+1\right)^{3/2}-7c^2\sqrt{c^2x^2+1}+\frac{8\sqrt{c^2x^2+1}}{x^2}\right)dx^2}{60c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^{12}}$$

$$\downarrow 2009$$

$$-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^{12}}+$$
$$\frac{b\sqrt{1-c^2x^2}\left(-16\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{2}{3}\left(c^2x^2+1\right)^{9/2}-\frac{18}{7}\left(c^2x^2+1\right)^{7/2}+\frac{26}{5}\left(c^2x^2+1\right)^{5/2}-\frac{14}{3}\left(c^2x^2+1\right)^{3/2}+\right.}{60c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}$$

input
```
Int[(x^11*(a + b*ArcSec[c*x]))/Sqrt[1 - c^4*x^4],x]
```

output
```
-1/2*(Sqrt[1 - c^4*x^4]*(a + b*ArcSec[c*x]))/c^12 + ((1 - c^4*x^4)^(3/2)*(
a + b*ArcSec[c*x]))/(3*c^12) - ((1 - c^4*x^4)^(5/2)*(a + b*ArcSec[c*x]))/(
10*c^12) + (b*Sqrt[1 - c^2*x^2]*(16*Sqrt[1 + c^2*x^2] - (14*(1 + c^2*x^2)^
(3/2))/3 + (26*(1 + c^2*x^2)^(5/2))/5 - (18*(1 + c^2*x^2)^(7/2))/7 + (2*(1
 + c^2*x^2)^(9/2))/3 - 16*ArcTanh[Sqrt[1 + c^2*x^2]]))/(60*c^13*Sqrt[1 - 1
/(c^2*x^2)]*x)
```

3.170.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 5769
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Simp[(a + b*ArcSec[c*x])   v, x] - Simp[b/c   Int[SimplifyIntegran
d[v/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]]
/; FreeQ[{a, b, c}, x]
```

rule 7272
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
   Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !
IntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

3.170.4 Maple [F]

$$\int \frac{x^{11}(a+b\ \text{arcsec}\,(cx))}{\sqrt{-c^4x^4+1}}dx$$

input
```
int(x^11*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x)
```

output
```
int(x^11*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x)
```

3.170.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.59

$$\int \frac{x^{11}(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$$
$$= \frac{(35\,bc^8x^8+5\,bc^6x^6+78\,bc^4x^4+36\,bc^2x^2+768\,b)\sqrt{-c^4x^4+1}\sqrt{c^2x^2-1}-840\,(bc^2x^2-b)\arctan\left(\frac{\sqrt{-c^4x^4}}{\sqrt{c^2x^2-}}\right.}{}$$

input
```
integrate(x^11*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3150*((35*b*c^8*x^8 + 5*b*c^6*x^6 + 78*b*c^4*x^4 + 36*b*c^2*x^2 + 768*b)
*sqrt(-c^4*x^4 + 1)*sqrt(c^2*x^2 - 1) - 840*(b*c^2*x^2 - b)*arctan(sqrt(-c
^4*x^4 + 1)/sqrt(c^2*x^2 - 1)) - 105*(3*a*c^10*x^10 - 3*a*c^8*x^8 + 4*a*c^
6*x^6 - 4*a*c^4*x^4 + 8*a*c^2*x^2 + (3*b*c^10*x^10 - 3*b*c^8*x^8 + 4*b*c^6
*x^6 - 4*b*c^4*x^4 + 8*b*c^2*x^2 - 8*b)*arcsec(c*x) - 8*a)*sqrt(-c^4*x^4 +
 1))/(c^14*x^2 - c^12)
```

3.170.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \text{Timed out}$$

input
```
integrate(x**11*(a+b*asec(c*x))/(-c**4*x**4+1)**(1/2),x)
```

output
```
Timed out
```

3.170.7 Maxima [F]

$$\int \frac{x^{11}(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x^{11}}{\sqrt{-c^4x^4+1}}\,dx$$

input
```
integrate(x^11*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/30*a*(3*(-c^4*x^4 + 1)^(5/2)/c^12 - 10*(-c^4*x^4 + 1)^(3/2)/c^12 + 15*s
qrt(-c^4*x^4 + 1)/c^12) - 1/30*(30*c^12*integrate((30*sqrt(c*x + 1)*c^10*x
^11*log(c) + (3*c^8*x^9 + 4*c^6*x^7 + 3*(10*c^10*log(c) + c^10)*x^11 + 4*c
^4*x^5 + 8*c^2*x^3 + 8*x)*e^(3/2*log(c*x + 1) + log(c*x - 1)) + 30*(c^10*x
^11*e^(3/2*log(c*x + 1) + log(c*x - 1)) + sqrt(c*x + 1)*c^10*x^11)*log(x))
/((c^10*e^(2*log(c*x + 1) + log(c*x - 1) + 1/2*log(-c*x + 1)) + c^10*e^(lo
g(c*x + 1) + 1/2*log(-c*x + 1)))*sqrt(c^2*x^2 + 1)), x) + (3*c^8*x^8 + 4*c
^4*x^4 + 8)*sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan(sqrt(c*x
 + 1)*sqrt(c*x - 1)))*b/c^12
```

3.170.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^{11}(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^11*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.170.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11}(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \int \frac{x^{11}\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

input
```
int((x^11*(a + b*acos(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
```

output
```
int((x^11*(a + b*acos(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

3.171 $\int \frac{x^7\left(a+b\sec^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$

3.171.1 Optimal result 1196
3.171.2 Mathematica [A] (verified) 1197
3.171.3 Rubi [A] (warning: unable to verify) 1197
3.171.4 Maple [F] 1200
3.171.5 Fricas [A] (verification not implemented) 1200
3.171.6 Sympy [F(-1)] 1201
3.171.7 Maxima [F] 1201
3.171.8 Giac [F(-2)] 1201
3.171.9 Mupad [F(-1)] 1202

3.171.1 Optimal result

Integrand size = 26, antiderivative size = 268

$$\begin{aligned}\int \frac{x^7(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx &= \frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{3c^9\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{b\sqrt{1-c^2x^2}(1+c^2x^2)^{3/2}}{18c^9\sqrt{1-\frac{1}{c^2x^2}}x} \\ &+ \frac{b\sqrt{1-c^2x^2}(1+c^2x^2)^{5/2}}{30c^9\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^8} \\ &+ \frac{\left(1-c^4x^4\right)^{3/2}(a+b\sec^{-1}(cx))}{6c^8} \\ &- \frac{b\sqrt{1-c^2x^2}\operatorname{arctanh}\left(\sqrt{1+c^2x^2}\right)}{3c^9\sqrt{1-\frac{1}{c^2x^2}}x}\end{aligned}$$

output

```
1/6*(-c^4*x^4+1)^(3/2)*(a+b*arcsec(c*x))/c^8-1/18*b*(c^2*x^2+1)^(3/2)*(-c^
2*x^2+1)^(1/2)/c^9/x/(1-1/c^2/x^2)^(1/2)+1/30*b*(c^2*x^2+1)^(5/2)*(-c^2*x^
2+1)^(1/2)/c^9/x/(1-1/c^2/x^2)^(1/2)-1/3*b*arctanh((c^2*x^2+1)^(1/2))*(-c^
2*x^2+1)^(1/2)/c^9/x/(1-1/c^2/x^2)^(1/2)+1/3*b*(-c^2*x^2+1)^(1/2)*(c^2*x^2
+1)^(1/2)/c^9/x/(1-1/c^2/x^2)^(1/2)-1/2*(a+b*arcsec(c*x))*(-c^4*x^4+1)^(1/
2)/c^8
```

3.171.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.59

$$\int \frac{x^7(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$$

$$= \frac{-15a\sqrt{1-c^4x^4}(2+c^4x^4) + \frac{bc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1-c^4x^4}(28+c^2x^2+3c^4x^4)}{-1+c^2x^2} - 15b\sqrt{1-c^4x^4}(2+c^4x^4)\sec^{-1}(cx) + 30b\,a}{90c^8}$$

input
```
Integrate[(x^7*(a + b*ArcSec[c*x]))/Sqrt[1 - c^4*x^4],x]
```

output
```
(-15*a*Sqrt[1 - c^4*x^4]*(2 + c^4*x^4) + (b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt
[1 - c^4*x^4]*(28 + c^2*x^2 + 3*c^4*x^4))/(-1 + c^2*x^2) - 15*b*Sqrt[1 - c
^4*x^4]*(2 + c^4*x^4)*ArcSec[c*x] + 30*b*ArcTan[(c*Sqrt[1 - 1/(c^2*x^2)]*x
)/Sqrt[1 - c^4*x^4]])/(90*c^8)
```

3.171.3 Rubi [A] (warning: unable to verify)

Time = 1.23 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.58, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5769, 27, 7272, 1388, 1579, 517, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$$

$$\downarrow 5769$$

$$-\frac{b\int -\frac{\sqrt{1-c^4x^4}(c^4x^4+2)}{6c^8\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{c} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{6c^8} - \frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^8}$$

$$\downarrow 27$$

$$\frac{b\int \frac{\sqrt{1-c^4x^4}(c^4x^4+2)}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{6c^9} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{6c^8} - \frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^8}$$

$$\downarrow 7272$$

$$\frac{b\sqrt{1-c^2x^2}\int\frac{\sqrt{1-c^4x^4}(c^4x^4+2)}{x\sqrt{1-c^2x^2}}dx}{6c^9x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{6c^8}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^8}$$

$$\downarrow 1388$$

$$\frac{b\sqrt{1-c^2x^2}\int\frac{\sqrt{c^2x^2+1}(c^4x^4+2)}{x}dx}{6c^9x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{6c^8}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^8}$$

$$\downarrow 1579$$

$$\frac{b\sqrt{1-c^2x^2}\int\frac{\sqrt{c^2x^2+1}(c^4x^4+2)}{x^2}dx^2}{12c^9x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{6c^8}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^8}$$

$$\downarrow 517$$

$$\frac{b\sqrt{1-c^2x^2}\int-\frac{x^4\left(c^4x^8-2c^4x^4+3c^4\right)}{1-x^4}d\sqrt{c^2x^2+1}}{6c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{6c^8}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^8}$$

$$\downarrow 25$$

$$-\frac{b\sqrt{1-c^2x^2}\int\frac{x^4\left(c^4x^8-2c^4x^4+3c^4\right)}{1-x^4}d\sqrt{c^2x^2+1}}{6c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{6c^8}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^8}$$

$$\downarrow 1584$$

$$-\frac{b\sqrt{1-c^2x^2}\int\left(-c^4x^8+c^4x^4-2c^4+\frac{2c^4}{1-x^4}\right)d\sqrt{c^2x^2+1}}{6c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{6c^8}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^8}$$

$$\downarrow 2009$$

$$\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{6c^8}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^8}+\frac{b\sqrt{1-c^2x^2}\left(-2c^4\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{c^4x^{10}}{5}-\frac{c^4x^6}{3}+2c^4\sqrt{c^2x^2+1}\right)}{6c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}$$

input
```
Int[(x^7*(a + b*ArcSec[c*x]))/Sqrt[1 - c^4*x^4],x]
```

3.171. $\int\frac{x^7\left(a+b\sec^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}dx$

output
```
-1/2*(Sqrt[1 - c^4*x^4]*(a + b*ArcSec[c*x]))/c^8 + ((1 - c^4*x^4)^(3/2)*(a
 + b*ArcSec[c*x]))/(6*c^8) + (b*Sqrt[1 - c^2*x^2]*(-1/3*(c^4*x^6) + (c^4*x
^10)/5 + 2*c^4*Sqrt[1 + c^2*x^2] - 2*c^4*ArcTanh[Sqrt[1 + c^2*x^2]]))/(6*c
^13*Sqrt[1 - 1/(c^2*x^2)]*x)
```

3.171.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 517
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.),
 x_Symbol] :> Simp[2*(e^m/d^(m + 2*p + 1))   Subst[Int[x^(2*n + 1)*(-c + x^
2)^m*(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4)^p, x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && ILtQ[m, 0] && IntegerQ[n + 1/2]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5769
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Simp[(a + b*ArcSec[c*x])   v, x] - Simp[b/c   Int[SimplifyIntegran
d[v/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]]
/; FreeQ[{a, b, c}, x]
```

rule 7272
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
   Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !
IntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

3.171.4 Maple [F]

$$\int \frac{x^7(a+b\,\text{arcsec}\,(cx))}{\sqrt{-c^4x^4+1}}dx$$

input
```
int(x^7*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x)
```

output
```
int(x^7*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x)
```

3.171.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.68

$$\int \frac{x^7(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$$
$$= \frac{(3\,bc^4x^4+bc^2x^2+28\,b)\sqrt{-c^4x^4+1}\sqrt{c^2x^2-1}-30\,(bc^2x^2-b)\arctan\left(\frac{\sqrt{-c^4x^4+1}}{\sqrt{c^2x^2-1}}\right)-15\,(ac^6x^6-ac^4x^4+}{90\,(c^{10}x^2-c^8)}$$

input
```
integrate(x^7*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/90*((3*b*c^4*x^4 + b*c^2*x^2 + 28*b)*sqrt(-c^4*x^4 + 1)*sqrt(c^2*x^2 - 1
) - 30*(b*c^2*x^2 - b)*arctan(sqrt(-c^4*x^4 + 1)/sqrt(c^2*x^2 - 1)) - 15*(
a*c^6*x^6 - a*c^4*x^4 + 2*a*c^2*x^2 + (b*c^6*x^6 - b*c^4*x^4 + 2*b*c^2*x^2
 - 2*b)*arcsec(c*x) - 2*a)*sqrt(-c^4*x^4 + 1))/(c^10*x^2 - c^8)
```

3.171.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^7(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \text{Timed out}$$

input
```
integrate(x**7*(a+b*asec(c*x))/(-c**4*x**4+1)**(1/2),x)
```

output
```
Timed out
```

3.171.7 Maxima [F]

$$\int \frac{x^7(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \int \frac{(b\,\text{arcsec}\,(cx)+a)x^7}{\sqrt{-c^4x^4+1}}\,dx$$

input
```
integrate(x^7*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
1/6*a*((-c^4*x^4 + 1)^(3/2)/c^8 - 3*sqrt(-c^4*x^4 + 1)/c^8) - 1/6*(6*sqrt(
c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*c^8*integrate((6*sqrt(c*x + 1)*c
^6*x^7*log(c) + (c^4*x^5 + (6*c^6*log(c) + c^6)*x^7 + 2*c^2*x^3 + 2*x)*e^(
3/2*log(c*x + 1) + log(c*x - 1)) + 6*(c^6*x^7*e^(3/2*log(c*x + 1) + log(c*
x - 1)) + sqrt(c*x + 1)*c^6*x^7)*log(x))/((c^6*e^(2*log(c*x + 1) + log(c*x
 - 1) + 1/2*log(-c*x + 1)) + c^6*e^(log(c*x + 1) + 1/2*log(-c*x + 1)))*sqr
t(c^2*x^2 + 1)), x) - (c^8*x^8 + c^4*x^4 - 2)*arctan(sqrt(c*x + 1)*sqrt(c*
x - 1)))*b/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*c^8)
```

3.171.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^7(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^7*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.171.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \int \frac{x^7\left(a+b\operatorname{acos}\left(\frac{1}{cx}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

input
```
int((x^7*(a + b*acos(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
```

output
```
int((x^7*(a + b*acos(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

3.172 $\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$

3.172.1 Optimal result 1203
3.172.2 Mathematica [A] (verified) 1203
3.172.3 Rubi [A] (verified) 1204
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3.172.5 Fricas [A] (verification not implemented) 1207
3.172.6 Sympy [F] 1207
3.172.7 Maxima [F] 1208
3.172.8 Giac [F] 1208
3.172.9 Mupad [F(-1)] 1208

3.172.1 Optimal result

Integrand size = 26, antiderivative size = 126

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \frac{bx\sqrt{1-c^4x^4}}{2c^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}} - \frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^4} - \frac{bx\arctan\left(\frac{\sqrt{1-c^4x^4}}{\sqrt{-1+c^2x^2}}\right)}{2c^3\sqrt{c^2x^2}}$$

output
```
-1/2*b*x*arctan((-c^4*x^4+1)^(1/2)/(c^2*x^2-1)^(1/2))/c^3/(c^2*x^2)^(1/2)-
1/2*(a+b*arcsec(c*x))*(-c^4*x^4+1)^(1/2)/c^4+1/2*b*x*(-c^4*x^4+1)^(1/2)/c^
3/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)
```

3.172.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \frac{\frac{\left(a+bc\sqrt{1-\frac{1}{c^2x^2}}x-ac^2x^2\right)\sqrt{1-c^4x^4}}{-1+c^2x^2} - b\sqrt{1-c^4x^4}\sec^{-1}(cx) + b\arctan\left(\frac{c\sqrt{1-\frac{1}{c^2x^2}}x}{\sqrt{1-c^4x^4}}\right)}{2c^4}$$

input
```
Integrate[(x^3*(a + b*ArcSec[c*x]))/Sqrt[1 - c^4*x^4],x]
```

output
```
(((a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x - a*c^2*x^2)*Sqrt[1 - c^4*x^4])/(-1 + c
^2*x^2) - b*Sqrt[1 - c^4*x^4]*ArcSec[c*x] + b*ArcTan[(c*Sqrt[1 - 1/(c^2*x^
2)]*x)/Sqrt[1 - c^4*x^4]])/(2*c^4)
```

3.172.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5769, 27, 1896, 1388, 243, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(a+b\sec^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$$

$$\downarrow 5769$$

$$-\frac{b\int -\frac{\sqrt{1-c^4x^4}}{2c^4\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{c}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^4}$$

$$\downarrow 27$$

$$\frac{b\int \frac{\sqrt{1-c^4x^4}}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{2c^5}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^4}$$

$$\downarrow 1896$$

$$\frac{b\sqrt{1-c^2x^2}\int \frac{\sqrt{1-c^4x^4}}{x\sqrt{1-c^2x^2}}dx}{2c^5x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^4}$$

$$\downarrow 1388$$

$$\frac{b\sqrt{1-c^2x^2}\int \frac{\sqrt{c^2x^2+1}}{x}dx}{2c^5x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^4}$$

$$\downarrow 243$$

$$\frac{b\sqrt{1-c^2x^2}\int \frac{\sqrt{c^2x^2+1}}{x^2}dx^2}{4c^5x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^4}$$

$$\downarrow 60$$

$$\frac{b\sqrt{1-c^2x^2}\left(\int \frac{1}{x^2\sqrt{c^2x^2+1}}dx^2+2\sqrt{c^2x^2+1}\right)}{4c^5x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^4}$$

$$\downarrow 73$$

$$\frac{b\sqrt{1-c^2x^2}\left(\frac{2\int \frac{1}{\frac{x^4}{c^2}-\frac{1}{c^2}}d\sqrt{c^2x^2+1}}{c^2}+2\sqrt{c^2x^2+1}\right)}{4c^5x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^4}$$

$$\downarrow 221$$

$$\frac{b\sqrt{1-c^2x^2}\left(2\sqrt{c^2x^2+1}-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{4c^5x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{\sqrt{1-c^4x^4}\left(a+b\sec^{-1}(cx)\right)}{2c^4}$$

input
```
Int[(x^3*(a + b*ArcSec[c*x]))/Sqrt[1 - c^4*x^4],x]
```

output
```
-1/2*(Sqrt[1 - c^4*x^4]*(a + b*ArcSec[c*x]))/c^4 + (b*Sqrt[1 - c^2*x^2]*(2
*Sqrt[1 + c^2*x^2] - 2*ArcTanh[Sqrt[1 + c^2*x^2]]))/(4*c^5*Sqrt[1 - 1/(c^2
*x^2)]*x)
```

3.172.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 1896
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_)*((a_) + (c_.)*(x_)^(n2_.))^(
p_.), x_Symbol] :> Simp[(e^IntPart[q]*((d + e*x^mn)^FracPart[q]/(1 + d*(1/(
x^mn*e)))^FracPart[q]))/x^(mn*FracPart[q])   Int[x^(m + mn*q)*(1 + d*(1/(x^
mn*e)))^q*(a + c*x^n2)^p, x], x] /; FreeQ[{a, c, d, e, m, mn, p, q}, x] &&
EqQ[n2, -2*mn] &&  !IntegerQ[p] &&  !IntegerQ[q] && PosQ[n2]
```

rule 5769
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Simp[(a + b*ArcSec[c*x])   v, x] - Simp[b/c   Int[SimplifyIntegran
d[v/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]]
/; FreeQ[{a, b, c}, x]
```

3.172.4 Maple [F]

$$\int \frac{x^3(a+b\,\text{arcsec}\,(cx))}{\sqrt{-c^4x^4+1}}dx$$

input
```
int(x^3*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x)
```

output
```
int(x^3*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x)
```

3.172.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.99

$$\int \frac{x^3(a+b\,\text{sec}^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$$
$$= \frac{\sqrt{-c^4x^4+1}\sqrt{c^2x^2-1}b-(bc^2x^2-b)\arctan\left(\frac{\sqrt{-c^4x^4+1}}{\sqrt{c^2x^2-1}}\right)-\sqrt{-c^4x^4+1}(ac^2x^2+(bc^2x^2-b)\,\text{arcsec}\,(cx)-}{2\left(c^6x^2-c^4\right)}$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(-c^4*x^4 + 1)*sqrt(c^2*x^2 - 1)*b - (b*c^2*x^2 - b)*arctan(sqrt(
-c^4*x^4 + 1)/sqrt(c^2*x^2 - 1)) - sqrt(-c^4*x^4 + 1)*(a*c^2*x^2 + (b*c^2*
x^2 - b)*arcsec(c*x) - a))/(c^6*x^2 - c^4)
```

3.172.6 Sympy [F]

$$\int \frac{x^3(a+b\,\text{sec}^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \int \frac{x^3(a+b\,\text{asec}\,(cx))}{\sqrt{-(cx-1)(cx+1)(c^2x^2+1)}}\,dx$$

input
```
integrate(x**3*(a+b*asec(c*x))/(-c**4*x**4+1)**(1/2),x)
```

output
```
Integral(x**3*(a + b*asec(c*x))/sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))
, x)
```

3.172.7 Maxima [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \int \frac{(b\,\text{arcsec}\,(cx)+a)x^3}{\sqrt{-c^4x^4+1}}\,dx$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(2*c^4*integrate((2*sqrt(c*x + 1)*c^2*x^3*log(c) + ((2*c^2*log(c) + c
^2)*x^3 + x)*e^(3/2*log(c*x + 1) + log(c*x - 1)) + 2*(c^2*x^3*e^(3/2*log(c
*x + 1) + log(c*x - 1)) + sqrt(c*x + 1)*c^2*x^3)*log(x))/(sqrt(c^2*x^2 + 1
)*(c^2*e^(2*log(c*x + 1) + log(c*x - 1) + 1/2*log(-c*x + 1)) + c^2*e^(log(
c*x + 1) + 1/2*log(-c*x + 1)))), x) + sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt
(-c*x + 1)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)))*b/c^4 - 1/2*sqrt(-c^4*x^4
+ 1)*a/c^4
```

3.172.8 Giac [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \int \frac{(b\,\text{arcsec}\,(cx)+a)x^3}{\sqrt{-c^4x^4+1}}\,dx$$

input
```
integrate(x^3*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)*x^3/sqrt(-c^4*x^4 + 1), x)
```

3.172.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \int \frac{x^3\left(a+b\,\text{acos}\left(\frac{1}{cx}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

input
```
int((x^3*(a + b*acos(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
```

output
```
int((x^3*(a + b*acos(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

3.173 $\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$

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3.173.2 Mathematica [N/A] 1209
3.173.3 Rubi [N/A] 1210
3.173.4 Maple [N/A] (verified) 1210
3.173.5 Fricas [N/A] 1211
3.173.6 Sympy [N/A] 1211
3.173.7 Maxima [N/A] 1211
3.173.8 Giac [N/A] 1212
3.173.9 Mupad [N/A] 1212

3.173.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \text{Int}\left(\frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}},x\right)$$

output
```
Unintegrable((a+b*arcsec(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

3.173.2 Mathematica [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x*Sqrt[1 - c^4*x^4]),x]
```

output
```
Integrate[(a + b*ArcSec[c*x])/(x*Sqrt[1 - c^4*x^4]), x]
```

3.173.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \sec^{-1}(cx)}{x\sqrt{1 - c^4x^4}}\, dx$$
$$\downarrow 5771$$
$$\int \frac{a + b \sec^{-1}(cx)}{x\sqrt{1 - c^4x^4}} dx$$

input
```
Int[(a + b*ArcSec[c*x])/(x*Sqrt[1 - c^4*x^4]),x]
```

output
```
$Aborted
```

3.173.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.173.4 Maple [N/A] (verified)

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{a + b \operatorname{arcsec}(cx)}{x\sqrt{-c^4x^4 + 1}} dx$$

input
```
int((a+b*arcsec(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

output
```
int((a+b*arcsec(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

3.173.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \int \frac{b\,\text{arcsec}\,(cx)+a}{\sqrt{-c^4x^4+1}x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^4*x^4 + 1)*(b*arcsec(c*x) + a)/(c^4*x^5 - x), x)
```

3.173.6 Sympy [N/A]

Not integrable

Time = 11.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \int \frac{a+b\,\text{asec}\,(cx)}{x\sqrt{-(cx-1)(cx+1)(c^2x^2+1)}}\,dx$$

input
```
integrate((a+b*asec(c*x))/x/(-c**4*x**4+1)**(1/2),x)
```

output
```
Integral((a + b*asec(c*x))/(x*sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))),
 x)
```

3.173.7 Maxima [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.35

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \int \frac{b\,\text{arcsec}\,(cx)+a}{\sqrt{-c^4x^4+1}x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*a*(log(sqrt(-c^4*x^4 + 1) + 1) - log(sqrt(-c^4*x^4 + 1) - 1)) + b*int
egrate(arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1
)*sqrt(-c*x + 1)*x), x)
```

3.173.8 Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \int \frac{b\,\text{arcsec}\,(cx)+a}{\sqrt{-c^4x^4+1}x}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/(sqrt(-c^4*x^4 + 1)*x), x)
```

3.173.9 Mupad [N/A]

Not integrable

Time = 1.81 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \int \frac{a+b\,\text{acos}\left(\frac{1}{cx}\right)}{x\sqrt{1-c^4\,x^4}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(x*(1 - c^4*x^4)^(1/2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x*(1 - c^4*x^4)^(1/2)), x)
```

3.174 $\int \frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$

3.174.1 Optimal result 1213
3.174.2 Mathematica [N/A] 1213
3.174.3 Rubi [N/A] 1214
3.174.4 Maple [N/A] (verified) 1214
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3.174.8 Giac [N/A] 1216
3.174.9 Mupad [N/A] 1216

3.174.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx = \text{Int}\left(\frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}},x\right)$$

output
```
Unintegrable((a+b*arcsec(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

3.174.2 Mathematica [N/A]

Not integrable

Time = 8.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx = \int \frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$$

input
```
Integrate[(a + b*ArcSec[c*x])/(x^5*Sqrt[1 - c^4*x^4]),x]
```

output
```
Integrate[(a + b*ArcSec[c*x])/(x^5*Sqrt[1 - c^4*x^4]), x]
```

3.174.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \sec^{-1}(cx)}{x^5 \sqrt{1 - c^4 x^4}} \, dx$$
$$\downarrow 5771$$
$$\int \frac{a + b \sec^{-1}(cx)}{x^5 \sqrt{1 - c^4 x^4}} dx$$

input
```
Int[(a + b*ArcSec[c*x])/(x^5*Sqrt[1 - c^4*x^4]),x]
```

output
```
$Aborted
```

3.174.3.1 Defintions of rubi rules used

rule 5771
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcSec[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.174.4 Maple [N/A] (verified)

Not integrable

Time = 3.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{a + b \operatorname{arcsec}(cx)}{x^5 \sqrt{-c^4 x^4 + 1}} dx$$

input
```
int((a+b*arcsec(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

output
```
int((a+b*arcsec(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

3.174.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int \frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx = \int \frac{b\,\text{arcsec}\,(cx)+a}{\sqrt{-c^4x^4+1}x^5}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^4*x^4 + 1)*(b*arcsec(c*x) + a)/(c^4*x^9 - x^5), x)
```

3.174.6 Sympy [N/A]

Not integrable

Time = 82.45 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int \frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx = \int \frac{a+b\,\text{asec}\,(cx)}{x^5\sqrt{-(cx-1)(cx+1)(c^2x^2+1)}}\,dx$$

input
```
integrate((a+b*asec(c*x))/x**5/(-c**4*x**4+1)**(1/2),x)
```

output
```
Integral((a + b*asec(c*x))/(x**5*sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1)
)), x)
```

3.174.7 Maxima [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 111, normalized size of antiderivative = 4.27

$$\int \frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx = \int \frac{b\,\text{arcsec}\,(cx)+a}{\sqrt{-c^4x^4+1}x^5}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/8*(c^4*log(sqrt(-c^4*x^4 + 1) + 1) - c^4*log(sqrt(-c^4*x^4 + 1) - 1) +
2*sqrt(-c^4*x^4 + 1)/x^4)*a + b*integrate(arctan(sqrt(c*x + 1)*sqrt(c*x -
1))/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^5), x)
```

3.174.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx = \int \frac{b\,\text{arcsec}\,(cx)+a}{\sqrt{-c^4x^4+1}x^5}\,dx$$

input
```
integrate((a+b*arcsec(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arcsec(c*x) + a)/(sqrt(-c^4*x^4 + 1)*x^5), x)
```

3.174.9 Mupad [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx = \int \frac{a+b\,\text{acos}\left(\frac{1}{cx}\right)}{x^5\,\sqrt{1-c^4\,x^4}}\,dx$$

input
```
int((a + b*acos(1/(c*x)))/(x^5*(1 - c^4*x^4)^(1/2)),x)
```

output
```
int((a + b*acos(1/(c*x)))/(x^5*(1 - c^4*x^4)^(1/2)), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions 1217

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                           convert(leaf_count_result,string),"$ vs. $2(",
                           convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,`^`) then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,`+`) or type(expn,`*`) then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
    return 1
  elif isinstance(expn,list):
    return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
  elif  is_sqrt(expn):
    if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
      return 1
    else:
      return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
  elif isinstance(expn,Pow):   #type(expn,'`^`')
    if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
      return expnType(expn.args[0])   #ExpnType(op(1,expn))
    elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
        return 1
      else:
        return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    else:
      return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
  elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
    m1 = expnType(expn.args[0])
    m2 = expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
    return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
  elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
  elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
    return max(7,m1)
  elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:    #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if  not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```